INITIAL VALUE PROBLEM FOR AN FEL DRIVEN BY AN ASYMMETRIC ELECTRON BEAM

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Abstract

FEL configurations in which the driving electron beam is not axially symmetric (round) are important in the study of novel concepts (such as TGU-based FELs, [1]- [2]) but also become relevant when one wishes to explore the degree to which the deviation from symmetry - inevitable in practical cases - affects the performance of more conventional FEL schemes [3]. In this paper, we present a technique for solving the initial value problem of such an asymmetric FEL. Extending an earlier treatment of ours [4], we start from a self-consistent, fully 3D, evolution equation for the complex amplitude of the electric field of the FEL radiation, which is then solved by expanding the radiation amplitude in terms of a set of orthogonal transverse modes. The numerical results from such an analysis are in good agreement with simulation and provide a full description of the radiation in the linear regime. Moreover, when the electron beam sizes are constant, this approach can be used to verify the predictions of the standard eigenmode formalism.

INTRODUCTION

In most theoretical treatments of the free electron laser (FEL), it is assumed that certain characteristics of the electron beam (such as size and angular divergence) and the undulator system (such as focusing strength) are the same in both transverse directions, a premise which defines the so-called round beam case. There exist, however, novel FEL concepts whose treatment requires a definite departure from the round beam scenario. A particularly intriguing example of the latter is an FEL based on a transverse gradient undulator (TGU), where the addition of dispersion may cause the horizontal size of the electron beam to become much larger than its vertical size. Moreover, non-symmetric FEL examples may become relevant even in the context of more conventional configurations since asymmetry is an inherent feature in many key FEL components (we note, for instance, the absence of horizontal focusing in a flat-pole undulator). In this work, we adopt a model that covers both cases and present a semi-analytical method for solving the initial value problem of the FEL in the linear regime.

THEORY

We begin our analysis by presenting a slightly generalized version of an already established analytical result regarding a TGU-based FEL. In particular, using the methods outlined in [4]- [5], one can derive a 3D equation which governs the evolution of the radiation amplitude $E_\nu(\mathbf{x},z)$ throughout the linear regime of the interaction. The result - in its most general form - can be stated as

$$\begin{aligned}&\left(\frac{\partial}{\partial z}+\frac{\nabla_\perp^2}{2ik_r}\right)E_\nu(\mathbf{x},z)+\frac{8i\rho_T^3k_u^3}{2\pi\sigma_x'\sigma_y'}\int_0^z d\zeta\xi e^{-i\Delta\nu k_u\xi}\\&\times\exp\left[-2(\sigma_\delta^{ef})^2k_u^2\xi^2\right]\int_{-\infty}^{\infty}dp_xdp_yE_\nu(x_+,y_+,\zeta)\\&\times\exp[-2ik_u\xi\left(C_p\frac{x}{\eta}+\left(\bar{\alpha}\frac{\xi}{2}+\frac{\eta\sigma_\delta^2}{\sigma_T^2}z_x\right)p_x\right)]\\&\times\exp\left[-\frac{(x-p_xz_x)^2}{2\sigma_T^2}-\frac{1}{2}\left(\frac{1}{\sigma_x'^2}+ik_r\xi\right)p_x^2\right.\\&\left.-\frac{1}{2}\left(\frac{1}{\sigma_y'^2}+ik_r\xi\right)(p_y^2+k_n^2y^2)\right]\equiv FE_\nu(\mathbf{x},z)=0\,.\end{aligned}\tag{1}$$

