Suppress the CSR-induced emittance growth in achromats using 2D point-kick analysis

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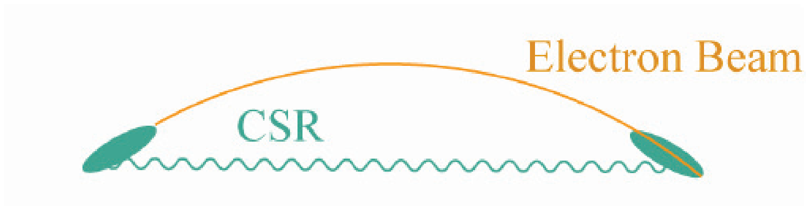
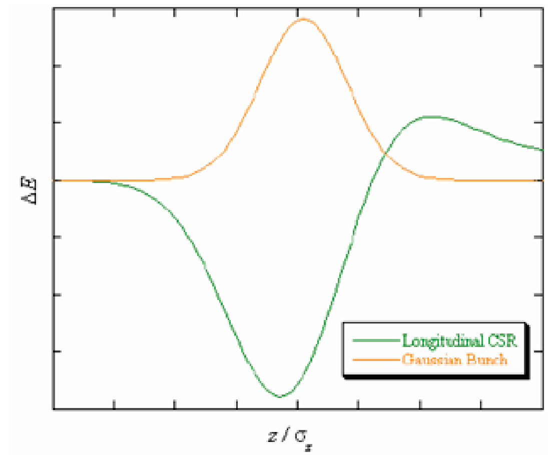
Transverse emittance dilution due to CSR

Courtesy of R. Hajima

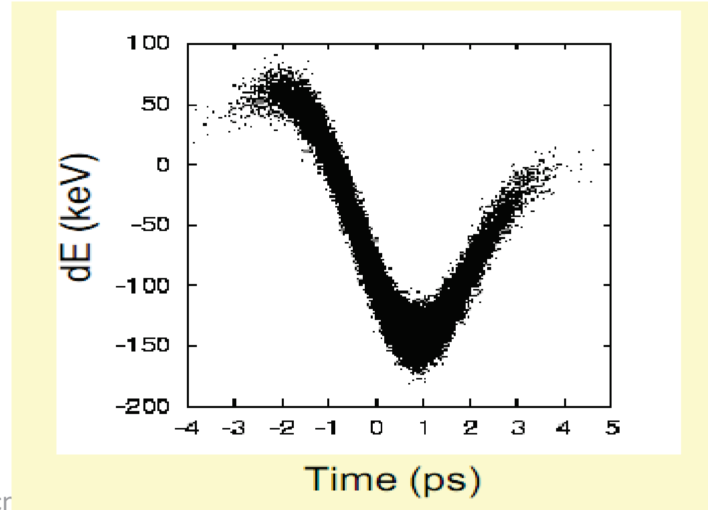


(1) CSR emission from the bunch tail catches up with the bunch head

(2) Energy change depending longitudinal position



(3) Displacement of bunch slices



Linearization of the CSR effect

$$\Delta E_{rms} \cong 0.22 \frac{eQL_b}{4\pi\varepsilon_0 \rho^{2/3} \sigma_z^{4/3}}$$

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- Bunch charge: Q
- Bending path: L_b
- Bending radius: ρ
- Bunch length: σ_z

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Bunch charge — Bending path — Bunch length — Bending radius

If we assume that:

1. **The bunch length σ_z does not change a lot along the transport line**
2. **The transient CSR effect is not large**
3. **Bending angles of the dipoles are not very large, $< 10^\circ$**

The CSR induced energy spread can be linearized

$$\Delta E(csr) / E_0 \cong k\rho^{1/3}\theta$$

$k = f(Q, \sigma_z)$, unit: $m^{-1/3}$

CSR-induced orbit deviation in a bending magnet

Betatron transfer matrix of a dipole:

$$M_d = \begin{pmatrix} \cos\theta & \rho\sin\theta \\ -\sin\theta/\rho & \cos\theta \end{pmatrix}$$

After passage through a sector bending magnet, expect the betatron transportation and momentum dispersive effect, a particle experiences "CSR dispersive" effect.

$$X_f = \underbrace{M_d X_i}_{\text{Betatron motion}} + \underbrace{\begin{pmatrix} D \\ D' \end{pmatrix}\delta_i + \begin{pmatrix} \zeta \\ \zeta' \end{pmatrix} k}_{\text{Orbit deviation}},$$

R. Hajima, R-matrix analysis, NIMA , 528, 2004.

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momentum dispersion (x-δ correlation terms), $D = \rho(1-\cos\theta)$, $D' = \sin\theta$.

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Betatron motion ($M_d X_i$)

Orbit deviation

"CSR dispersion" (x-k correlation terms), $\zeta = \rho^{4/3}(\theta - \sin\theta)$, $\zeta' = \rho^{1/3}(1-\cos\theta)$.

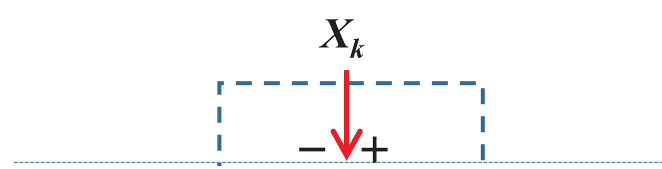
R. Hajima, R-matrix analysis, NIMA , 528, 2004.

CSR effect in dipole described with a 2D point kick

- The kick occurs at the dipole center;
- The kick contains two elements, related to δ and k, respectively;

$$X_k = \begin{pmatrix} x_k \\ x'_k \end{pmatrix} = \begin{pmatrix} 0 \\ 2\sin(\theta/2) \end{pmatrix} \delta + \begin{pmatrix} \rho^{4/3}[\theta\cos(\theta/2) - 2\sin(\theta/2)] \\ \sin(\theta/2)\rho^{1/3}\theta \end{pmatrix} k.$$

- After each kick, the particle coordinates increase by X_k and energy deviation increases by $k\rho^{1/3}\theta$.

