# FULL ELECTROMAGNETIC SIMULATION OF COHERENT SYNCHROTRON RADIATION VIA THE LORENTZ-BOOSTED FRAME APPROACH *

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*Abstract*

Numerical simulation of some systems containing charged particles with highly relativistic directed motion can by speeded up by orders of magnitude by choice of the proper Lorentz-boosted frame[1]. Orders of magnitude speedup has been demonstrated for simulations from first principles of laser-plasma accelerator, free electron laser, and particle beams interacting with electron clouds. Here we address the application of the Lorentz-boosted frame approach to coherent synchrotron radiation (CSR), which can be strongly present in bunch compressor chicanes. CSR is particularly relevant to the next generation of x-ray light sources and is simultaneously difficult to simulate in the lab frame because of the large ratio of scale lengths. It can increase both the incoherent and coherent longitudinal energy spread, effects that often lead to an increase in transverse emittance. We have adapted the WARP code [2] to to simulate CSR emission along a simple dipole bend. We present some scaling arguments for the possible computational speed up factor in the boosted frame and initial 3D simulation results.

## INTRODUCTION

Many problems of interest involving radiation production by highly relativistic charged particle beams can extremely difficult to model via full electromagnetic simulation due to the large ratio of length scales $L_{sim}/c\Delta t_C$ where $\Delta t_C$ is the Courant-condition limited maximum time step correpsonding to the wanted temporal and spatial resolution. Recently, Vay [1] pointed out that for some of these problems performing the simulation in a Lorentz-boosted frame offers potentially orders of magnitude reduction in computation time. Applications to date of boosted-frame (BF) modeling include laser-plasma wakefield accelerators, electron cloud dynamics, and free-electron lasers [3].

Coherent synchrotron emission (CSR) is another problem for which BF methods offer great promise given that the longitudinal interaction lengths in the laboratory are of order meters and the radiation emission wavelengths can be microns or less. Previously, the Trafic-4 simulation code [4] applied Leonard-Wiechart retarded potential methods for CSR modeling with a great complexity related to orbit history memory management issues.

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In this paper, we describe our modification and use of the WARP simulation code [2] with its standard full EM solver operating in full 3D geometry to study via direct calculation CSR emission by a short relativistic electron beam pulse traversing a simple, static magnetic dipole bend. The combination of Lorentz-boosted frame transformation and the use of a moving computation window in space helps strongly reduce both the number of needed time steps and the size of the spatial simulation grid in order to make full 3D simulation possible, including good transverse resolution of the electron beam. In the next sections we describe our approach to CSR simulation with BF methods and present some initial test case results.

## GEOMETRY AND APPLICATION OF BF METHODS TO CSR SIMULATION

Ignoring energy loss or gain, a magnetic dipole will bend a particle by an angle $\theta_B = L_M/R_C$ where $L_M$ and $R_c$ are the effective length and radius of curvature of the magnet. For $\theta_B \ll 1$, let us adopt a geometry system where the electron beam centroid lies in the $\hat{x} - \hat{z}$ plane and has its normalized $x$-momentum bent by the dipole from $-p_\perp^0$ to $+p_\perp^0$ where $p_\perp^0 \equiv \gamma_0\beta_0 \sin\theta_B/2$ . A natural choice for the boosted frame is one that moves with velocity $\beta_0\, c\cos\theta_B/2$ in the positive $z$ direction. In the boosted frame, time dilates, the dipole magnet length shrinks, and the beam length expands by an identical factor $\gamma_T \equiv \gamma_0/(1 + [p_\perp^0]^2)^{1/2}$. Within the dipole, the resultant trajectory in the boosted frame is similar to the Roman letter "U" with the width $z_M^{BF}$ being slightly larger than the depth$X_M^{BF} \approx R_c\theta_B^2/8$. In the limit $p_\perp^0 \gg 1$, $\gamma_T \approx \csc\theta_B/2 = 2R_C/L_M$ and numerically is $\sim 5 - 500$ for many bunch compressor systems relevant to short wavelength free-electron lasers.