Here, F is meant as an operator, $\nabla_\perp^2=\partial^2/\partial\mathbf{x}^2$, $\xi=\zeta-z$, $z_x=z-z_0$, $x_+=x+p_x\xi$ and $y_+=y\cos(k_n\xi)+(p_y/k_n)\sin(k_n\xi)$, where z_0 is a constant offset and k_n is the undulator natural focusing strength in the y-direction, $k_r=\omega_r/c=2\pi/\lambda_r$ and $k_u=2\pi/\lambda_u$ - where λ_r is the resonant wavelength, ω_r is the resonant frequency and λ_u is the undulator period - while $\Delta\nu=\nu-1=\omega/\omega_r-1$ is the detuning (ω is a frequency variable). On the other hand, σ_y and σ_y' are the rms values for the vertical size and angular divergence of the electron beam while σ_T and σ_x' are their horizontal counterparts at $z_x=0$. The former of the last two parameters includes the contribution of the - constant - dispersion η and is given by $\sigma_T=(\sigma_x^2+\eta^2\sigma_\delta^2)^{1/2}$, where σ_x is the non-dispersive horizontal beam size (at $z=z_0$) and σ_δ is the rms relative energy spread. It should be emphasized that - unlike the horizontal beam size, which attains a minimum at $z=z_0$ - the vertical beam size is assumed to be constant, so the matching condition $\sigma_y'/\sigma_y=k_n$ holds in the y-direction. Moreover, ρ_T and σ_δ^{ef} are, respectively, the effective Pierce parameter and energy spread of the FEL, quantities that are expressed by $\rho_T=\rho(1+\eta^2\sigma_\delta^2/\sigma_x^2)^{-1/6}$ and $\sigma_\delta^{ef}=\sigma_\delta(1+\eta^2\sigma_\delta^2/\sigma_x^2)^{-1/2}$, where ρ is the Pierce parameter for $\eta=0$. The non-dispersive FEL parameter is in turn given by $\rho=(K_0^2[JJ]^2I_p/(16I_A\gamma_0^3\sigma_x\sigma_yk_u^2))^{1/3}$, where γ_0 is the average electron energy in units of its rest mass m_0c^2, K_0 is the on-axis undulator parameter, $[JJ]=J_0(K_0^2/(4+2K_0^2))-J_1(K_0^2/(4+2K_0^2))$, $I_A\approx 17$ kA is the Alfven current and I_p is the peak current of the electron beam. As far as the remaining parameters are concerned, $C_p=\sigma_x^2/\sigma_T^2+\bar{\alpha}\eta-1$ with $\bar{\alpha}=K_0^2\alpha/(2+K_0^2)$, α being the transverse gradient of the undulator field. Finally, we should also note that the expression for C_p given above is a generalization of the one contained in [5], which only covered the case with $\bar{\alpha}=1/\eta$.

ISBN 978-3-95450-133-5

The latter is referred to as the TGU resonance condition. In the limit of $\alpha \to 0$ and $\eta \to 0$, $\rho_T \to \rho$, $\sigma_\delta^{ef} \to \sigma_\delta$ and the exponent in the third line of Eq. (1) vanishes (note that $C_p/\eta \to 0$) so the evolution equation now describes a standard FEL with vertical - but not horizontal - natural focusing.

Mode Expansion

Our goal is to obtain a solution to Eq. (1) that is compatible with a given input amplitude $E_\nu(\mathbf{x},0)$. To this end, we introduce a set of orthogonal transverse modes [6] given by $\psi_{mn}(\mathbf{x},z) = \chi_m(x,z)\varphi_n(y,z)$, where

$$\chi_m(x,z) = (2^m m!)^{-1/2} H_m\left(\frac{\sqrt{2}\Delta x}{w_x}\right) e^{-imu_x}\chi_0(x,z)\,,$$
$$\varphi_n(y,z) = (2^n n!)^{-1/2} H_n\left(\frac{\sqrt{2}y}{w_y}\right) e^{-inu_y}\varphi_0(y,z) \qquad (2)$$

and

$$\chi_0(x,z) = (k_r\beta_{xr}/\pi)^{1/4}(\beta_x+iz)^{-1/2}e^{-k_r\alpha_{xi}^2/(2\beta_{xr})} \times \exp\left(-\frac{k_r(x-\alpha_x)^2}{2(\beta_x+iz)}\right),$$
$$\varphi_0(y,z) = (k_r\beta_{yr}/\pi)^{1/4}(\beta_y+iz)^{-1/2} \times \exp\left(-\frac{k_r y^2}{2(\beta_y+iz)}\right). \qquad (3)$$