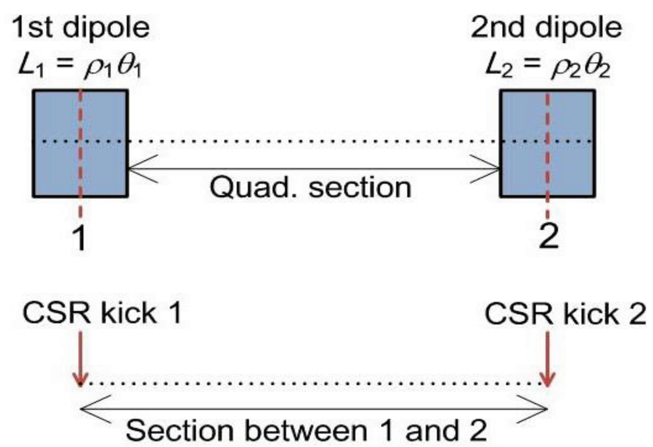


$$X_+ = X_- + X_k$$
$$\delta_+ = \delta_- + k\rho^{1/3}\theta$$

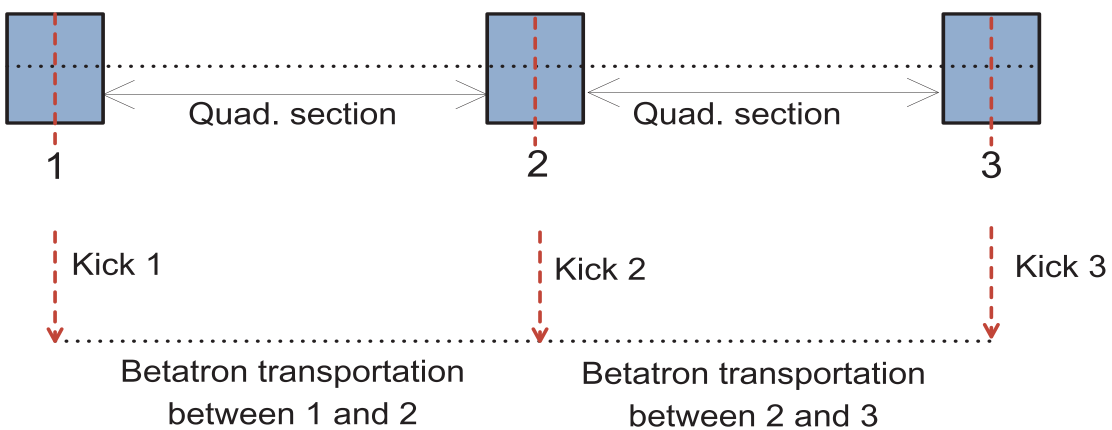
See Y. Jiao, et al., PRST-AB, 060701, 2014 for detail.

2D point-kick analysis for achromats

- For an n-dipole achromat, it needs only to analyze the horizontal betatron motion with n-point kicks, explicit formulation;
- The beam line between adjacent dipole centers is treated as a whole, so the obtained "zero CSR-kick" solutions predicts general requirements on optics design, generic CSR-kick cancellation conditions.



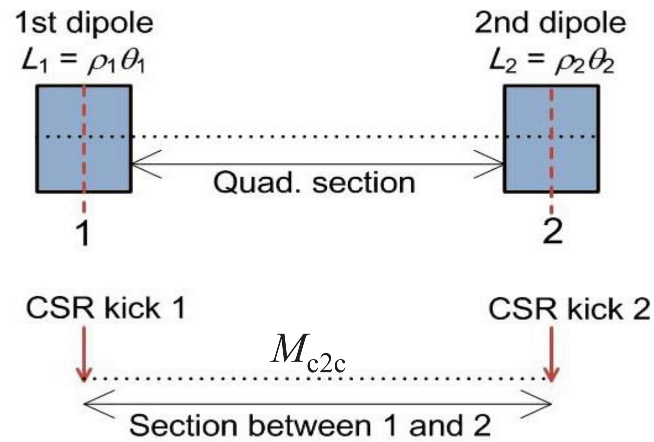
The bending radii and angles can be different.



CSR-cancellation conditions for a two-dipole achromat

- The transfer matrix of the quad. section between dipole centers is described in a general form:

$$M_{c2c} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}$$



- The net CSR kick:

$$\Delta X = X_{k2} + M_{c2c} X_{k1}$$

- The achromatic condition [$\Delta X(\delta) = 0$]:

$$m_{11} = -S_1 / S_2$$
$$m_{12} = 0$$
$$m_{22} = -S_2 / S_1$$

Phase advance between dipole centers: $n\pi$

$$S_1 = \sin(\theta_1 / 2)$$
$$S_2 = \sin(\theta_2 / 2)$$

- CSR-kick cancellation in linear regime [$\Delta X(k) = 0$]:

$$L_1\theta_1^2 \cong L_2\theta_2^2$$

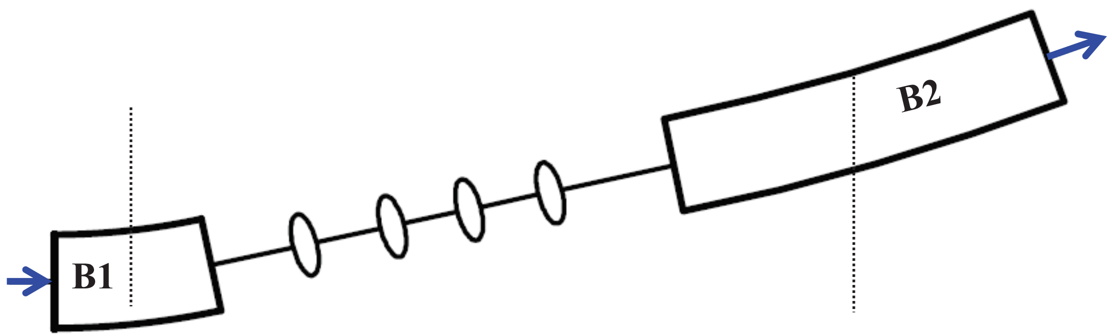
Automatically satisfied in DBA or dogleg with $L_1 = L_2$ and $\theta_1 = \theta_2$

$$m_{21} \cong \frac{12}{L_1}\frac{S_2}{S_1}$$

Design a “CSR-cancellation” two-dipole achromat

- Consider a two-dipole achromat, with θ_1 = 6 deg., θ_2 = 4 deg., ρ_1 = 8 m.

➡ $\rho_2^* \cong 27$ m, and $M^*_{c2c}(2, 1) = 30/\pi$



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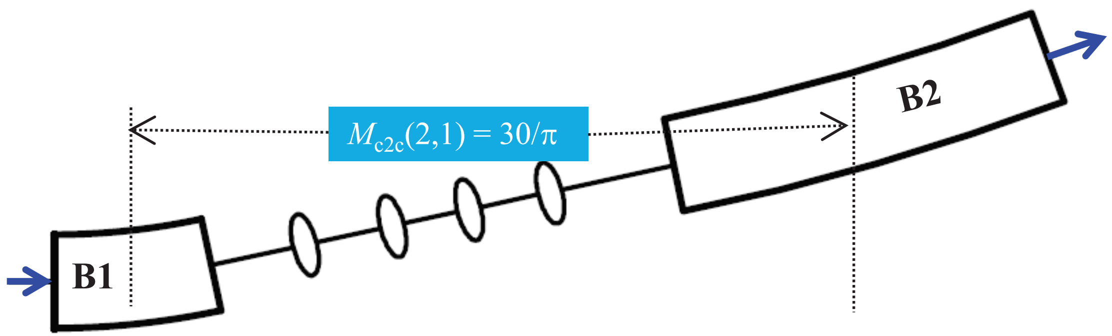

