RELATIVISTIC EFFECTS IN MICRO-BUNCHING

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Abstract

In this paper we present our theoretical studies of limits on bunching using magnetic systems. We discuss the connection of this limit with plasma oscillations in electron beams and present simple formulae for an additional limit of micro-bunching amplification.

INTRODUCTION

Bunching and microbunching are very popular beam manipulation techniques. They are used (or planed to be used) for creating high peak current beams for X-ray FELs [1-4], for controlling or amplifying the shot noise in electron beams [5-12], or even using it for cooling of hadron beams [13-14].

Majority of these applications rely on magnetic bunchers providing time-of-flight dependence on particle energy, which is usually described by the R_{56} coefficient of the transport matrix. One underlying assumption in many of these papers is that the bunching parameters are limited by kinematics of the motion, e.g. by R_{56} and the beam energy spread. For example, there is no assumed limit on the maximum amplification of the shot noise (micro-bunching) in electron beam.

On other hand, it is known from plasma physics that in ballistic compression case the energy oscillates between the space charge and the kinetic energy of the particles. Specifically, it is shown in [15,16], placing an external point charge q into a cold plasma (an electron beam) or warm plasma with κ-2 velocity distribution [16] will cause oscillations of the screening charge with maximum value not exceed $-2q$. This limit does not depend on the value of R_{56} or beam energy spread. It also means that in such system the shot noise can not be amplified. Nevertheless, the micro-bunching using magnetic chicanes with gain exceeding unity was both predicted theoretically and demonstrated experimentally [5-12].

Hence, there are fundamental questions about the attainable bunching in electron beam:

(a) what energy is available to compensate for the space charge energy acquired during the process?

(b) what is the maximum attainable microbunching gain?

In this paper we show that this is purely relativistic effect and are given a simple answer on the maxim amplification for micro-bunching.

We are not considering here any dynamic issues related to bunching such as coherent synchrotron radiation and focus only on the fundamental limit.

STANDARD APPROACH

Standard approach used on modern theory of micro-bunch (see for example [11]) uses the bunching and energy modulation factors, defined as follows:

$$b_k = \frac{1}{N}\sum_{n=1}^{N} e^{-ikz_n};\mu_k = \frac{i}{N}\sum_{n=1}^{N}\eta_n e^{-ikz_n};\eta_n = \frac{\delta\gamma_n}{\gamma_o}, \quad (1)$$

to describe the microbunching process using matrix formalism [7]:

$$\begin{bmatrix} b_k(s_2) \\ \mu_k(s_2) \end{bmatrix} = R\left(s_1|s_2\right)\begin{bmatrix} b_k(s_1) \\ \mu_k(s_1) \end{bmatrix} \quad (2)$$

Propagation through a straight section is described as a simple (plasma) oscillation between the bunching and the energy modulation, which is a good approximation:

$$R\left(s_1|s_2\right) = \begin{bmatrix} \cos\varphi_p & -\alpha\sin\varphi_p \\ \alpha\sin\varphi_p & \cos\varphi_p \end{bmatrix}; \quad (3)$$

$$\varphi_p = k_p\left(s_2 - s_1\right);\ k_p = \sqrt{4\pi r_e n_e/\gamma^3};\ \alpha = k/\gamma^2 k_p.$$

The other, and much stronger approximation, is used for propagating the beam through a chicane (buncher) with longitudinal compaction factor, R_{56}, yielding in case of longitudinally cold e-beam [7]:

$$R_b = \begin{bmatrix} 1 & -kR_{56} \\ 0 & 1 \end{bmatrix}. \quad (4)$$

Taking into account Gaussian energy spread, yields an exponential suppression factor, which is well know from theory of optical klystron [17,18]:

$$R_b = \begin{bmatrix} 1 & -kR_{56}\cdot e^{-\frac{1}{2}\left(\frac{kR_{56}\sigma_\gamma}{\gamma}\right)^2} \\ 0 & 1 \end{bmatrix}. \quad (5)$$

formally limiting amplification to $g_{mn} \le \gamma/\sqrt{e}\sigma_\gamma$. With high quality e-beam having $\sigma_\gamma/\gamma \sim 10^{-3} \div 10^{-4}$, Eq. (5) predicts a possibility if very high microbunching gain. Hence, standards treatment assumes that the bunching amplification in a chicane is determined by the values of R_{56} and is limited by the relative energy spread in the

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beam. We will show that there is a fundamental limitation, which require reconsidering the use of Eqs. (4) and (5).