If the necessary grid resolution for EM simulation in the boosted frame is set by the transverse beam size $\sigma_x$, the computational speedup relative to the laboratory frame scales as $\gamma_T^2$, with one factor from time dilation and another from the shrinkage of the dipole length in $z$. Note for a given bend angle, $\gamma_T$ is independent of $\gamma_0$ and in general we will have $\gamma_T\sigma_z > \sigma_x$. If on the other hand, the necessary spatial resolution in the laboratory frame is set by the smallest longitudinal CSR scale length of physical interest $\lambda_{min} \approx \sigma_z/10$ is smaller than $\sigma_x/\gamma_T$, one can achieve additional speedup factors in boosted frame calculation due to a larger permitted transverse grid spacing too.

Various Python scripts were written to implement directly into WARP the static (in the lab frame) magnetic

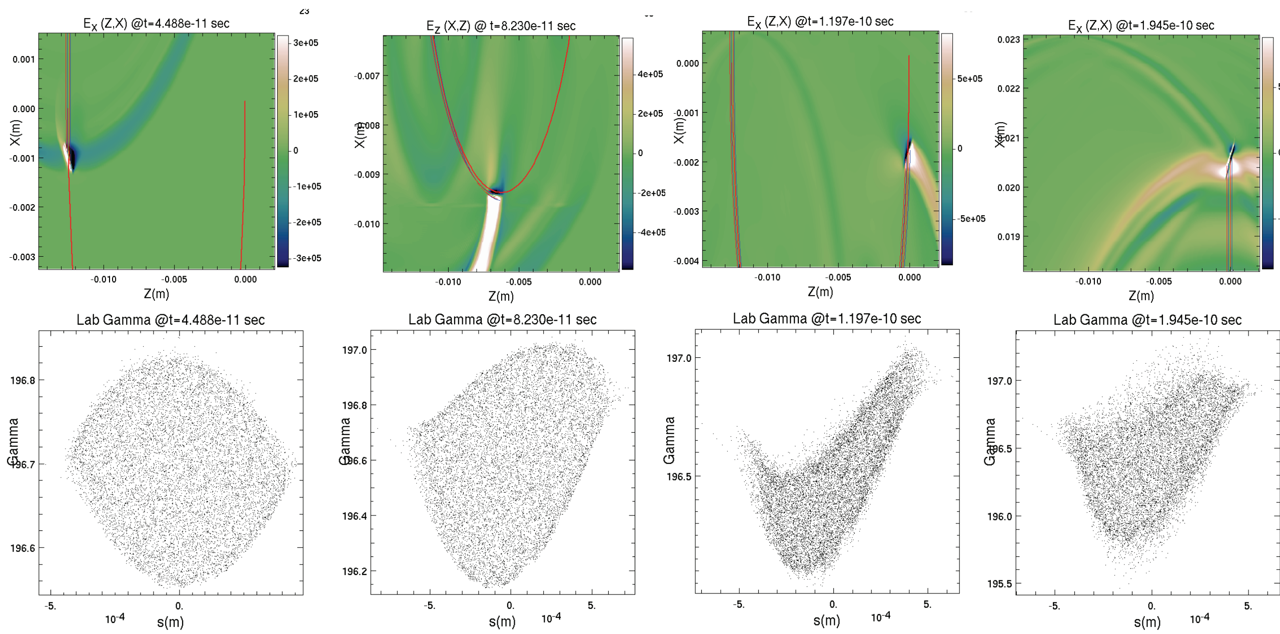

Figure 1: Top row: longitudinal electric field plots in the $x-z$ boosted frame plane with $y=0$ for a 100 MeV, 500A, 150 fs pulse transversing a 0.5-m long dipole with a 3.33 radius of curvature. The four positions correspond approximately to dipole entrance, mid-dipole, dipole exit, and 20 mm beyond dipole exit. The solid red line corresponds to the nominal trajectory of a constant (lab-)energy test particle in the magnetic dipole. Bottom row: macroparticle scatterplots for $\gamma_{LAB}(s)$ where $s$ is the local longitudinal position in the boosted frame measured relative to the beam centroid.