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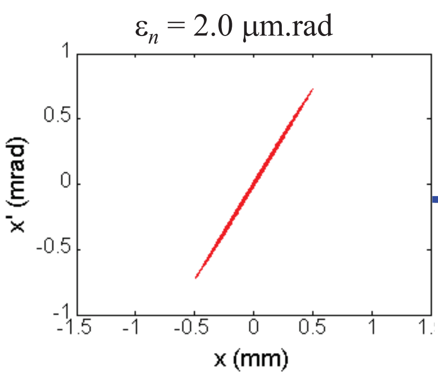

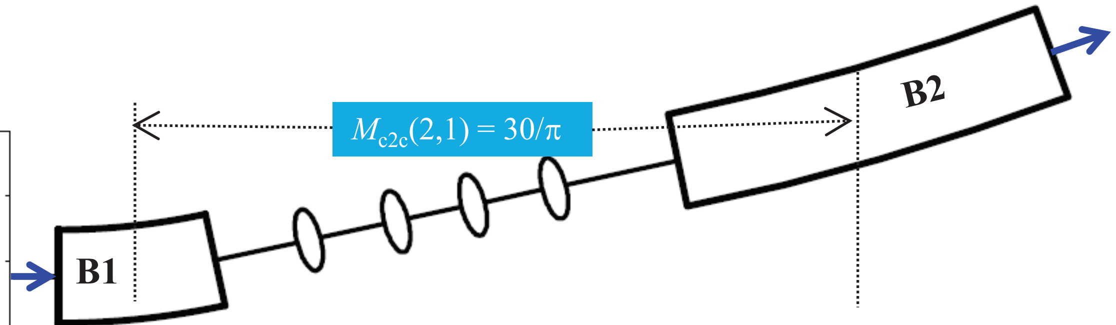

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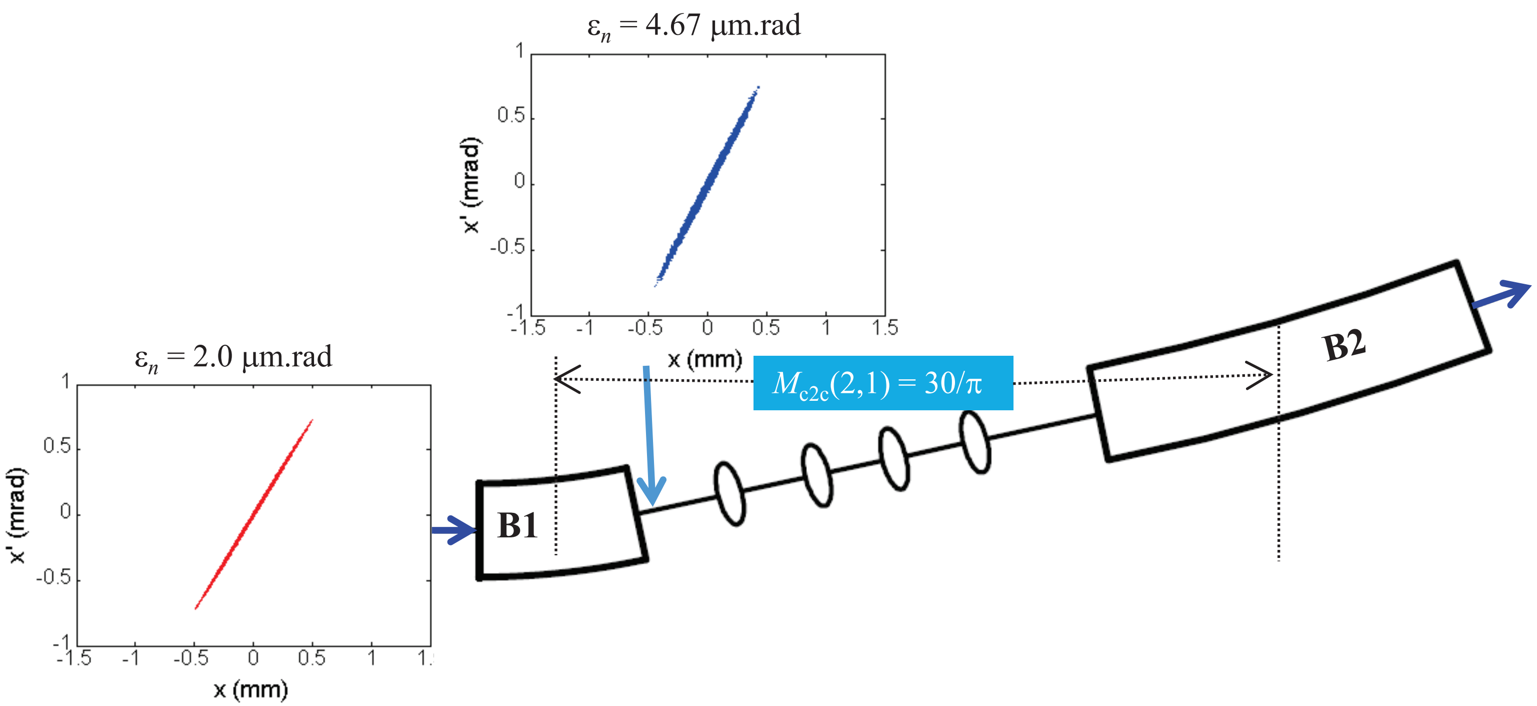