A SIMPLE CHICANE MODEL

Let's simplify the buncher into a chicane with short magnets, as illustrated in Fig.1. In this case interaction between particles and the bunching occurs in the legs of the chicane where their trajectories are straight. The later allows to simplify the model and to get an analytical expression for limits of attainable compression.

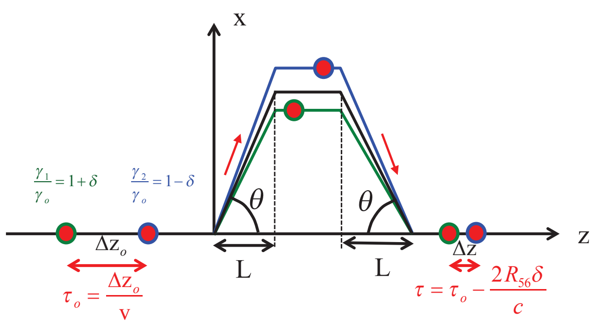

Figure 1: A simplified schematic of a chicane buncher with short dipole magnets, $R_{56} \cong L\theta^2$, where L is the length of each leg of the chicane. Some distances, sizes and angles are exaggerated for visibility: for example, in practice $\theta << 1$, $\Delta z <<< L$.

Let's also choose magnets with translation symmetry in x-direction whose the magnetic field can be represented by a simple one-component vector potential (see Fig. 2):

$$A_x(x,0,z) = \left\{ \begin{array}{c} A_o; \quad 0 < z < L \\ -A_o; \; L+l < z < 2L+l \\ 0; \quad z<0, \; z > 2L+l \\ L < z < L+l \end{array} \right\}; \; A_o = \frac{p_o c}{e}\sin\theta; \tag{6}$$

where we took into account that electron has negative charge $-e$. For the rest of the paper we are interested in plane trajectories and, unless specified otherwise, we assume y=0. Since the vector potential (and the Hamiltonian) does not depend on x, in the absence of other forces the x component of the canonical momentum is an invariant. It gives us a change of the horizontal momenta when electron passes each dipole

$$\begin{aligned} &P_x = p_x - \frac{e}{c}A_x = inv; \\ &p_x(0+\varepsilon) - p_x(0-\varepsilon) = +\frac{eA_o}{c} = p_o \sin\theta; \quad 0 < \varepsilon << L \\ &p_x(2L+l+\varepsilon) - p_x(2L+l-\varepsilon) = p_o \sin\theta. \end{aligned} \tag{7}$$

Our model uses a very short kicks and drifts whose length much longer than the distance between charges. Hence, we neglect the interaction of the charges at the corners of the trajectory. We intentionally the conservation law (7) at the boundaries of the vector potential dependence, with $0 < \varepsilon << L$, to identify the change imposed by the dipole magnet, and to separate it from the EM fields induced by beam itself. Specifically, Eq. (7) states, that the change of the horizontal momentum by a dipole does not depend neither on particle's energy nor its horizontal position.

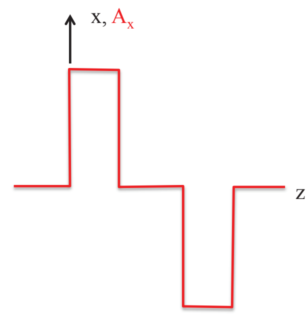

Figure 2: Sketch of beam trajectory and horizontal component of the vector potential of a simplified chicane.

We are using simple energy conservation law for deriving our limit. Naturally, we use the fact that magnetic field does not changes neither energy of the particle not the energy of the beam. It also applies to the total particle momentum. It means that changes of the horizontal momentum will cause corresponding change in the z-component of the momentum:

$$p_z = \sqrt{p^2 - p_x^2}. \tag{8}$$

Quite naturally, we focus on a second leg of chicane, where the bunch becomes shorter and the beam-induced field is increased.

CM ENERGY

To illustrate the method, we start from a simple model of two identical charged particles (electrons) interacting with each other. The idea is to find how close particles can approach each other (e.g. a minimum Δz) with a given energy deviation δ). In other words, when "space charge" stops "the bunching"?