In the above definitions, $m,n = 0,1,2,3,...$, H_k are the Hermite polynomials while β_x, β_y and α_x are complex-valued functions of z. The real parts of these quantities are denoted by β_{xr}, β_{yr} and α_{xr} - respectively - while their imaginary parts are β_{xi}, β_{yi} and α_{xi}. This convention - an index r/i denoting real/imaginary part - is adopted for other variables in this paper as well. Moreover, $w_x = (2/(k_r\beta_{xr}))^{1/2}|\beta_x+iz|$, $w_y = (2/(k_r\beta_{yr}))^{1/2}\left|\beta_y+iz\right|$, $u_x = \arctan((z+\beta_{xi})/\beta_{xr})$, $u_y = \arctan((z+\beta_{yi})/\beta_{yr})$ and $\Delta x = x - x_c$, where $x_c = \alpha_{xr} + \alpha_{xi}(z+\beta_{xi})/\beta_{xr}$. These modes satisfy the orthonormality condition $\int d^2\mathbf{x}\psi^*_{m'n'}(\mathbf{x},z)\psi_{mn}(\mathbf{x},z) = \delta_{mm'}\delta_{nn'}$ and form a complete set. Thus, the radiation amplitude $E_\nu(\mathbf{x},z)$ can be expanded in terms of the generalized Gauss-Hermite modes described above. Our strategy is to construct an approximate solution to the initial value problem by retaining only the fundamental ($m = n = 0$ or 00) mode in such an expansion. In other words, we would like to have $E_\nu(\mathbf{x},z) \approx E_\nu^{00}(\mathbf{x},z) = a_{00}C_{00}(z)\psi_{00}(\mathbf{x},z)$, where a_{00} is a constant and $C_{00}(z)$ is a dimensionless mode coefficient to be determined along with the basis parameters $\beta_x(z)$, $\alpha_x(z)$ and $\beta_y(z)$. Since $E_\nu^{00}(\mathbf{x},z)$ is not an exact solution, $FE_\nu^{00}(\mathbf{x},z)$ is a non-zero function. Our solution is based on imposing the condition that the projections of FE_ν with respect to the 00, 10, 20 and 02 modes are zero i.e. $(\psi_{00},FE_\nu^{00}) = (\psi_{10},FE_\nu^{00}) = (\psi_{20},FE_\nu^{00}) = (\psi_{02},FE_\nu^{00}) = 0$, where we define the inner product $(f,g) = \int d^2\mathbf{x} f^* g$. The projections with respect to the 01 and 11 modes are identically zero. Using Eqs. (1)-(3) along with the aforementioned conditions, we can obtain a set of four integro-differential equations for C_{00}, β_x, α_x and β_y. In order to cast these analytical results in a more useful form, we first introduce the scaled variables $\hat{z} = z/\beta_{ex}$, $\hat{\beta}_x = \beta_x/\beta_{ex}$, $\hat{\alpha}_x = \alpha_x/\sigma_x$ and $\hat{\beta}_y = \beta_y/\beta_{ey}$, where $\beta_{ex} = \sigma_x/\sigma'_x$ is the minimum horizontal electron beta and $\beta_{ey} = \sigma_y/\sigma'_y = 1/k_n$ the vertical beta function. The resulting equations are

$$\frac{dC_{00}}{d\hat{z}} = i\left\{\frac{1}{4\hat{\beta}_{xr}}\frac{d\hat{\beta}_{xi}}{d\hat{z}} + \frac{1}{4\hat{\beta}_{yr}}\frac{d\hat{\beta}_{yi}}{d\hat{z}} + B_x\frac{\hat{\alpha}_{xi}}{\hat{\beta}_{xr}} \times\left(\frac{d\hat{\alpha}_{xr}}{d\hat{z}} + \frac{\hat{\alpha}_{xi}}{2\hat{\beta}_{xr}}\frac{d\hat{\beta}_{xi}}{d\hat{z}}\right)\right\}C_{00} + \int_0^{\hat{z}} d\hat{\zeta}C_{00}(\hat{\zeta})L_1(\hat{z},\hat{\zeta},\hat{\beta}_x,\hat{\beta}_{x,\zeta},\hat{\alpha}_x,\hat{\alpha}_{x,\zeta},\hat{\beta}_y,\hat{\beta}_{y,\zeta})\,, \qquad (4)$$