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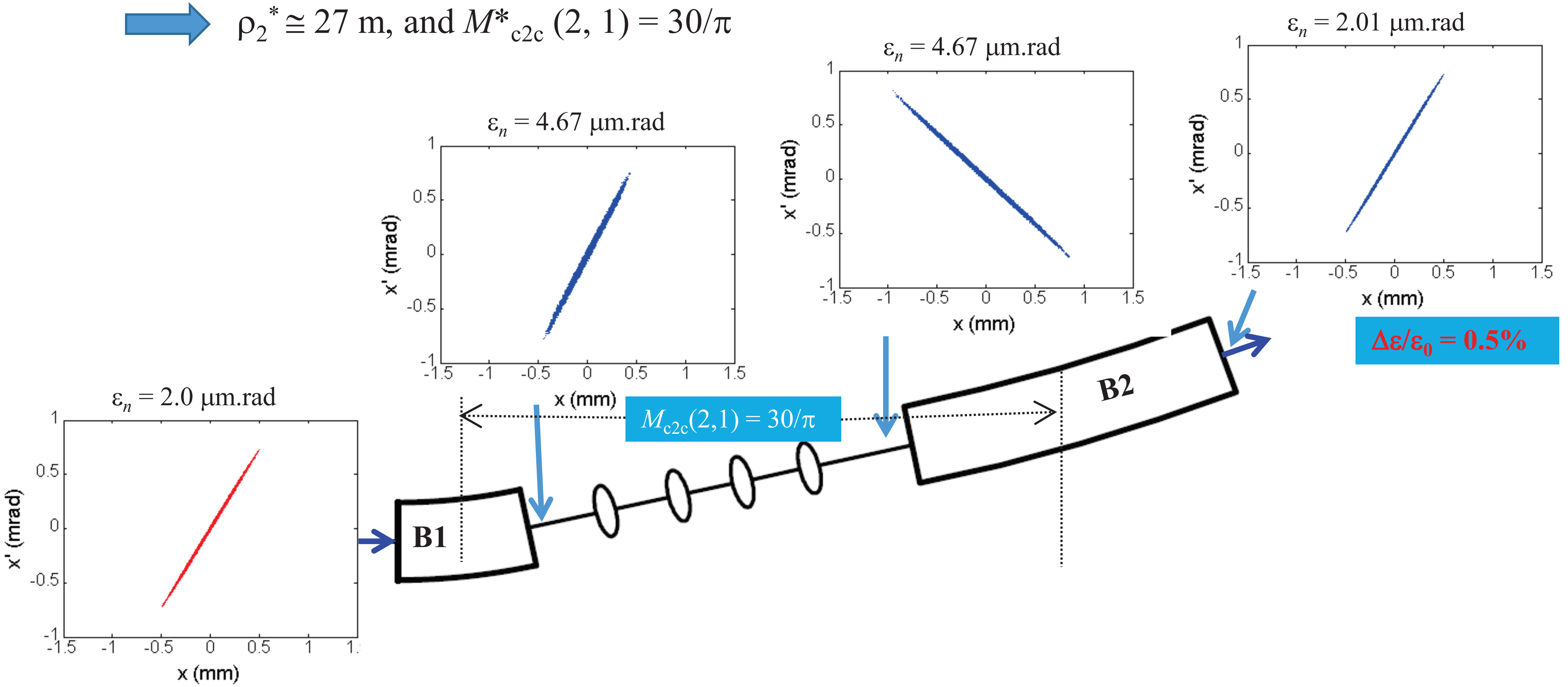

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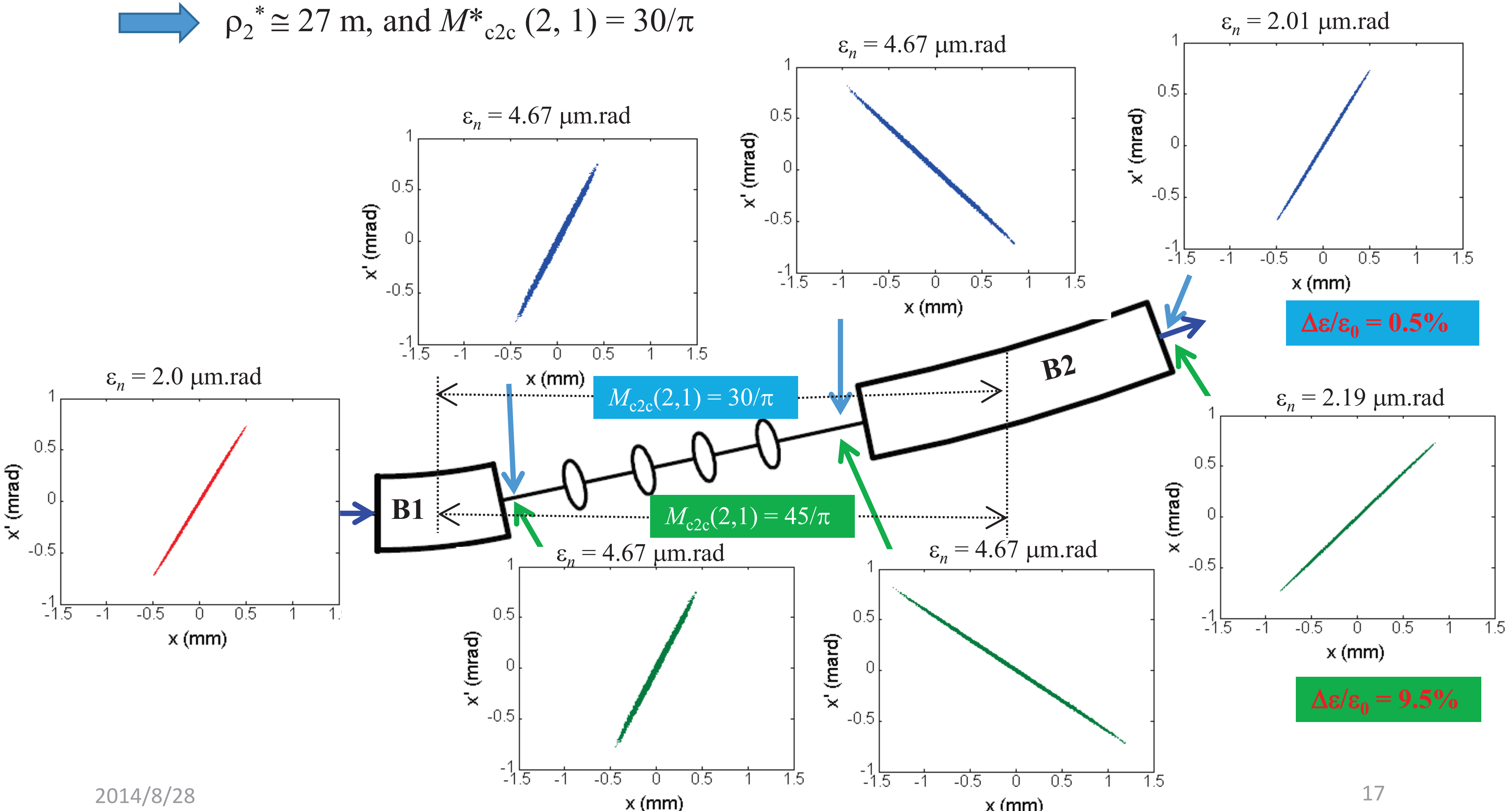
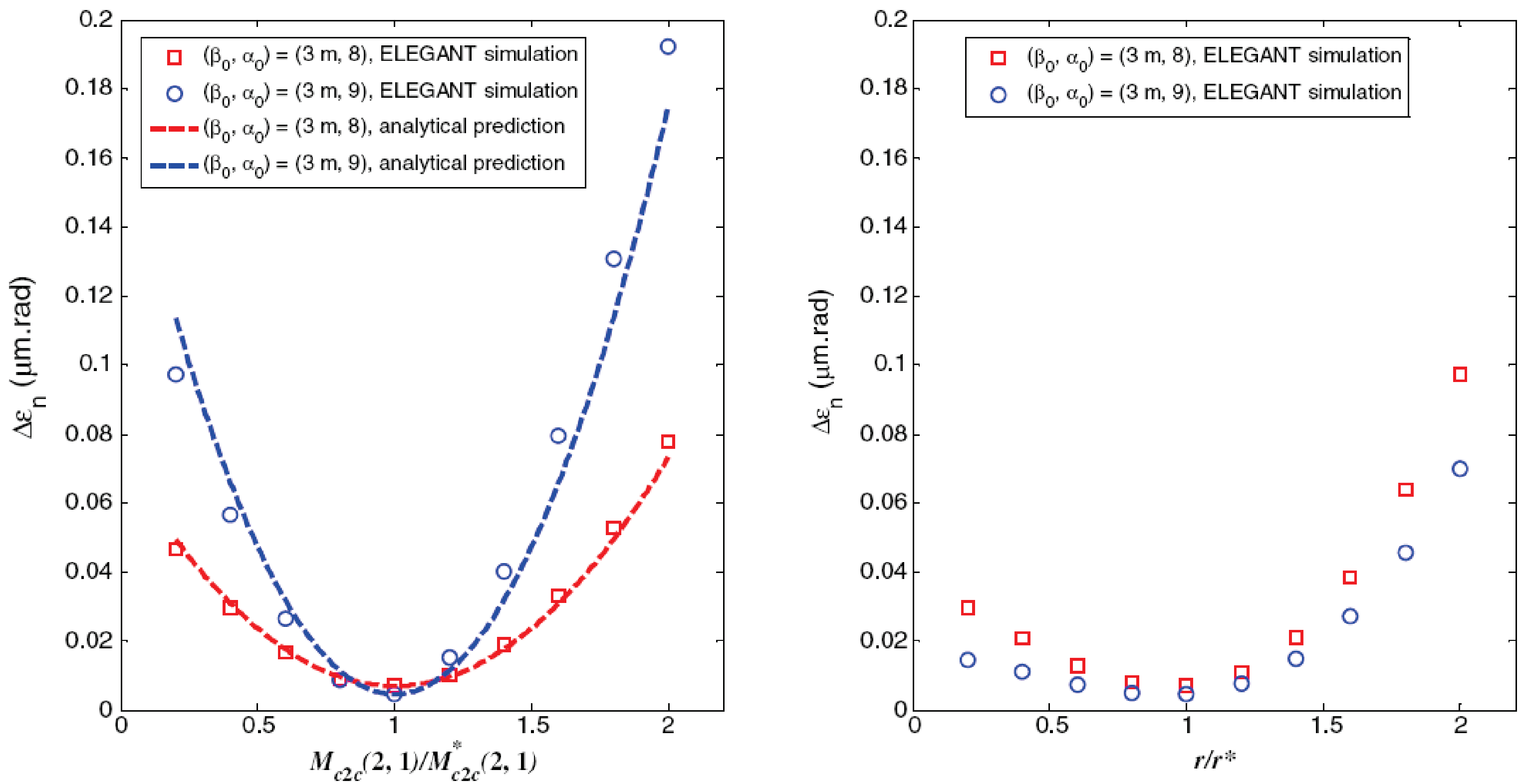


