CAVITY LENGTH CHANGE VS. MIRROR STEERING IN A RING CONFOCAL RESONATOR*

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Abstract

In principle, a ring confocal resonator allows for the use of a short Rayleigh length without the extreme sensitivity to mirror steering typical in a near-concentric resonator [1]. One possible weakness of such a resonator is that the cavity length is no longer independent of the mirror steering. This is one of the strengths of a linear resonator. In this presentation, it is shown that, in a simple 2-dimensional corner cube type ring confocal resonator, the cavity length is, in fact, not dependent on the mirror steering to first order in the mirror angles. Thus the ring-confocal resonator might be a very easy-to-operate and stable resonator for short Rayleigh range operation in FEL oscillators.

INTRODUCTION

It is well known that the optical mode in a confocal resonator has much lower sensitivity to mirror steering than a near-concentric design. Since the mode size in the return path of the confocal resonator is rather large it is more practical to make the confocal resonator for an FEL in a ring configuration so that the mode does not have to pass through a narrow wiggler gap on the return leg. This may lead to a couple of problems. The first is astigmatism, which may be addressed using cylindrical or toroidal optics. The second is a potential coupling of mirror steering to cavity length changes. Decoupling of cavity length and mirror steering, a given in linear resonators, makes laser optimization very straightforward. A dependence of cavity length on mirror steering would greatly complicate operation of the FEL. This note will derive the change in cavity length as each mirror is steered in a ring confocal resonator. As will be seen, the properties of the ring resonator tend to make the cavity length extremely insensitive to mirror steering.

DEFINITION OF VARIABLES

Let us first describe the ring confocal resonator and define our variables. The resonator is shown in Fig. 1. The two flat mirrors that deflect the beam away from the electron beam axis are called the flat deflecting optics or FDOs. The curved mirrors that bring the beam back towards the wiggler are called the fold mirrors or FMs. The angle that the FDOs deflect the beam (twice the angle of incidence on the FMs) will be referred to as θ. The transverse separation between the forward and reverse legs of the resonator will be represented by B. The distance between the FDO and the FM is equal to $B/\sin\theta = C$. The distance from the cavity center (assumed also to be the wiggler center) to the FDO is A. The FMs are concave mirrors that collimate the beam that emerges from the wiggler. The radius of curvature for a ring confocal resonator should be $R_{\mathrm{eff}} = 2(A+C)$. The actual radius of curvature will be longer by $\sec(\theta/2)$. It is useful to know the effective radius of curvature as a function of the round trip distance in the resonator. Since the distance in the backleg between the FDO and the FM is $C \bullet \cos(\theta)$, the round trip cavity length $L_{\mathrm{rt}}=2(2A+C(1+\cos(\theta)))$. Using this we have $R_{\mathrm{eff}} = L_{\mathrm{rt}}/2 + C(1-\cos(\theta))$.

NORMAL VIBRATION MODES

To calculate the change in cavity length for a given mirror angle tilt it is useful to define normal modes of the resonator that are linear combinations of all four mirrors. Since the angular shifts are all linear one can represent a change in the angle of any one mirror as a linear combination of these four modes. Since there are four mirrors, it is possible to define four normal modes related to the four angles via a non-singular matrix to be derived below. There is a default beam path that goes through the center of the wiggler, hits the centers of each mirror, and returns a distance B away from the wiggler in the backleg. The default beam path is parallel in the forward and backwards directions in the resonator. Define the following four modes of the beam path with respect to the default beam path. Assume that the mirrors rotate about the point that the default beam path intersects the mirror surface:

- Mode 1 – In this mode the light paths are still parallel in the forward and backward directions but they move either towards the center of the ring or away from the center.
- Mode 2 – In this mode the light path is parallel in the forward and backward directions but the two paths both move in the same direction with respect to the default beam path.
- Mode 3 – In this mode the light path goes through the center of the wiggler and is a distance B away in the backleg at that point. The path goes through the wiggler at an angle and returns with the opposite angle so the mode looks wedged from above.
- Mode 4 – In this mode the light path again goes through the center of the wiggler at an angle but the angle in the backleg is the same as in the forward leg. The mode as a whole is tilted.

