HELICAL UNDULATOR RADIATION IN INTERNALLY COATED METALLIC PIPE*

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Abstract

The vacuum chambers of many advanced undulator sources are coated internally in order to reduce the impedance of the vacuum chamber or improve the vacuum performance. Although the impedances and radiation properties of the internally coated metallic pipes for straightforward moving charge are well studied, the peculiarities of the particles wiggling motion on the radiation characteristics in such structure are missed. In this paper we obtain exact expressions for the fields of a particle moving along a spiral path, as in the single-layer resistive as well as in the two-layer metallic waveguides, modelling NEG coating of the waveguide walls. Based on these results, it will be possible to obtain the necessary characteristics of the radiation of helical undulators, very close to reality. The solution is obtained as a superposition of a particular solution of inhomogeneous Maxwell's equations in a waveguide with perfectly conducting walls, and the solutions of the homogeneous Maxwell equations in the single-layer and double-layer resistive waveguides. Solution in the form of the multipole expansion for inhomogeneous Maxwell's equations for a waveguide with perfectly conducting walls, are also obtained in this study.

INTRODUCTION

We consider the spiral motion of a point charge in a resistive circular waveguide, modeling the charge motion in the vacuum chamber of a helical undulator. Helical undulator radiation, which has a number of important specific properties (axial symmetry of the distribution of the radiation power and its narrow directional and narrow band character) is widely used in modern synchrotron radiation sources [1,2].

The problem was considered earlier in the approximation of the spiral motion of the bunch in the free space [3] and in the presence of cylindrical [3-5] or rectangular [6] vacuum chamber with perfectly conducting walls. Limitation on the transverse dimensions of the vacuum chamber by the magnet poles causes a significant impact on the character of the chamber wall radiation. In connection with this it is necessary to take into account the finite conductivity of vacuum chamber walls, which is made in this paper. In this paper we consider also the case of a two-layer metal vacuum chamber, the presence of which, under certain conditions, causes the presence of narrow-band resonance of wakefield radiation in case of rectilinear motion of a particle [7].

In this paper, the method of expansion of the field in multipoles [8,9] is used, in contrast to the method of eigenfunction expansion of the waveguide, used previously to solve similar problems. The work examines the particle radiation, moving along a spiral trajectory in perfectly conducting cylindrical waveguide, in the resistive cylindrical waveguide and in the two-layer metal waveguide.

RADIATION IN A PERFECTLY CONDUCTING CIRCULAR WAVEGUIDE

Density distribution of charge and current in a cylindrical coordinate system in time domain are represented as follows:

$$\rho(z,r,\varphi,t)=\frac{1}{\sqrt{ra}}\delta(r-a)\delta(\varphi-\omega_b)\delta(z-vt),$$
$$j(z,r,\varphi,t)=\left(v\vec{e}_z+\omega_b a\,\vec{e}_\varphi\right)\rho(z,r,\varphi,t) \quad (1)$$

and in frequency domain:

$$\tilde{\rho}(\omega,r,\varphi,t)=\frac{1}{2\pi\sqrt{ra}}\delta(r-a)\exp\{j(z-vt+\varphi-jn\omega_b t)\} \quad (2)$$
$$\tilde{\vec{j}}(\omega,r,\varphi,t)=\left(v\vec{e}_z+\omega_b a\,\vec{e}_\varphi\right)\rho(\omega,r,\varphi,t),\ n=1,2,3,\dots .$$

Here a is the radius of the helical curve, ω_b the gyroscopic rotation frequency, v the longitudinal velocity of the particle, $\vec{e}_z,\vec{e}_\varphi$ the unit vectors of cylindrical coordinate system. Solutions for the radiation fields are searched using the pair of combinations of vector-valued functions [8,9]:

$$\vec{e}_{I,K}(\alpha)=\exp\{j(z-vt+\varphi-jn\omega_b t)\}\begin{Bmatrix} nI_n(\alpha r)/\alpha r, & jI'_n(\alpha r), & 0 \\ nK_n(\alpha r)/\alpha r, & jK_n(\alpha r), & 0 \end{Bmatrix} \quad (3)$$

Here I_n, K_n are the modified Bessel functions of the first and second kind, respectively and $\alpha=k/\gamma$, where γ is a Lorentz factor and $k=\omega/v$ with frequency ω.