Scaling of the emittance growth due to CSR

—ELEGANT simulation

- The CSR wake in dipoles included in the tracking
  - The found conditions predicts minimum emittance growth,
  - The found conditions are robust against variation of the initial beam distribution,
  - Quadratic increase of Δε as $M_{c2c}(2,1)$ moves away from the optimal value.



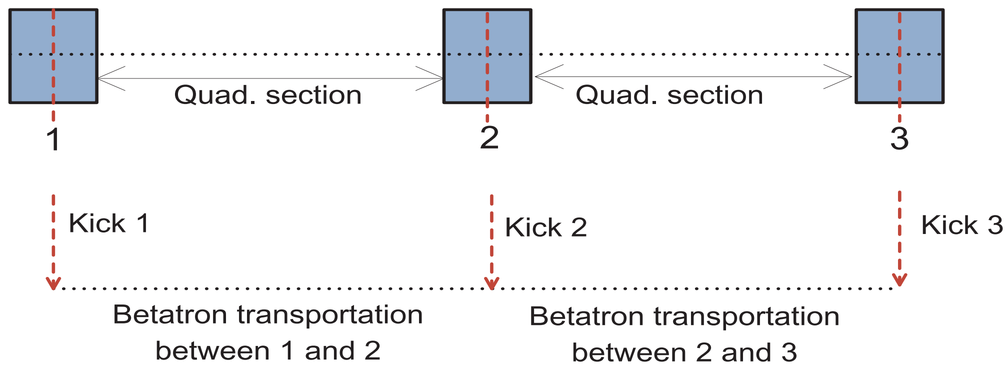
$$\Delta\varepsilon_n |_{r=r^*} \approx \frac{1}{2}\gamma\beta k_{rms}^2 S_1^2 (\theta_1 + r^{*1/3}\theta_2)^2 \rho_1^{2/3}\beta_1[1 - M_{c2c}(2,1)/M^*_{c2c}(2,1)]^2.$$

$r \equiv \rho_2/\rho_1$, and $r^* = 27/8$ in this case

CSR-cancellation conditions for a symmetric TBA

- To control the No. of variables, a TBA with symmetric layout is considered.
  - The 1st and 3rd dipole: θ_1, ρ
  - The 2nd dipole: θ_2, ρ
- The transfer matrix of the beam line from 1 to 2 and from 2 to 3:

$$M_{12}=\begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix},\quad M_{23}=\begin{pmatrix} m_{22} & m_{12} \\ m_{21} & m_{11} \end{pmatrix}.$$



- Net CSR kick

$$\Delta X = M_{23}(M_{12}X_{k1}+X_{k2})+X_{k3},$$

- Achromatic condition and CSR-kick cancellation condition:

$$M_{12}=\begin{pmatrix} -\dfrac{q_2\rho+2m_{12}(\theta_1+\theta_2)S_1}{2q_1\rho} & m_{12} \\ \dfrac{1}{m_{12}}\left(\dfrac{q_2S_2}{4q_1S_1}+\dfrac{m_{12}(\theta_1+\theta_2)S_2}{2q_1\rho}-1\right) & -\dfrac{S_2}{2S_1} \end{pmatrix},$$

$q_1 = 2S_1\text{-}C_1q_1$ and $q_2 = 2S_2\text{-}C_2q_2$.