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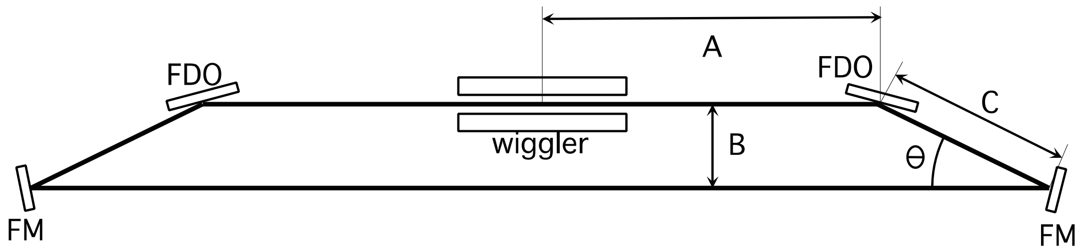

Figure 1: Layout of the ring confocal resonator with the leg lengths and mirror labels defined.

Any mode can be made from these four. It is possible to define a matrix that relates the modes to the individual mirror angles. This matrix can then be inverted to give the individual mode changes for a given tilt in one mirror.

In this note, the discussion is limited to one plane. Any changes in cavity length due to vertical steering should be second order in the angle (which is a very small quantity) and so should be negligible.

To derive the normal modes in terms of the mirror angles it is useful to review the nature of a two-dimensional corner cube. The beam will exit parallel to the input beam unless an angular error between the two mirrors is introduced. Movement of the beam to one side on the entrance automatically leads to movement in the same direction on the output (i.e. beam left motion stays beam left unlike a mirror where beam left turns into beam right). The curvature in the FM does lead to behavior different from a corner cube. Changes in position on the FM lead to a relative change in the angular orientation between the mirrors. This has the effect of stabilizing the mode so that it tends to move towards the position where the input and output angles are the same.

Mode 1

In this mode one only rotates the two FMs. If both of these mirrors turn the opposite direction then the separation between the forward and backward beams increases and decreases but the beam path is parallel to the default path. Devine the transverse shift as δ, where a positive δ is away from the ring center. To first order, the FDO and the FM act as a two-dimensional corner cube. Thus, the exit beam from the two mirrors is also shifted away from the center by the same amount. Since the beam has moved on the FM however, it is necessary to steer the FM slightly to maintain the exit beam parallel to the input beam to correct for the angle caused by the mirror curvature. The angle necessary to correct the angle is δ/R_{eff}. The geometry of this is shown in Fig. 2 where all offsets have been greatly exaggerated. In fact, the mirror steering angles are in µrad or less and the bend angles are in rad. The mode movements are in hundreds of microns while the beam separation is on the order of a meter.

To see how sensitive this is, let us consider an example. Assume a ring confocal resonator with an angle $\theta = 30°$ and a distance B = 68.6 cm. Assume that the radius of curvature in the plane of the ring is given by the formula above and that L_{rt}=64.084 m. The effective ROC should therefore be 32.23 meters and the actual ROC must be 33.36 m. In this case a one microradian tilt in both FMs in the opposite directions will move the mode by 32.2 µm both in the wiggler and in the backleg (both away from the ring center). The $1/e^2$ waist size for a Rayleigh range of 70 cm and a wavelength of 1.06 µm is 486 µm so this is quite a small shift compared to the mode size.

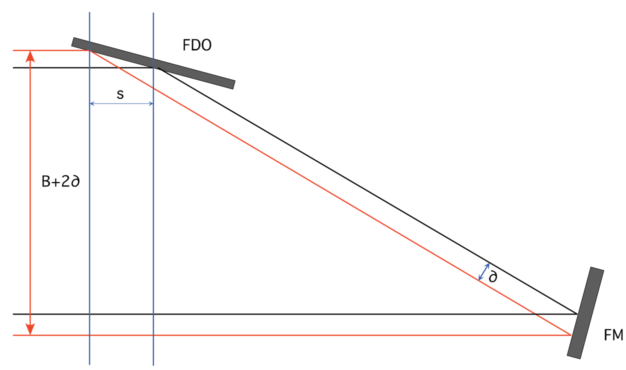

Figure 2: Geometry for Mode 1 shift. Both parallel rays move out from the center by a distance δ. The black curve is the default path and the red is the position for mode 1.