$$\frac{d\hat{\alpha}_x}{d\hat{z}} = i\frac{\hat{\alpha}_{xi}}{\hat{\beta}_{xr}}\frac{d\hat{\beta}_x}{d\hat{z}} + C_{00}^{-1} \times\int_0^{\hat{z}} d\hat{\zeta}C_{00}(\hat{\zeta})L_2(\hat{z},\hat{\zeta},\hat{\beta}_x,\hat{\beta}_{x,\zeta},\hat{\alpha}_x,\hat{\alpha}_{x,\zeta},\hat{\beta}_y,\hat{\beta}_{y,\zeta})\,, \qquad (5)$$

$$\frac{d\hat{\beta}_x}{d\hat{z}} = \frac{\int_0^{\hat{z}} d\hat{\zeta}C_{00}(\hat{\zeta})L_3(\hat{z},\hat{\zeta},\hat{\beta}_x,\hat{\beta}_{x,\zeta},\hat{\alpha}_x,\hat{\alpha}_{x,\zeta},\hat{\beta}_y,\hat{\beta}_{y,\zeta})}{C_{00}} \qquad (6)$$

and

$$\frac{d\hat{\beta}_y}{d\hat{z}} = \frac{\int_0^{\hat{z}} d\hat{\zeta}C_{00}(\hat{\zeta})L_4(\hat{z},\hat{\zeta},\hat{\beta}_x,\hat{\beta}_{x,\zeta},\hat{\alpha}_x,\hat{\alpha}_{x,\zeta},\hat{\beta}_y,\hat{\beta}_{y,\zeta})}{C_{00}}\,. \qquad (7)$$

In the above equations (and elsewhere), an extra subscript ζ assigned to a variable denotes a dependence upon the integration variable $\hat{\zeta}$, i.e. $\hat{\beta}_{x,\zeta} = \hat{\beta}_x(\hat{\zeta})$, $\hat{\alpha}_{x,\zeta} = \hat{\alpha}_x(\hat{\zeta})$ and $\hat{\beta}_{y,\zeta} = \hat{\beta}_y(\hat{\zeta})$ etc. The functions L_1-L_4 are given by

$$\begin{aligned}
L_1 &= -8iA^3\hat{\xi}\exp(-2iA\hat{\nu}\hat{\xi} - 2A^2(\hat{\sigma}_\delta^{ef})^2\hat{\xi}^2) \\
&\times(\hat{\beta}_{xr,\zeta}/\hat{\beta}_{xr})^{1/4}(\hat{\beta}_{yr,\zeta}/\hat{\beta}_{yr})^{1/4}(\hat{\bar{\beta}}_x/\hat{\bar{\beta}}_{x,\zeta})^{1/2} \\
&\times(\hat{\bar{\beta}}_y/\hat{\bar{\beta}}_{y,\zeta})^{1/2}G_{pp}^{-1/2}\hat{T}_2^{-1/2}\exp\left(\hat{T}_0\right)C_{pp}^{-1/2}\hat{Y}_2^{-1/2}\,, \\
L_2 &= (\hat{\beta}_{xr}/B_x)^{1/2}e^{iu_x}(\hat{T}_1/\hat{T}_2)L_1\,, \\
L_3 &= 2\hat{\beta}_{xr}e^{2iu_x}[\hat{T}_2^{-1}(1+\hat{T}_1^2/(2\hat{T}_2)) - 1]L_1\,, \\
L_4 &= 2\hat{\beta}_{yr}e^{2iu_y}(\hat{Y}_2^{-1} - 1)L_1\,,
\end{aligned} \qquad (8)$$