If $\theta_2 = 0$, reduces to a DBA

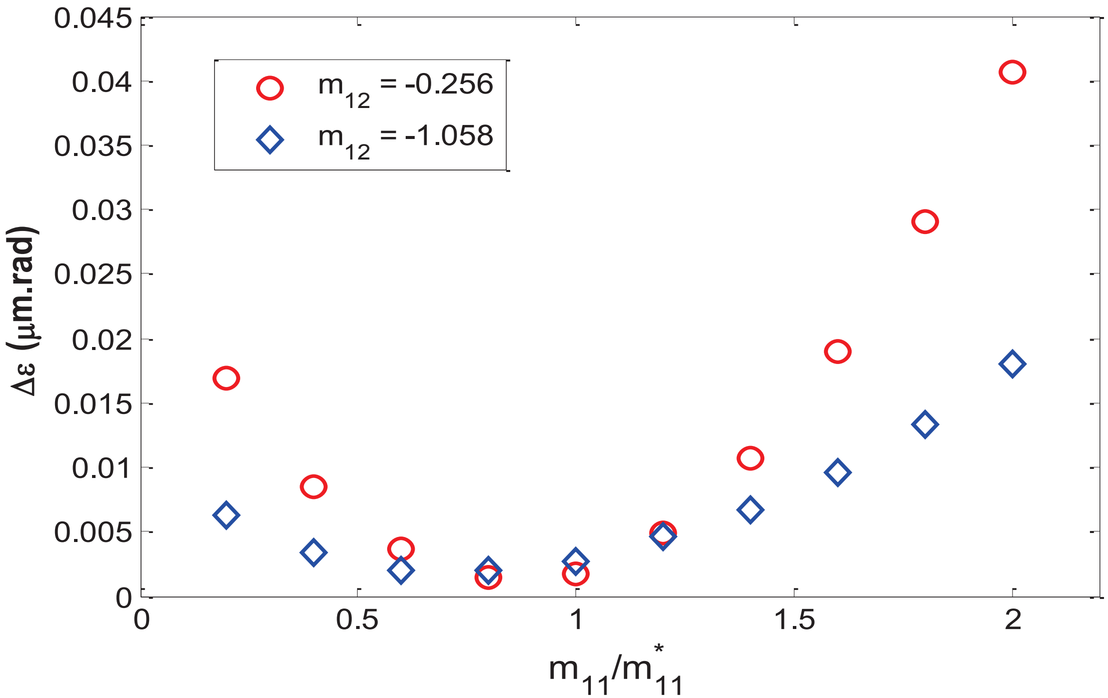
$$M_{13}|_{\theta_2=0} = M_{23}M_{12}|_{\theta_2=0} = \begin{pmatrix} -1 & 0 \\ 2\theta_1S_1/q_1\rho & -1 \end{pmatrix} \cong \begin{pmatrix} -1 & 0 \\ 12/L_1 & -1 \end{pmatrix}$$

Verification of the conditions for a symmetric TBA

—ELEGANT simulation

- Consider a symmetric TBA with three identical dipoles
  - ρ = 7 m and θ = 3 deg.
  - For the matrix M_{12}, fix m_{12}, the optimal value of m_{11} can be determined
  - Symmetric optics

$$m_{11}^{*} = -\frac{q_2\rho + 2m_{12}(\theta_1+\theta_2)S_1}{2q_1\rho}$$



Parameter	Value	Units
Bunch charge	500	pC
Norm. emittance	2	mm.rad
Beam energy	1000	MeV
Energy spread	0.05	%
Bunch length	30	mm
Dipole bending radius	7	m
Dipole bending angle	3	degree

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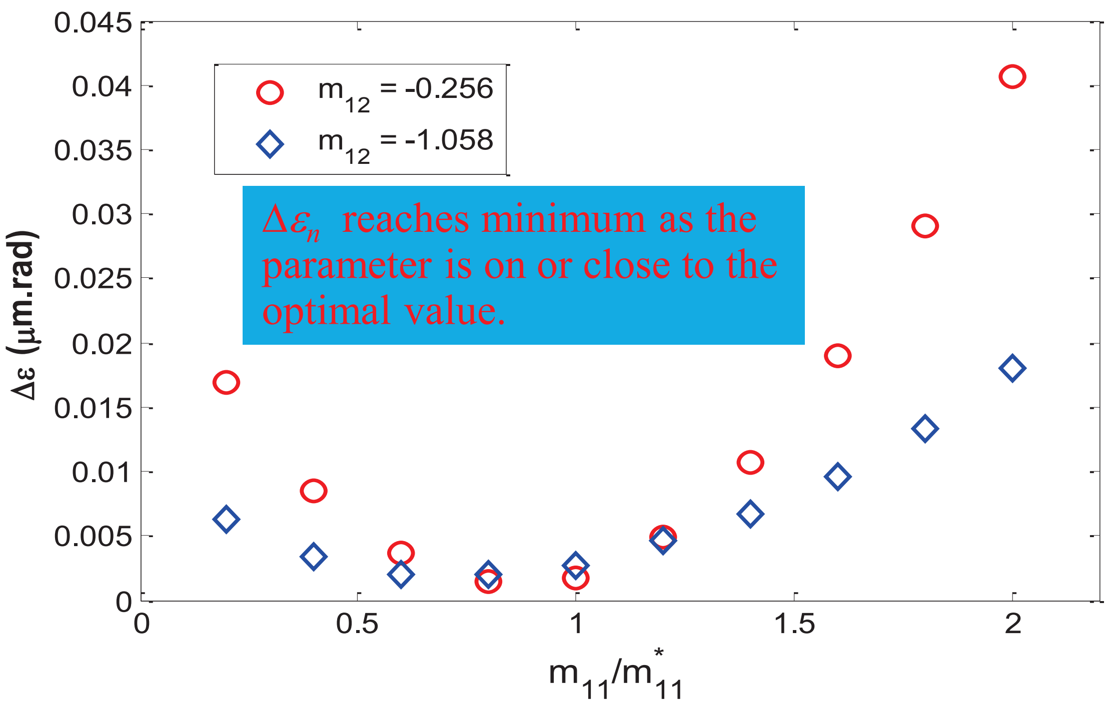

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Conclusions

Consider long. CSR wake in free space, short bunch (tens of μm), low emittance

- 1, 2D point-kick analysis promises explicit formulation of the net CSR kick in achromats;
- 2, this method results in generic conditions to cure the CSR kick in linear regime and minimizes the CSR-induced geometric emittance growth;
- 3, the obtained conditions are robust against the variation of the initial beam distribution;
- 4, it suggests easily-applied CSR-suppression scheme. Most times it needs only to vary the strengths of the quadrupoles. An demonstration experiment has been suggested on SDUV-FEL in Shanghai.

Presently the solutions are applicable to spreaders of FELs, recirculation loops of ERLs, where the bunch length does not have significant change. In near future, this method can be potentially expanded to suppress the CSR effect in specified functional bunch compressors.

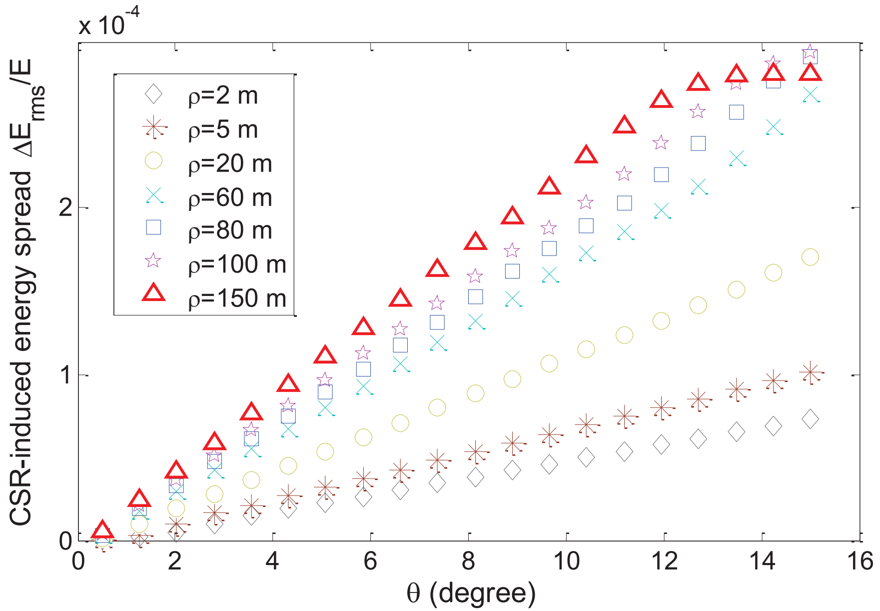
Thanks for your attention!