Mode 2

This mode requires all four mirrors to move. The mode is a bit unnatural since one is fighting the tendency of the mode to move the same direction on the output as at the input. One wants the FDO and the FM to both rotate approximately the same amount but the FM must rotate a bit less to account for its ROC. For a common axis shift of distance δ, the FDO must shift by an angle $\phi_{FDO} = \delta/C$. This actually moves the return leg up by 2δ so that the beam moves in the opposite direction instead of the same direction. It also changes the angle of the beam by $2\phi_{FDO}$ and this must be compensated for by also rotating the FM by about the same amount. Since we have moved up on the FM however, there is an extra angle provided by the curvature so the FM does not have to move quite so far. The net result is:

$$\phi_{FM} = \frac{\delta}{C} - \frac{\delta}{R_{eff}} = \frac{\delta}{C}\left(1 - \frac{C}{R_{eff}}\right) = \phi_{FDO}\left(1 - \frac{C}{R_{eff}}\right)$$

The final expression in parentheses is equal to 0.957 for the numerical example above. Thus the two mirrors move almost the same amount. The mirrors at the opposite end of the cavity rotate in the opposite direction, with the FM again rotating a bit less than the FDO. The geometry for

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this configuration is shown in Fig. 3. Note that for the numerical example above, the distance in microns is close to the tilt angle of the mirrors in micro-radians. The mode moves by 1.37 µm for 1 µrad of mirror tilt. This is 23.5 times less sensitive than the movement in Mode 1.

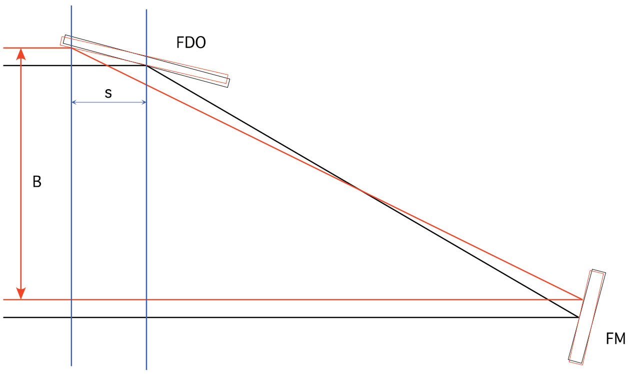

Figure 3: Geometry of Mode 2. Here both mirror rotate by a positive angle. The FM rotates slightly less than the FDO to account for the radius of curvature. The angles shown are exaggerated by about a factor of 1000 to show that they occur. The black path is the default position and the red path is the rotated position.

Mode 3

In this mode we again only need rotate the FMs. If they are rotated in the same direction we find that the separation between the forward and back branches is smaller on the upstream end than at the downstream end for a negative (clockwise) angle. The geometry looks the same as Fig. 2 except that the input and return ray for mode 3 are slightly converging so that they intersect with the default rays at the position of the center of the wiggler. If a ray leaves the wiggler center at an angle ψ with respect to the default ray, it will be offset by a distance $\psi R_{eff}/2$ at the FM. Since the purpose of this mirror is to collimate the beam, the exit ray will be parallel to the default ray if the FM is not rotated. To get the return ray to intersect the default ray at the position of the wiggler center one must rotate the FM by an angle:

$$\phi_{FM} = \frac{\psi R_{eff}}{4(A + C\cos\theta)} = \frac{\psi(A + C)}{2(A + C\cos\theta)}$$

So to create a mode with rays at an angle ψ_{Mode3} with respect to the default ray, one must rotate the FMs by a little more than half the mode angle

$$\phi_{FM} = \psi_{Mode3}\frac{A + C}{2(A + Ccos\theta)}$$

Remember that in this case the two FMs rotate in the same sense, i.e. both positive or both negative angles. For the numerical example above the fraction in this equation is 0.506 so the wedge angle is essentially twice the mirror rotation.