With the help of these functions solutions of the inhomogeneous Maxwell's equations for the radiation fields in the waveguide with a perfectly-conducting walls can be constructed. The result is a combination of TM $(H_z\equiv 0)$ and TE $(E_z\equiv 0)$ waves:

$$\vec{E}^i=\vec{E}^{TM}+\vec{E}^{TE} \quad (4)$$

with

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$$\vec{E}^{TM}=\begin{cases}-jA_1Z_0k^{-1}\,rot\,\vec{e}_I(\alpha), & 0\le r\le a\\ -jA_2Z_0k^{-1}\,rot\,\{\vec{e}_K(\alpha)+u\vec{e}_I(\alpha)\}, & a\le r\le b\end{cases}$$

$$\vec{H}^{TM}=\begin{cases}A_1k^{-2}\,rot\,rot\,\vec{e}_I(\alpha), & 0\le r\le a\\ A_2k^{-2}\,rot\,rot\,\{\vec{e}_K(\alpha)+u\vec{e}_I(\alpha)\}, & a\le r\le b\end{cases}$$

$$u=-K_n(\alpha a)/I_n(\alpha a)$$

$$\vec{E}^{TE}=\begin{cases}-jB_1Z_0\,\vec{e}_I(\alpha), & 0\le r\le a\\ -jB_2Z_0\,\{\vec{e}_K(\alpha)+u\vec{e}_I(\alpha)\}, & a\le r\le b\end{cases}$$

$$\vec{H}^{TE}=\begin{cases}B_1k^{-1}\,rot\,\vec{e}_I(\alpha), & 0\le r\le a\\ B_2k^{-1}\,rot\,\{\vec{e}_K(\alpha)+u\vec{e}_I(\alpha)\}, & a\le r\le b\end{cases} \qquad (5)$$

$$u=-K_n(\alpha a)/I'_n(\alpha a)$$

Coefficients u are determined from the condition of the zero-equal tangential component of the electric field on the waveguide surface: $E_z=E_\varphi=0$ at $r=a$ (a is a waveguide radius). The case $u=0$ corresponds to the free space. Amplitudes $A_{1,2}$ and $B_{1,2}$ are determined from the condition of continuity of the magnetic components H_r^{TM} and H_r^{TE} at $r=a$ and from the discontinuity of component H_φ^{TM} and H_z^{TE}, equal to $gv/2\pi$ and $g\omega_b a/2\pi$, (g is the normalization factor) respectively, for the same value of radius,

$$A_1=-j\exp\{-jn\omega_b t\}\frac{g\alpha v}{\pi\beta^2}\left(K_n(\alpha a)+uI_n(\alpha a)\right),$$

$$A_2=-j\exp\{-jn\omega_b t\}\frac{g\alpha v}{\pi\beta^2}I_n(\alpha a), \qquad (6)$$

$$B_1=j\exp\{-jn\omega_b t\}\frac{gka\omega_b}{\pi}\left(K'_n(\alpha a)+uI'_n(\alpha a)\right),$$

$$B_2=j\exp\{-jn\omega_b t\}\frac{gka\omega_b}{\pi}I'_n(\alpha a).$$

Thus, the longitudinal component of the current generates TM wave, while the transverse ones generates TE mode.

RADIATION IN RESISTIVE-WALL CIRCULAR WAVEGUIDE

The electromagnetic properties of the metal walls of the waveguide are characterized by the metal conductivity σ and its dielectric permittivity $\varepsilon=\varepsilon_0-j\sigma/\omega$. The radius of the waveguide is still b, and the wall thickness is considered infinite. The solution is sought in the form of a sum of the particular solution $\vec{E}^i$ of inhomogeneous Maxwell equations (4)-(6), obtained for a waveguide with perfectly conducting walls, and the general solution $\vec{E}^r$ of the homogeneous Maxwell equations for the resistive-wall waveguide:

$$\vec{E}=\vec{E}^r+\vec{E}^i \qquad (7)$$

Partial solutions for the internal region within the waveguide $(0\le r\le b)$ and in its walls $(b\le r<\infty)$ can be written as follows:

for $0\le r\le b$:

$$\vec{E}=C_1\vec{E}_0^{TM}+C_2\vec{E}_0^{TE}+\vec{E}^i,\quad \vec{H}=C_1\vec{H}_0^{TM}+C_2\vec{H}_0^{TE}+\vec{H}^i \qquad (8)$$

where

$$\vec{E}_0^{TM}=-jZ_0k^{-1}\,rot\,\vec{e}_I(\alpha),\ \vec{E}_0^{TE}=-jZ_0\,\vec{e}_I(\alpha),$$

$$\vec{H}_0^{TM}=k^{-2}\,rot\,rot\,\vec{e}_I(\alpha),\ \vec{H}^{TE}=k^{-1}\,rot\,\vec{e}_I(\alpha)\ . \qquad (9)$$