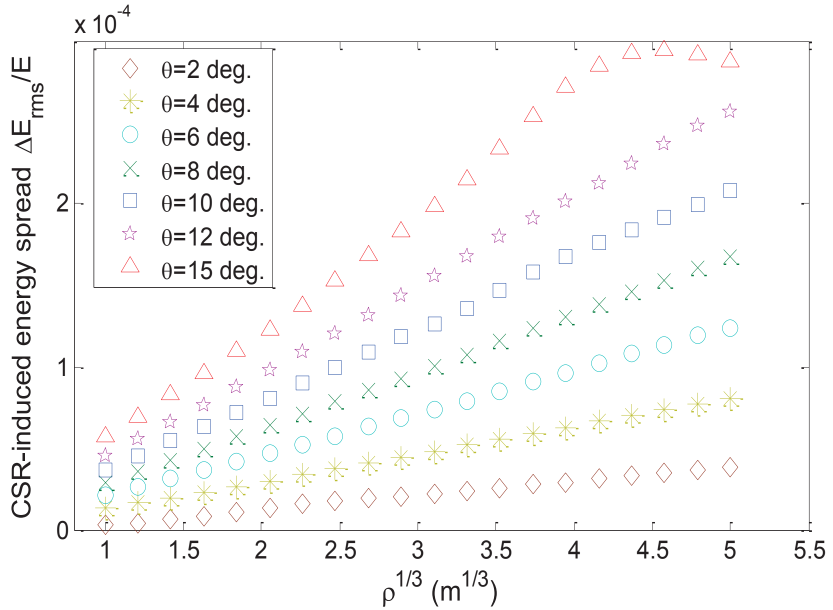
Backup slides

Linear dependency of the energy spread vs. $\rho^{1/3}$ & θ

$$\text{If fix } \theta,\ \Delta E(csr)/E_0 \propto \rho^{1/3}$$

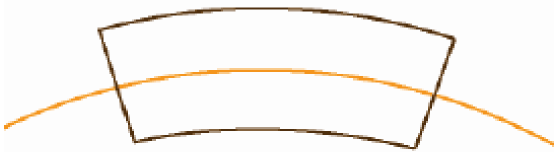
$$\text{If fix } \rho,\ \Delta E(csr)/E_0 \propto \theta$$





This linear relation applies well to the cases with θ from 1 to 12 degrees and ρ from 1 to 150 m.

CSR-induced orbit deviation in a bending magnet



Bending Magnet

Betatron transfer matrix :

$$M_d = \begin{pmatrix} \cos\theta & \rho\sin\theta \\ -\sin\theta/\rho & \cos\theta \end{pmatrix}$$

$\delta_i = 0$, w/o CSR effect :

$$X_f = \begin{pmatrix} x_f \\ x'_f \end{pmatrix} = MX_i = M\begin{pmatrix} x_i \\ x'_i \end{pmatrix},$$

$$x_f = m_{11}x_i + m_{12}x'_i, \quad x'_f = m_{21}x_i + m_{22}x'_i.$$

$\delta_i \neq 0$, w/o CSR effect :

$$X_f = MX_i + \begin{pmatrix} D \\ D' \end{pmatrix}\delta_i,$$

$$x_f = m_{11}x_i + m_{12}x'_i + D\delta_i, \quad x'_f = m_{21}x_i + m_{22}x'_i + D'\delta_i.$$

(D, D') : momentum dispersion (x-δ correlation terms), $D = \rho(1-\cos\theta)$, $D' = \sin\theta$.

$\delta_i \neq 0$, w/ CSR effect :

$$X_f = MX_i + \begin{pmatrix} D \\ D' \end{pmatrix}\delta_i + \begin{pmatrix} \zeta \\ \zeta' \end{pmatrix}k,$$

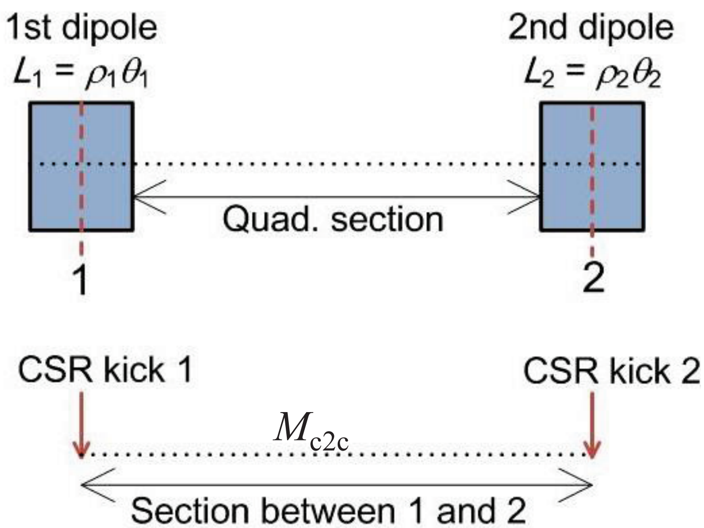
$$x_f = m_{11}x_i + m_{12}x'_i + D\delta_i + \zeta k, \quad x'_f = m_{21}x_i + m_{22}x'_i + D'\delta_i + \zeta' k.$$

(ζ, ζ') : "CSR dispersion" (x-k correlation terms), $\zeta = \rho^{4/3}(\theta - \sin\theta)$, $\zeta' = \rho^{1/3}(1-\cos\theta)$.