where $A = \rho_T k_u\beta_{ex}$, $\hat{\xi} = \hat{\zeta} - \hat{z}$, $\hat{\nu} = \Delta\nu/(2\rho_T)$, $\hat{\sigma}_\delta^{ef} = \sigma_\delta^{ef}/\rho_T$, $\hat{\bar{\beta}}_x = \hat{\beta}_x + i\hat{z}$, $\hat{\bar{\beta}}_y = \hat{\beta}_y + i(\beta_{ex}/\beta_{ey})\hat{z}$ and $B_x = k_r\sigma_x\sigma'_x$. Furthermore, $\hat{T}_1 = G_\theta + G_{\theta p}G_p/G_{pp}$, $\hat{T}_2 = (1/2)(G_{\theta\theta} - G_{\theta p}^2/G_{pp})$, $\hat{Y}_2 = (1/2)(C_{tt} - C_{tp}^2/C_{pp})$ and

$$\hat{T}_0 = \frac{G_p^2}{2G_{pp}} + \frac{\hat{T}_1^2}{4\hat{T}_2} - \frac{\hat{x}_c^2}{2R^2} - 2i\bar{p}_0\hat{\xi}\hat{x}_c - \frac{B_x}{2}\left\{\frac{\hat{\alpha}_{xi}^2}{\hat{\beta}_{xr}} + \frac{\hat{\alpha}_{xi,\zeta}^2}{\hat{\beta}_{xr,\zeta}} + \frac{(\hat{x}_c - \hat{\alpha}_x^*)^2}{\hat{\bar{\beta}}_x^*} + \frac{(\hat{x}_c - \hat{\alpha}_{x,\zeta})^2}{\hat{\bar{\beta}}_{x,\zeta}}\right\}, \qquad (9)$$

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Table 1: Undulator and Electron Beam Parameters

Parameter	LCLS	USR
Undulator parameter K_0	3.7	3.68
Undulator period λ_u	3 cm	2 cm
Beam energy $\gamma_0 m_0 c^2$	14.3 GeV	4.5 GeV
Resonant wavelength λ_r	0.15 nm	1 nm
Peak current I_p	3 kA	200 A
Energy spread σ_δ	10^{-4}	1.5×10^{-3}
Norm. emittance $\gamma_0\sigma_x\sigma'_x$	0.5 μm	0.0123 μm
Norm. emittance $\gamma_0\sigma_y\sigma'_y$	0.5 μm	1.23 μm
Horizontal size σ_x	23.1 μm	8.3 μm
Vertical size σ_y	30.2 μm	38.7 μm
FEL parameter ρ	5×10^{-4}	6×10^{-4}

where $R = \sigma_T/\sigma_x$, $\hat{x}_c = x_c/\sigma_x = \hat{\alpha}_{xr} + \hat{\alpha}_{xi}(\hat{z}+\hat{\beta}_{xi})/\hat{\beta}_{xr}$ and $\bar{p}_0 = k_u\beta_{ex}\sigma_x(C_p/\eta)$. The remaining quantities to be defined are

$$\begin{aligned}
G_{pp} &= 1 + \hat{z}_x^2/R^2 + iB_x\hat{\xi} + B_x\hat{\xi}^2/\hat{\hat{\beta}}_{x,\zeta}\,,\\
G_{\theta\theta} &= \hat{\hat{\beta}}_x/\hat{\beta}_{xr} + \tilde{w}_x^2[1/\hat{\hat{\beta}}_{x,\zeta} + (R^2B_x)^{-1}]\,,\\
G_{\theta p} &= \tilde{w}_x[\hat{z}_x/(R^2B_x^{1/2}) - B_x^{1/2}\hat{\xi}/\hat{\hat{\beta}}_{x,\zeta}]\,,\\
G_p &= \hat{x}_c\hat{z}_x/R^2 - B_x(\hat{x}_c - \hat{\alpha}_{x,\zeta})\hat{\xi}/\hat{\hat{\beta}}_{x,\zeta}\\
&\quad - 2i\hat{\xi}(\bar{Q}_1\hat{\xi}/2 + \bar{Q}_2\hat{z}_x)\,, \qquad (10)\\
G_\theta &= -\tilde{w}_x[B_x^{1/2}(\hat{x}_c - \hat{\alpha}_x^*)/\hat{\hat{\beta}}_x^* + \hat{x}_c/(R^2B_x^{1/2})\\
&\quad + B_x^{1/2}(\hat{x}_c - \hat{\alpha}_{x,\zeta})/\hat{\hat{\beta}}_{x,\zeta} + 2i\bar{p}_0\hat{\xi}/B_x^{1/2}]
\end{aligned}$$