For $b\le r<\infty$:

$$\vec{E}=D_1\vec{E}_1^{TM}+D_2\vec{E}_1^{TE},\ \vec{H}=D_1\vec{H}_1^{TM}+D_2\vec{H}_1^{TE^i}, \qquad (10)$$

where

$$\vec{E}_1^{TM}=-jZ_0k^{-1}\,rot\,\vec{e}_K(\delta),\ \vec{E}_1^{TE}=-jZ_0\,\vec{e}_K(\delta),$$

$$\vec{H}_1^{TM}=k^{-2}\,rot\,rot\,\vec{e}_K(\delta),\ \vec{H}_1^{TE}=k^{-1}\,rot\,\vec{e}_K(\delta). \qquad (11)$$

Here $\delta=\alpha^2-j\omega\sigma\mu_0$ with μ_0 magnetic permeability of vacuum.

Amplitudes $C_{1,2}$ and $D_{1,2}$ are determined from the condition of continuity of the tangential component of the electric and magnetic fields (8) and (10) on the boundary between two media (at $r=b$).

We write down the value of the tangential component of the radiation fields in an ideal waveguide on the boundary between two media (at $r=b$):

$$E_\varphi^i=E_z^i=0,\ H_\varphi^i=-j\frac{\beta^2A_2}{\alpha bI_n(\alpha b)}+j\frac{nB_2}{\alpha^2b^2I'_n(\alpha b)},$$

$$H_z^i=j\frac{B_2}{bkI'_n(\alpha b)} \qquad (12)$$

In matrix form, the system of equations for amplitudes written as follows:

$$\begin{Bmatrix}\frac{Z_0nI_n}{\alpha b} & Z_0I'_n & -\frac{Z_0nK_n}{\delta b} & -Z_0K'_n\\ \frac{Z_0I_n\alpha}{k} & 0 & -\frac{Z_0K_n\delta}{k} & 0\\ j\frac{(k^2-\alpha^2)I'_n}{k^2} & j\frac{nI_n}{\alpha b} & j\frac{(k^2-\delta^2)K'_n}{k^2} & -j\frac{nK_n}{\delta b}\\ 0 & j\frac{\alpha I_n}{k} & 0 & -j\frac{\delta K_n}{k}\end{Bmatrix}\bullet\begin{Bmatrix}C_1\\C_2\\D_1\\D_2\end{Bmatrix}=\begin{Bmatrix}0\\0\\-H_{\varphi,b}^i\\-H_{z,b}^i\end{Bmatrix} \qquad (13)$$

Here $I_n=I_n(\alpha b), K_n=K_n(\delta b), I'_n=I'_n(\alpha b), K'_n=K'_n(\delta b)$.

Thus, the problem reduces to solving a linear system of four algebraic equations with four unknown amplitudes.

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RADIATION IN THE TWO LAYER RESISTIVE-WALL CIRCULAR WAVEGUIDE

Here we consider the spiral motion of a particle in a two-layer circular waveguide. Ideally conducting waveguide walls are coated from inside with a non-magnetic metal layer. The inner radius of the waveguide and electromagnetic parameters of the inner coating are the same as in the previous case. An additional parameter $b_1 = b + d$ is the outer radius of the inner coating, where d is its thickness. Solution is sought, as before, by the method of partial waves. In this case, there are two boundaries between media: at $r = b$ and $r = b_1$.