dipole field and to perform the various transformations of particle positions and momenta to the boosted frame. In order to allow the electromagnetic fields to relax (approximately) to the asymptotic values corresponding to those surrounding a relativistic beam of constant velocity, we started the beam from rest a distance in $x$ approximately twice or more $X_M^{BF}$. Over a startup ramp distance $\approx X_M^{BF}$, we artificially increased $p_x$ to $-p_\perp^0$, simultaneously not allowing the generated EM fields to act back upon the beam. By adopting this approach, we tried to limit any unphysical modulation of particle momenta or positions before the beam entered the bend to a level a few per cent or less of the effects caused by physically real CSR fields associated with the bend. For most simulations, we also employed a moving window in the $\hat{x}$ direction. Initially, this window moves at $-c$ until the beam reaches approximately halfway down the bend "U". At that point, we freeze the window while the beam goes through the bottom of the "U" and until it reaches the halfway point on its upward trajectory in the "U", after which point we restart the window motion but now with $\vec{v}=+c\,\hat{x}$. At present we have adopted open (radiating) boundary conditions in all directions. We note that for laboratory situations where conducting surfaces are used to suppress CSR, further algorithmic development are needed to model such surfaces which will be moving with respect to the calculation frame, which is not the case with standard algorithms for treating internal conductors.

## COMPUTATIONAL EXAMPLES

In our initial exploration of CSR simulation done in a boosted frame, we have concentrated on situations typical of bunch compressor magnets used for short wavelength FEL sources. As a first example, we chose an electron beam with 100-MeV energy, 500-A current, 0.5-mm-mrad normalized transverse emittance, and a full 3D Gaussian profile with $\sigma_{x,y}=50\,\mu$m and $\sigma_z=45\,\mu$m. The magnetic dipole had $R_c=3.33$ m and a length $L_M=0.5$ m corresponding to a full bend angle of 150 mrad and an "overtaking" (or slippage) length of $L_M^3/(24R_c^2)=470\,\mu$m. With $\gamma_T=13.3$, the initial beam energy in the boosted frame is $\approx 7$ MeV and dips down to $\approx 3.3$ MeV at the middle of the bend (where $p_x=0$). We adopted a 20$\mu$m grid resolution for all 3 dimensions and a moving window that spanned 16 mm in $z$ (the magnet length in the boosted frame is 12 mm), 4.2 mm in $x$, and 1 mm in $y$, for a total of $\sim$ 3M grid cells. Typically we ran this type of problem in parallel on 64 processors with domain decomposition being done in the $x-z$ plane over a maximum time of $\sim$ 200 ps (3000 time steps), equivalent to a beam propagation distance of 60 mm in the boosted frame.

Figure 1 displays longitudinal field plots and macroparticle energy scatterplots at various locations along the trajectory through the magnetic dipole. By the mid-point in the dipole, there is an obvious lagging behind of the field, with stronger fields lying outward from the arc. By the time the particle is 20 mm beyond the dipole (farthest right plot), the field has become far more symmetrical and has dropped by more than an order of magnitude from the peak values

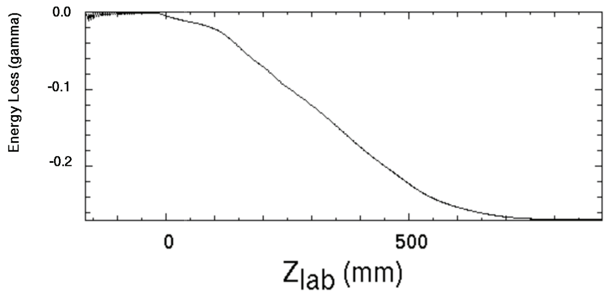


Figure 2: Mean energy loss in units of $mc^2$ versus lab position in $z$ for the 500 A, 150-fs duration pulse of Fig. 1. The 0.5-m long dipole magnet begins at $z = 0\,\mathrm{m}$.