where $\hat{z}_x = z_x/\beta_{ex}$, $\tilde{w}_x = \left|\hat{\hat{\beta}}_x\right|/\hat{\beta}_{xr}^{1/2}$, $\bar{Q}_1 = k_u\beta_{ex}^2\sigma'_x\bar{\alpha}$, $\bar{Q}_2 = k_u\beta_{ex}^2\sigma'_x(\eta\sigma_\delta^2/\sigma_T^2)$ and

$$\begin{aligned}
C_{pp} &= 1 + iB_y\bar{\xi} + B_y\sin^2\bar{\xi}/\hat{\hat{\beta}}_{y,\zeta}\,, \qquad (11)\\
C_{tt} &= \hat{\hat{\beta}}_y/\hat{\beta}_{yr} + \tilde{w}_y^2[B_y^{-1} + i\bar{\xi} + \cos^2\bar{\xi}/\hat{\hat{\beta}}_{y,\zeta}]\,,\\
C_{tp} &= -B_y^{1/2}(\tilde{w}_y/\hat{\hat{\beta}}_{y,\zeta})\cos\bar{\xi}\sin\bar{\xi}\,,
\end{aligned}$$

with $B_y = k_r\sigma_y\sigma'_y$, $\bar{\xi} = (\beta_{ex}/\beta_{ey})\hat{\xi}$ and $\tilde{w}_y = \left|\hat{\hat{\beta}}_y\right|/\hat{\beta}_{yr}^{1/2}$. Two important special cases are worth mentioning. The first one refers to the limit $\alpha, \eta \to 0$, which describes a conventional FEL with a flat-pole undulator and an undispersed beam. In this case, we have $\bar{p}_0 = 0$, $\bar{Q}_1 = \bar{Q}_2 = 0$, $R = 1$ and - provided that $\hat{\alpha}_x(0) = 0$ - the radiation profile remains on-axis, i.e. $\hat{\alpha}_x = 0$ so $\hat{x}_c = 0$, $G_p = G_\theta = 0$, $\hat{T}_0 = \hat{T}_1 = 0$ and $L_2 = 0$ (Eq. (5) then becomes trivial). The second case is a TGU-based FEL in which the horizontal emittance effect can be neglected. This scenario can be accommodated by the general equations given above if we set $G_{pp} = 1$ and $G_{\theta p} = G_p = 0$.