Field inside the waveguide (at $0 \le r \le b$, as in the previous case, is defined by expressions (6) - (9), while the field inside the metal coating $(b \le r \le b_1)$, in addition to the modified Bessel functions of the second kind $K_n(\delta r)$ also contains the modified Bessel functions of the first kind $K_n(\delta r)$:

$$\vec{E}' = D_1 \vec{E}_1^{TM} + D_2 \vec{E}_1^{TE} + F_1 \vec{E}_2^{TM} + F_2 \vec{E}_2^{TE},$$
$$\vec{H}' = D_1 \vec{H}_1^{TM} + D_2 \vec{H}_1^{TE} + F_1 \vec{H}_2^{TM} + F_2 \vec{H}_2^{TE^i}, \qquad (14)$$

where

$$\vec{E}_2^{TM} = -jZ_0 k^{-1}\, rot\, \vec{e}_I(\delta),\; \vec{E}_2^{TE} = -jZ_0\, \vec{e}_I(\delta),$$
$$\vec{H}_2^{TM} = k^{-2}\, rot\, rot\, \vec{e}_K(\delta),\; \vec{H}_2^{TE} = k^{-1}\, rot\, \vec{e}_I(\delta). \qquad (15)$$

Six equations for the amplitudes $C_{1,2}$, $D_{1,2}$ and $F_{1,2}$ determined from the matching conditions of the tangential electric and magnetic components of fields (8) and (14) on the vacuum-metal boundary $(r = b)$ and the condition of zero-equal tangential electric field components (14) at the metal-perfect conductor boundary $(r = b_1)$:

$$E_\varphi = E'_\varphi,\, E_z = E'_z,\quad H_\varphi = H'_\varphi,\, H_z = H'_z \quad at\; r = b,$$
$$E'_\varphi = 0,\, E'_z = 0,\quad at\; r = b_1. \qquad (16)$$

In matrix form, the system of equations for amplitudes may be written as follows:

$$\hat{D}\cdot\hat{X} = \hat{A} \qquad (17)$$

where

$$\hat{D} = \begin{Bmatrix} \frac{Z_0 n I_n(\alpha b)}{\alpha b} & Z_0 I'_n(\alpha b) & \frac{Z_0 n K_n(\delta b)}{\delta b} & -Z_0 K'_n(\delta b) & -\frac{Z_0 n I_n(\delta b)}{\delta b} & -Z_0 I'_n(\delta b) \\ \frac{Z_0 \alpha I_n(\alpha b)}{k} & 0 & -\frac{Z_0 \delta K_n(\delta b)}{k} & 0 & -\frac{Z_0 \delta I_n(\delta b)}{k} & 0 \\ j\frac{(k^2-\alpha^2) I'_n(\alpha b)}{k^2} & j\frac{n I_n(\alpha b)}{\alpha b} & -j\frac{(k^2-\alpha^2) K'_n(\alpha b)}{k^2} & -j\frac{n K_n(\delta b)}{\delta b} & -j\frac{(k^2-\alpha^2) I'_n(\alpha b)}{k^2} & -j\frac{n I_n(\delta b)}{\delta b} \\ 0 & j\frac{\alpha I_n(\alpha b)}{k} & 0 & -j\frac{\delta K_n(\delta b)}{k} & 0 & -j\frac{\delta I_n(\delta b)}{k} \\ 0 & 0 & \frac{Z_0 n K_n(\delta b_1)}{\delta b_1} & Z_0 K'_n(\delta b_1) & \frac{Z_0 n I_n(\delta b_1)}{\delta b_1} & Z_0 I'_n(\delta b_1) \\ 0 & 0 & \frac{Z_0 \delta K_n(\delta b_1)}{k} & 0 & \frac{Z_0 \delta I_n(\delta b_1)}{k} & 0 \end{Bmatrix} \qquad (18)$$

and two single-column matrixes $\hat{X}$ and $\hat{A}$:

$$\hat{X} = \{C_1,\; C_2,\; D_1,\; D_2,\; F_1,\; F_2\}$$
$$\hat{A} = \{0,\; 0,\; -H^i_{\varphi,b},\; -H^i_{z,b},\; 0,\; 0\} \qquad (19)$$

CONCLUSION

Expressions are obtained for the radiation fields in the ideal-conductive, resistive, and double-layer metal waveguides for a charged particle moving along a spiral trajectory coaxial with the axis of the waveguide.

The form in which the solutions are obtained (superposition of solutions for inhomogeneous and homogeneous Maxwell equations) allows ones to select part of the radiation due to the resistivity of the walls of the waveguide, and to investigate its effect on the undulator radiation.

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