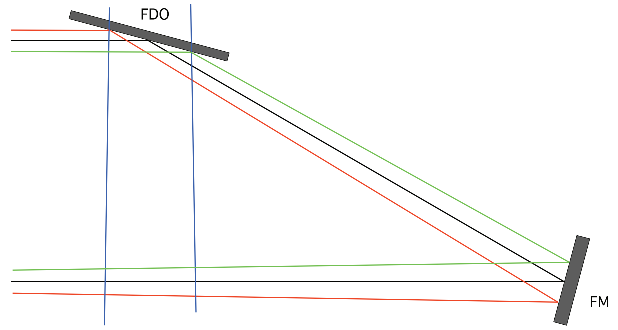

Figure 4: Geometry of Mode 3. The green rays are the upstream rays mirrored onto the downstream end. The red rays are the downstream rays. The blue lines are perpendicular to the return leg rays

Mode 4

Finally in Mode 4 all four mirrors rotate the same direction but the FMs rotate slightly less than the FDOs. This looks just like Fig. 3 except that the mode 4 rays are at a slight angle with respect to the default rays and they are in opposite directions on the upstream and downstream mirrors. The analysis is the same as for Mode 2 except that the quantity δ is equal to $\psi(A + C\cos\theta)$ where ψ is the angle of the mode with respect to the default mode. The ratio between the mode rotation ψ and the mirror tilt ϕ_{FDO} is found from

$$\psi = \frac{C\phi_{FDO}}{A + C\cos\theta}$$

For the parameters of the resonator detailed above, the ratio is about 12 to 1, so you have to tilt all the mirrors by 12 µrad to get 1 µrad of mode tilt.

CHANGE IN CAVITY LENGTH

Now that we have defined the modes let us derive the change in the cavity length for each mode compared to the default rays. The blue vertical lines in Fig. 2 show the intersections between a vertical line and the point where the rays hit the FDO. The distance between the left blue line and its corresponding line on the other end of the resonator is obviously the same for the default rays and the Mode 1 rays. Note that, by construction, the red lines are parallel to the black lines and therefore, all the angles of incidence and angles of reflection are the same. Any change in the cavity length would be due to a difference in the path length from the left blue line back to the left blue line. The distance along the red path is simply

$$L_{Mode1} = (B + 2\delta)(\csc\theta + \cot\theta)$$

The distance along the black path is the same but without the 2δ term and the extra distance 2s where s=$\delta\cot(\theta/2)$. The path difference is then:

$$L_{Mode1} - L_{default} = 2\delta\left[\csc\theta + \cot\theta - \cot\left(\frac{\theta}{2}\right)\right]$$

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It can be shown that the expression in brackets is identically zero via trigonometric identities. Mode 1 therefore does not change the cavity length with respect to the default path.

Mode 2 is more complicated due to the fact that the angles are now not the same. The opening angle for the Mode 2 path in Fig. 3 (red lines) is now $\theta_{M2} \equiv \theta - 2\phi_{FDO}$. The distance along the path for Mode 2 is then:

$$L_{Mode2} = B(\csc\theta_{M2} + \cot\theta_{M2})$$

The path for the default ray from the left blue line back to the left blue line is

$$L_{default} = 2\delta \cot\left(\frac{\theta_{M2}}{2}\right) + B(\csc\theta + \cot\theta)$$

It is useful to use the relations (true for small η) $\csc(\theta - \eta) \cong \csc\theta + \eta \csc\theta \cot\theta$ and $\cot(\theta - \eta) \cong \cot\theta + \eta(\csc\theta)^2$.

The change in path length is then

$$L_{Mode2} - L_{default} = 2B\phi_{FDO}\left[\csc\theta\cot\theta + (\csc\theta)^2 - \csc\theta\cot\left(\frac{\theta}{2}\right)\right]$$

where we have used the expression $C=B\csc(\theta)$. We now have:

$$L_{Mode2} - L_{default} = 2C\phi_{FDO}\left[\cot\theta + \csc\theta - \cot\left(\frac{\theta}{2}\right)\right]$$

which is again zero. So the path length change for mode 2 is also zero to first order in the angular offsets.