NUMERICAL RESULTS

In the previous section, we presented a semi-analytical, approximate method for solving the initial value problem of an asymmetric FEL in the linear regime. Here, we test this method by comparing numerical results derived from it with simulation data and the predictions of standard eigenmode analysis. In particular, we first consider the case of an FEL based on a conventional, LCLS-type undulator with a flat pole-face ($\bar{\alpha} = 0$) and no external focusing (Table 1). The driving electron beam is also characterized by LCLS-like parameters, namely a 14.3 GeV energy, a 3 kA peak current and a 0.5 μm normalized emittance in both x and y (of course, $\eta = 0$). The vertical natural focusing beta function $\beta_{ey} \approx \sqrt{2}\gamma_0/(K_0k_u)$ is approximately 51 m while - in the absence of horizontal focusing - we assume that the horizontal beta function attains a minimum value of $\beta_{ex} = 30$ m at $z = z_0 = 36$ m. For this configuration, we performed a steady-state GENESIS simulation [7] assuming a seed wavelength of 0.1500372 nm and an input Gaussian seed with a 9 m Rayleigh length and a waist located 6 m inside the undulator. At the same time, we derived the linearized solution by numerically solving Eqs. (4)-(7). Based on the seed parameters mentioned above, our initial values were $\hat{\beta}_x(0) = 0.3 - 0.2i$, $\hat{\alpha}_x(0) = 0$ and $\hat{\beta}_y(0) = 0.176 - 0.117i$. The comparison between the results obtained through the two approaches is shown in Figs. 1-2, where the FEL gain $G = \log(P/P_0)$ - P is the radiation power and P_0 its value at $z = 0$ - and the radiation beam size σ_r are plotted as functions of z. Since $P \propto \int d^2\mathbf{x}|E_\nu(\mathbf{x},z)|^2$, we find that $P = P_0|C_{00}(z)|^2$ if we select $C_{00}(0) = 1$. The analytical expression for σ_r - defined in the context of GENESIS as the square root of the average value of $x^2 + y^2$ - is $\sigma_r = (\sigma_{rx}^2 + \sigma_{ry}^2)^{1/2}$, where $\sigma_{rx} = w_x/2$ and $\sigma_{ry} = w_y/2$. Very good agreement is observed in the first 50 m of undulator, which is the approximate extent of the linear regime.

On the other hand, whenever the z-dependent effects introduced by the horizontal emittance can be disregarded, the solution of the initial value problem can be expressed in terms of the guided, FEL eigenmodes. This corresponds to

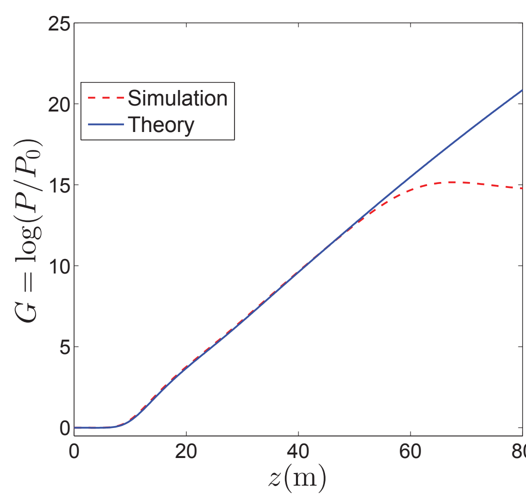

Figure 1: FEL gain as a function of z for the LCLS parameters (data from the linearized solution versus simulation results).

ISBN 978-3-95450-133-5

the previously mentioned model of a TGU-based FEL with $\sigma_x' = 0$ but $\sigma_y' \neq 0$. In this case, a single, Gaussian-like FEL mode typically dominates all the others in the high-gain part of the linear regime, i.e. $E_\nu(\mathbf{x}, z) \propto A_{00}(\mathbf{x})e^{i\mu_{00}z}$, where μ_{00} is the growth rate and $A_{00}(\mathbf{x})$ the transverse profile of the dominant, fundamental mode. As has been shown elsewhere [5], this fundamental growth rate can be accurately calculated through a variational technique. In view of this property, we should also expect the linearized solution to asymptotically converge into a guided mode. To facilitate a direct comparison with the results of the eigenmode formalism, we express the mode coefficient as $C_{00}(z) = \exp\left(i\int_0^z \mu(t)dt\right)$, where $\mu(z) = -iC_{00}^{-1}(dC_{00}/dz)$ is a z-dependent complex growth rate and compare the variational value of μ_{00} with the asymptotic value $\mu(\infty)$. For a specific example, we use the TGU parameters listed in Table 1, which refer to a concept based on the PEPX ultimate storage ring (USR). In this case, $\beta_{ey} \approx 11$ m and $\beta_{ex} = \sigma_x/\sigma_x' = 50$ m, though the latter quantity - like the horizontal emittance $\sigma_x\sigma_x'$ - is now merely a convenient scaling factor. We solve Eqs. (4)-(7) for $\eta = 3.5$ cm, $\bar{\alpha} = 1/\eta$, $\hat{\nu} = -0.15$, $\hat{\beta}_x(0) = 0.238+0.086i$, $\hat{\alpha}_x(0) = 0.167+0.992i$ and $\hat{\beta}_y(0) = 0.896 + 0.543i$. These initial values are selected so that the transverse profile of the input radiation field roughly matches that of the fundamental FEL mode. As is evident from Fig. 3, the z-dependent growth rate indeed attains a constant value in the high-gain portion of the linear regime. The same conclusion can be established for $\beta_x + iz$, $\beta_y + iz$ and α_x, so that ψ_{00} evolves into a guided mode with a z-invariant transverse profile. Lastly, we point out that our solution yields $\hat{\mu}(\infty) = \mu(\infty)/(2\rho_T k_u) = 0.398 - 0.312i$, which is very close to the variational result $\hat{\mu}_{00} = \mu_{00}/(2\rho_T k_u) = 0.394 - 0.308i$.