generated when the beam was in the middle region of the dipole. The scatterplots show the development of a fishhook shape of $\gamma(s)$ with the head region electrons showing net acceleration and tail electrons net deceleration as expected from CSR theory (see, *e.g.* [5]). However, while 1D CSR treatments can predict the overall energy loss and also the functional dependence of the longitudinal wake with $s$, a full 3D EM simulation can resolve transversely the energy loss and increase of energy spread. In Fig. 2 we plot the average particle energy loss as a function of beam centroid position in $z$ in the lab frame. The total loss evaluated $\approx$ 0.3 m beyond the magnet is about 50% greater than would be predicted in the asymptotic rigid beam, 1D limit of $E_{loss} \approx 0.35\,N^2 e^2 L_M/(R_c \sigma_z^2)^{2/3}$ [5] . A number of causes most likely underly this discrepancy including unphysical numerical macroparticle noise giving some enhancement of CSR emission at very short wavelengths, the fact that this beam is not in the ultrarelativistic, completely rigid regime, and the physically real effects of a non-zero transverse electron beam size (as indicated by the large spread in $\gamma(s)$ in Fig. 1) which in the 1D limit would vanish.

Currently, we suspect that the transverse size of the electron beam does play a significant role in affecting CSR energy loss. The left plot of Fig. 3 displays contours of local mean energy as a function of $(s, r_\perp)$ for the run corresponding to Figs. 1&2. One sees that the regions of maximum energy loss and gain are diagonally across from each other indicating that inner and outer transverse portions (relative to the arc motion) at a given $s$ had different energy loss/gain histories. Runs at higher beam energies (250 MeV) and larger radii of curvatures (26.7m) show less of such an effect. The right plot of Fig. 3 corresponds to a beam with $\sigma_z = 2.5\,\mu$m; although much shorter in the lab frame than the left plot's 100-MeV case, the lengths are essentially equal in the boosted frame. The contours of constant $\gamma$ are nearly vertical, indicating there is much less variation with transverse position. Furthermore, the total energy loss is within a few per cent of the asymptotic formula. For larger $\sigma_z$, $E_{loss}$ starts to diverge from the asymptotic limit and moreover shows evidence of a microbunching instability. The latter could well be an unphysical numerical artifact but the former may be due some

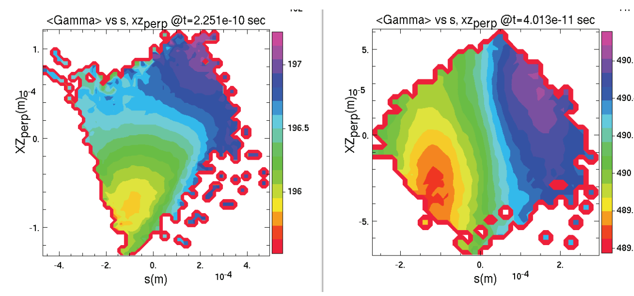


Figure 3: False color image showing $\bar{\gamma}(s, r_\perp)$ at the simulation end for both the 100-MeV case of Figs. 1&2 (left plot) and a much shorter pulse ($\sigma_z = 4.5\,\mu$m at 250-MeV traversing a 0.5-m long dipole with 26.7 m radius of curvature. Negative (positive) values of $r_\perp$ correspond to particles whose transverse position (presuming laminar flow) was on the outer (inner) portion of the arc.

higher order effect related to the ratio of $(\sigma_z \csc\theta_B/L_M)$, in addition to approaching the "overtaking" limit mentioned above.

## DISCUSSION

The calculation of CSR emission and energy loss is a straight forward application of the Lorentz-boosted-frame simulation approach, provided that internal conductors are ignored. Our initial work has confirmed that there can be a very large reduction in the required number of time steps for full EM simulation due to the relativistic shrinking of the magnet length and concurrent increase in the longitudinal scale length corresponding to the electron beam and radiation wavelengths of interest. For reasonable modern computational hardware, full 3D, EM CSR simulation with good transverse resolution is quite possible. Our results to date suggest that fully electromagnetic and accurate simulation of beam compressors is possible in regimes of great interest for design of linac-based short wavelength free-electron lasers.

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