For Mode 3 we can use the anti-symmetric nature of the mode to argue that any change in the mode orbit length in one half of the resonator is cancelled out in the other half. In Fig. 4 we show the geometry for Mode 3 with the upstream end reflected onto the downstream end. The upstream end is represented by the green rays and the downstream end is represented by the red rays.

One can see that each change from the default rays is mirrored in the opposite end. When the calculation is done carefully one finds, in fact, that the path difference from the default rays is $-B\psi_{Mode3}$ on the downstream end and $B\psi_{Mode3}$ on the upstream end to first order in ψ_{Mode3} so the net change in path is zero for Mode 3.

The same asymmetry argument can be used for Mode 4 to state that, since the path length change for Mode 2 is zero, the path length change for Mode 4 must also be zero.

Since any mode in the resonator can be derived from a linear combination of these four modes and since each of the modes has no change in the cavity length one comes to the startling conclusion that the ring confocal resonator round trip path length is independent of the mirror steering to first order in the mirror tilt angles.

RELATION BETWEEN INDIVIDUAL MIRROR STEERING AND MODE SHIFTS

It is useful to derive the relation between the mode changes, which may involve a change in all four mirrors to the mode changes when only one mirror is shifted. Let us first number the mirrors as 1 through 4 with the upstream FDO being 1 and the upstream FM as mirror 4. Let us also define the quantities $G=(A+C\cos\theta)/(A+C)$, $H=(A+C)$, $F=(1-C/R_{eff})$ and $J=C/(A+\cos\theta)$. As noted above F and G are usually very close to but slightly less than unity. The quantity 2H is the same as the effective radius of curvature of the FMs and J is usually small compared to one. We can represent the modes by the following steering operator:

$$S = \begin{pmatrix} FH & -FH & H & -H \\ -C/4 & C/4 & 0 & 0 \\ -FG & -FG & G & G \\ J/4 & J/4 & 0 & 0 \end{pmatrix}$$

When the angle vector $(\theta_1, \theta_2, \theta_3, \theta_4)$ is multiplied by this matrix it will give a set of mode amplitudes. Modes 1 and 2 are defined by the distances that the axes move while Modes 3 and 4 are defined by the angles that the rays tilt. One can also invert this matrix to get the angles for a given combination of modes:

$$S^{-1} = \begin{pmatrix} 0 & -2/C & 0 & 2/J \\ 0 & 2/C & 0 & 2/J \\ 1/2H & 2F/C & 1/2G & 2F/J \\ -1/2H & -2F/C & 1/2G & 2F/J \end{pmatrix}$$

When this is multiplied by a vector of the mode amplitudes (M_1, M_2, M_3, M_4) we get the angle vector $(\theta_1, \theta_2, \theta_3, \theta_4)$.

It is worth noting some features of this resonator.

1. Modes 2 and 4 are suppressed due to the nature of the end mirrors as corner cubes. In fact the fold mirrors do not change modes 2 and 4 at all.
2. The resonator mode position in the wiggler (via mode 1) is fairly sensitive to the tilt of any of the 4 mirrors. For the example resonator, the beam moves 161 μm in the wiggler for a 10 μrad shift in either of the fold mirrors and 150 μm for a 10 μrad shift in either FDO.
3. The mode is extremely stable in angle. To get 100 μrad of mode tilt one needs 2.2 mrad of tilt in mode 4 and 50 μrad in mode 3.

CONCLUSION

The conclusion is a bit counter-intuitive but is consistent with the use of corner cubes in interferometers, where path length changes are critical. Since each of the

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modes does not change the cavity length (to first order in the mirror tilts), and since any mode may be produced by a linear combination of the four modes, then the resonator mode is independent of the mirror tilts. The counter-intuitive nature comes from the nature of a corner cube. When one turns a corner cube, the beam comes back exactly along the original angle independent of the angle of the cube. The route through the cube also changes but in such a way that the length change in one leg is exactly compensated by the other. The net change is then exactly zero.

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