In addition, $\delta_f = \delta_i + k\rho^{1/3}\theta.$

R. Hajima, R-matrix analysis, NIMA 2004.

2D point-kick analysis for a two-dipole achromat



1, For simplicity, assume
$X_0 = (0, 0)^T$, $\delta = \delta_0$;

2, Right **before** the **1st** kick,
$X_{1-} = (0, 0)^T$, $\delta = \delta_0$;

3, Right **after** the **1st** kick,
$X_{1+} = X_{1-} + X_{k,1}$, $\delta = \delta_0 + k\rho_1^{1/3}\theta_1$;

4, Right **before** the **2nd** kick,
$X_{2-} = M_{c2c}X_{1+}$, $\delta = \delta_0 + k\rho_1^{1/3}\theta_1$;

5, Right **after** the **2nd** kick,
$X_{2+} = X_{2-} + X_{k,2}$, $\delta = \delta_0 + k\rho_1^{1/3}\theta_1 + k\rho_2^{1/3}\theta_2$;

Net CSR kick:

$$X_{2+} = X_{k,2} + M_{c2c}X_{k,1}$$

Final geometric emittance:

$$\varepsilon = \sqrt{(\varepsilon_0\beta_2 + x_{2+,rms}^2)(\varepsilon_0\gamma_2 + x'^2_{2+,rms}) - (\varepsilon_0\alpha_2 - x_{2+,rms}x'_{2+,rms})^2} = \sqrt{\varepsilon_0^2 + \varepsilon_0 d\varepsilon_1},$$

$$d\varepsilon_1 = \gamma_2 x_{2+,rms}^2 + 2\alpha_2 x_{2+,rms}x'_{2+,rms} + \beta_2 x'^2_{2+,rms}.$$

2D point-kick analysis for a two-dipole achromat

M_{c2c}: the betatron transfer matrix between two dipole centers

$$M_{c2c} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}$$

Net CSR kick:

$$X_{2+} = \begin{pmatrix} 2m_{12}S_1 \\ 2(m_{22}S_1 + S_2) \end{pmatrix}\delta_0 + \begin{pmatrix} m_{12}S_1\theta_1\rho_1^{1/3} + \rho_1^{4/3}[m_{11}(C_1\theta_1 - 2S_1) + r^{4/3}(C_2\theta_2 - 2S_2)] \\ (m_{22}S_1 + 2S_2)\theta_1\rho_1^{1/3} + m_{21}(C_1\theta_1 - 2S_1)\rho_1^{4/3} + S_2\theta_2\rho_2^{1/3} \end{pmatrix} k$$

with $S_1 = \sin(\theta_1/2)$, $C_1 = \cos(\theta_1/2)$, $S_2 = \sin(\theta_2/2)$, $C_1 = \cos(\theta_1/2)$.

For a two-dipole achromat:

The element $\propto \delta_0$ should be zero →

$$M_{c2c} = \begin{pmatrix} -S_1/S_2 & 0 \\ m_{21} & -S_2/S_1 \end{pmatrix}$$

For a two-dipole achromat, the horizontal phase advance between two dipole centers is π or 2π, only $M_{c2c}(2, 1)$ is variable.

2D point-kick analysis for a two-dipole achromat

With the achromatic condition, net CSR kick:

$$X_{2+} = \begin{pmatrix} \rho_1^{4/3} S_1 (2S_1 - C_1\theta_1)/S_2 - \rho_2^{4/3}(2S_2 - C_2\theta_2) \\ S_2(\theta_1\rho_1^{1/3} + \theta_2\rho_2^{1/3}) - m_{21}(2S_1 - C_1\theta_1)\rho_1^{4/3} \end{pmatrix} k$$

$with\ S_1 = \sin(\theta_1/2),\ C_1 = \cos(\theta_1/2),\ S_2 = \sin(\theta_2/2),\ C_1 = \cos(\theta_1/2).$

Δε = 0 if the element ∝ *k* becomes 0 in a two-dipole achromat

Keep the first significant terms with respect to θ_1 and θ_2

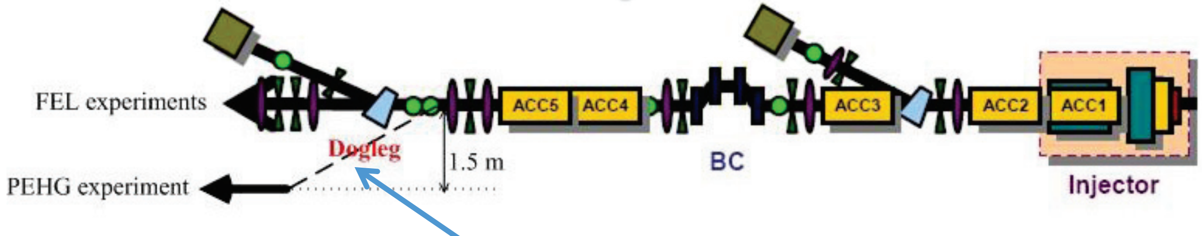
$$\left(\frac{\rho_2}{\rho_1}\right)^{4/3} = \frac{S_1(2S_1 - C_1\theta_1)}{S_2(2S_2 - C_2\theta_2)} \approx \left(\frac{\theta_1}{\theta_2}\right)^4$$

$$m_{21} = \frac{S_2}{S_1}\frac{(\theta_1\rho_1^{1/3} + \theta_2\rho_2^{1/3})}{(2 - C_1\theta_1/S_1)\rho_1^{4/3}} \approx \frac{12}{L_1}\frac{\theta_2}{\theta_1}$$

$$L_1\theta_1^2 \cong L_2\theta_2^2$$

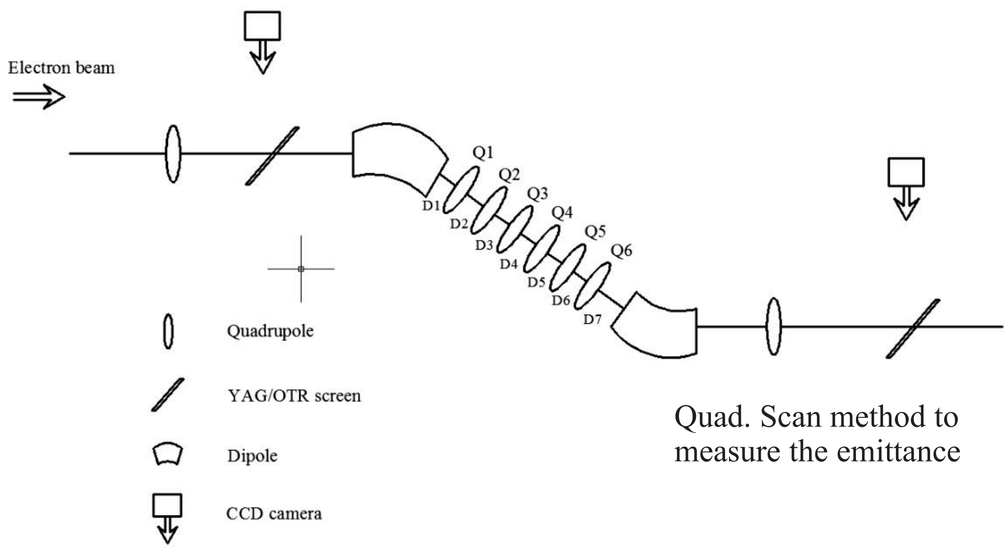
$$M_{c2c}(2,1) \cong \frac{12}{L_1}\frac{\theta_2}{\theta_1}$$

Demonstration experiment on SDUV-FEL



CSR suppress experiment

Experiment layout



Quad. Scan method to measure the emittance

Expected result, as m_{21}= -12/L_1, minimum emittance growth