The simplest way of finding this is use the center of mass system where the total momentum of two particles is zero and they approaching each other:

$$\pi_{1,2} = \pm\gamma_c\beta_c mc; \;\; \varepsilon_c = \gamma_c mc^2. \tag{9}$$

Then the total kinetic energy

$$E_k = 2\left(\varepsilon_c - mc^2\right) = 2\left(\gamma_c - 1\right)mc^2 \tag{10}$$

can be transferred into the potential energy, when electrons approach each other to the distance in c.m. frame of:

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$$r_f = \frac{r_e}{2(\gamma_c - 1)},\; r_e = \frac{e^2}{mc^2}, \tag{11}$$

and stop. Here we neglect initial electrostatic energy assuming $r_f << r_i$. The exact solution is

$$\frac{1}{r_f} = \frac{2(\gamma_c - 1)}{r_e} + \frac{1}{r_i} \to r_f = \frac{r_e}{2(\gamma_c - 1) + r_e / r_i}.$$

We note that since we are considering momentum along z-axis, this distance is compressed by factor γ_o.

Let's start from the case of ballistic compression, when the in c.m. two electrons have equal and opposite momenta along z-axis. Generally, their motion in c.m. is non-relativistic, but here we will use exact notations in order not to miss possible high order effects. In the lab frame boosted along the momenta of the electrons with $\gamma \equiv (1-\beta^2)^{-1/2}$ we have:

$$\begin{gathered} p_{1,2} = \gamma_o(\beta \pm \beta_c)mc;\; E_{1,2} = \gamma_o(1 \pm \beta\beta_c)mc^2; \\ \gamma_o = \gamma\gamma_c; \delta = \beta\beta_c;\; \beta_o^2 = \beta^2 + \beta_c^2 - \delta^2. \end{gathered} \tag{12}$$

Hence, the total energy available for "bunching" is

$$E_k = 2\frac{1-\sqrt{1-\delta^2/\beta^2}}{\sqrt{1-\delta^2/\beta^2}}mc^2 \tag{13}$$

which for small relative energy deviations $\delta << 1$, typical for example in microbunching, becomes:

$$E_k \cong \frac{\delta^2}{\beta^2}mc^2. \tag{14}$$

We note that for relativistic beams, which are of interest for this paper, $1-\beta << 1$, e.g. for $\delta \sim 10^{-3}$ we would have only about quarter of eV of kinetic energy per electron available for bunching. When the bunching is completed both electrons stop in the c.m. frame, and their energies in the lab-frame become equal:

$$p_{1,2} = \gamma\beta mc;\; E_{1,2} = \gamma mc^2;\; \delta = 0. \tag{15}$$

The "beam" of two electrons then looses some energy, which can be calculated by subtracting Eq. (15) from Eq. (12):

$$\Delta E = \gamma \cdot 2mc^2(\gamma_c - 1), \tag{16}$$

which is naturally equal to the energy of electrostatic field (potential energy) of electrons, Eq. (11), boosted into the lab-frame – hence the factor γ. Thus, as expected the total energy of the system is conserved.

Let's now consider how the chicane changes this situation. The processes in the both legs are similar to a degree, but bunching of interest is happening in the second leg where particles come closer in both the transverse and longitudinal directions. As we discussed in previous section, the translation invariance in x direction preserves the x-component of the Canonical momentum:

$$P_x = p_x - \frac{e}{c}A_x(z)$$

Let's start from the same initial conditions as in Eq. (12) and propagate particles through first dipole to get (using Eq. (8)):

$$\begin{gathered} \vec{p} = \hat{x}\frac{eA_o}{c} + \hat{z}\sqrt{p^2 - \left(\frac{eA_o}{c}\right)^2}; \\ \frac{\vec{p}_{1,2}}{mc} = \hat{x}\frac{eA_o}{mc^2} + \hat{z}\sqrt{\gamma_o^2(\beta \pm \beta_c)^2 - \left(\frac{eA_o}{mc^2}\right)^2}, \end{gathered} \tag{17}$$

with energies unchanged. The c.m. energy of the system is easiest to calculate using standard 4D product to the systems 4-momentum [19]:

$$\begin{gathered} E_{c.m.}^2 = c^2 p_i p^i = (p_1 + p_2)^2 = 2(mc^2)^2 + 2c^2 p_1 p_2; \\ c^2 p_1 p_2 = E_1 E_2 - c^2 \vec{p}_1 \vec{p}_2, \end{gathered} \tag{18}$$

where we used know 4D product [19] $p_1^2 = p_2^2 = m^2c^2$. Using Eqs. (12) and (17), $\gamma_o = \gamma\gamma_c$ and $\delta = \beta\beta_c$:

$$E_1 E_2 = (\gamma_o mc^2)^2(1-\delta^2);$$

$$\vec{p}_1\vec{p}_2 = (\gamma_o mc)^2 \left\{ \begin{gathered} \left(\frac{eA_o}{\gamma_o mc^2}\right)^2 + \sqrt{(\beta+\beta_c)^2 - \left(\frac{eA_o}{\gamma_o mc^2}\right)^2} \cdot \\ \sqrt{(\beta-\beta_c)^2 - \left(\frac{eA_o}{\gamma_o mc^2}\right)^2} \end{gathered} \right\};$$

yields final

$$\begin{gathered} \frac{E_1E_2 - c^2\vec{p}_1\vec{p}_2}{(\gamma_o mc^2)^2} = 1 - \delta^2 - a_o^2 - \sqrt{(\beta_o^2 + \delta^2 - a_o^2)^2 - 4\delta^2}; \\ E_{c.m.} = mc^2\sqrt{2 + 2\cdot\gamma_o^2\left(1 - \delta^2 - a_o^2 - \sqrt{(\beta_o^2 + \delta^2 - a_o^2)^2 - 4\delta^2}\right)}. \end{gathered} \tag{19}$$

where we used and introduced dimensionless vector potential (see (6)):

$$a_o = \frac{eA_o}{\gamma_o mc^2} = \beta_o \sin\theta. \tag{20}$$

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Expanding (19) with $\gamma_o >> 1$, $\delta^2 << 1$ to the leading order we obtain:

$$\frac{E_{c.m.}}{mc^2} \cong 2 + \delta^2\left(1+\gamma^2 tg^2\theta\right). \qquad (21)$$

with the kinetic energy available for the bunching of

$$\frac{E_k}{mc^2} = \frac{E_{c.m.}}{mc^2} - 2 \cong \delta^2\left(1+\gamma^2 tg^2\theta\right) \qquad (22)$$

For ballistic bunching with $\theta = 0$ we get already known result (14) $E_k \cong mc^2\delta^2$. In a chicane it is amplified by a factor of $1+\gamma^2 tg^2\theta \sim 1+(\gamma\theta)^2$. It is well known in accelerator theory is that factor $K=\gamma tg\theta$ is a measure of the relativity of the transverse motion:

$$\frac{p_x}{mc} = \gamma\beta\sin\theta \sim \gamma\theta.$$

For ultra-relativistic particles K-factor can be large even for modest bending angles. Hence, using a buncher with $K = \gamma\theta >> 1$ will boost by $(1+K^2)$ -fold the energy available to compensate the space charge potential energy accrued during the bunching.

Repeating similar calculations for an ensemble of particles gives an expected result:

$$E_k \cong \left(1+(\gamma\theta)^2\right)\sum_{n=1}^{N}\delta_n^2 mc^2, \qquad (23)$$

where δ_n is a relative energy deviation of nth particle. Thus, the available kinetic energy available in an ensemble of particles is also amplified by the same factor.

DISCUSIIONS AND CONCLUSIONS

From analogy with plasma oscillation during ballistic transport (where microbunching gain of shot noise can not exceed unity), one should expect that a buncher could provide a shot noise gain ~ K. Qualitatively this can be concluded from following observations:

(a) in case of ballistic motion after a half of a plasma oscillation any unshielded electric charge will be surrounded by a cloud of electron with about twice the external charge with the cloud size defined by Debye radius [15,16]. The potential energy of such cloud is naturally proportional to the square of its charge. At this moment all kinetic energy is exhausted and electrons starting moving in opposite direction. In terms of the bunching factor, it is the same a gain not exceeding unity;

(b) since the potential energy if a charge collected in a given pattern is proportional to the square of its value, means that its maximum value is proportional to the square root of the available kinetic energy;

(c) since the chicane amplifies the available kinetic energy (K^2+1) fold when compared with the ballistic motion, the charge which can be collected in the similar pattern could be $\sqrt{K^2+1} \sim K$ -fold larger than in a case of ballistic compression.

Hence, the conservation of the energy gives us a very straightforward way of limiting micro-bunching gain in a single chicane. First, we should notice that in the drift section the bunching is oscillating and "maximum gain" is simply equal 1. In the chicane, the available kinetic energy is amplified and hence the microbunching gain can exceed unity. Since the potential energy is proportional to the square of the bunching factor, the maximum micro-bunching gain (per chicane) must be limited by:

$$g_{\max} \le \sqrt{\left(1+(\gamma\theta)^2\right)} \sim \gamma\theta, \qquad (24)$$

We want to underline that this boost has pure relativistic nature and depends on the product of the Lorentz factor and the bending angle, but not on the value to R_{56}.

Overall, Eq. (24) represents an additional, frequently overlooked, limit on micro-bunching amplification.

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