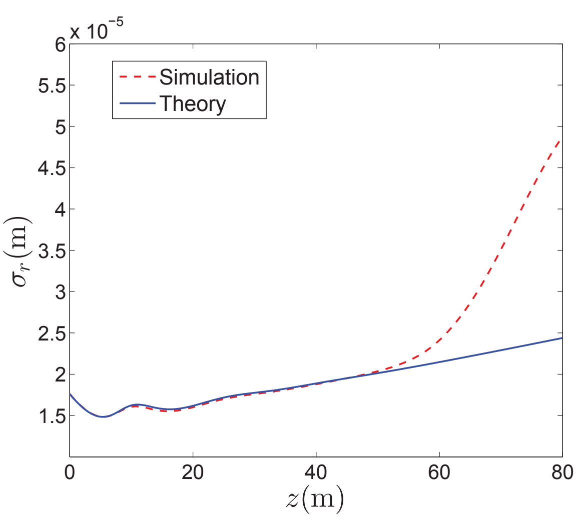

Figure 2: Radiation beam size as a function of z for the LCLS parameters (data from the linearized solution versus simulation results).

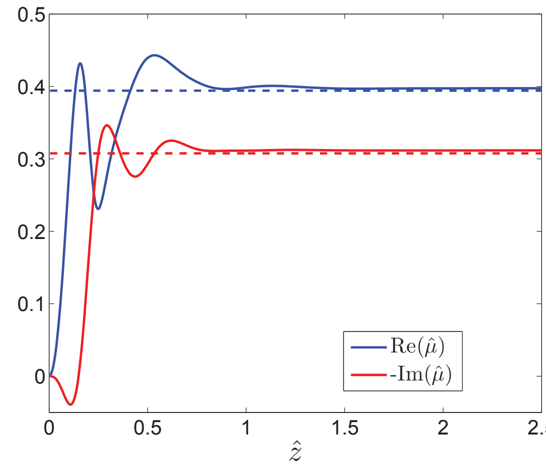

Figure 3: Real and imaginary part of the scaled growth rate $\hat{\mu} = \mu/(2\rho_T k_u)$ as a function of $\hat{z} = z/\beta_{ex}$ (USR parameters). The dashed lines correspond to the variational solution.

CONCLUSION

We have developed a technique for approximately solving the initial value problem of an asymmetric FEL in the linear regime of the interaction. Starting from a self-consistent, 3D evolution equation for the radiation amplitude, we constructed an approximation scheme in which the latter quantity can be adequately described by the fundamental (00) element in a basis of generalized Gauss-Hermite modes. In this way, the problem is ultimately reduced to a set of four integro-differential equations for a single mode coefficient and three basis parameters. Numerically solving this set yields results which provide a complete characterization of the radiation and are in good agreement both with simulation data and with the predictions of the eigenmode theory.

ACKNOWLEDGMENT

This work was supported by the Department of Energy Contract No. DE-AC02-76SF00515.

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