PARTICLE DENSITY EFFECTS IN THE TRANSITION RADIATION ENERGY SPECTRUM: THEORY AND EXPERIMENTAL INVESTIGATION AT PSI

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Abstract

The spectral and angular distribution of the radiation intensity by a single and individually radiating electron is in principle different from what expected from a high density electron beam. The beam particle density modifies via a charge form factor the angular and spectral distributions characterizing the radiation emission by a single electron. In particular, under high energy and high particle density conditions, the Transition Radiation (TR) energy spectrum by an electron beam is expected to be affected by the electron-transverse-density that at very short wavelength – even in the visible, in principle – can influence the number and the angle of photons emitted at a given wavelength (brightness increase with density). The investigation of such a phenomenon is relevant to beam diagnostics and to understand the bunch collective effects influencing TR emission. The status of the experimental investigation of the beam-transverse-size effects in the Optical Transition Radiation (OTR) at SLS and, in perspective, at the SwissFEL [1] will be presented and the main formal aspects of the model predicting them will be described.

INTRODUCTION

Particle density effects are commonly affecting the electromagnetic radiation emission by a relativistic electron beam. Compared to a single and individually radiating electron, both the number and the angular distribution of the photons emitted by an electron bunch at a given wavelength may be influenced by collective effects due to the bunch-particle-density. They can manifest themselves as a dramatic threshold increase of the radiation intensity in a quadratic proportion to the number of the radiating particles as the observed wavelength becomes comparable to the electron bunch length and/or as a smoother diffractive modification of the spectral and angular distribution of the radiation intensity as a function of the ratio of the beam transverse size to the observed wavelength. The investigation of particle-density modifications of the radiation emission is promising for an insight into the role of the bunch collective effects and for possible outcomes in the analysis of the experimental results. In particular, in the case of the Transition Radiation (TR) – emitted as a charge crosses a dielectric interface – [2, 3] it is worthy to investigate the role played by the particle density in the radiation emission as the observed wavelength is shorter than the longitudinal length of the charged beam, i.e., not sufficiently long to temporally resolve the duration and the temporal shape of the photon pulse but sufficiently short to spatially resolve the effect of the finite transverse dimension of the charged beam on the angular and spectral distribution of the radiation intensity. The case of an electron bunch colliding at a normal angle of incidence with a metallic screen, which, for convenience, is supposed to behave as an ideal conductor in the wavelength region of interest, is in the following considered. In the case of the TR by a bunch consisting of N electrons, under observation conditions of temporal incoherence, current theoretical predictions foresee that the radiation energy spectrum is simply equal to N times the radiation energy spectrum by a single and individually radiating electron. According to such a theoretical prediction, neither variations in the number of the TR photons emitted by the electron bunch at a given wavelength shorter than the electron bunch length nor modifications of their angular distribution are expected to be observed as a function of the relative distance of the electrons in the transverse plane even under very high density and energy conditions of the beam. The electromagnetic field produced by a relativistic high density electron beam is, in principle, a particle-density dependent function as well as the 4-potential, whose propagation equations in the space-time depend on the charge density and current density vector of the beam. The TR field by an electron beam being in turn a function of the virtual quanta field - i.e., the quasi-plane-wave approximation of the electromagnetic field traveling with the charged beam - has to show somehow the bunch-particle-density imprinting that also characterizes the primitive electromagnetic field of the electron bunch. As long as the density and the energy of the charged beam are sufficiently high so that the angular and spectral distribution of the TR intensity is sensitive - in a given wavelength band - to particle-density effects, the TR energy spectrum has somehow to depend on the charge form factor. Whatever is the charge form factor definition, the formulation of the TR energy spectrum by a charged beam is expected to evolve from the point-charge-like covariance - characterizing the radiation energy spectrum by a single and individually radiating electron - to the charge-density-like covariance characterizing instead the electromagnetic field produced by a high density charged distribution. Besides the charge-density-like covariance constraint, the TR energy spectrum by an electron beam is expected to fulfil as well the temporal causality principle that constrains the relative phase of the N single electron field amplitudes composing the

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radiation field to the temporal sequence of the N particle collisions with the metallic screen.

Diffractive effects due to the finite transverse size of the electron beam can be predicted to affect the TR energy spectrum at very short wavelengths [4, 5, 6]. Provided that the beam energy and particle density are sufficiently high, beam-transverse-size dependent diffractive modification of the angular and spectral distribution of the TR intensity are expected to be observed at a very short wavelength, in particular, in the visible optical region. With the decrease of the beam transverse size, an increase of the number of photons emitted at a given wavelength and a broadening of their angular distribution can be foreseen at a wavelength even shorter than the electron bunch length.

The investigation of beam-transverse-size effects in the TR and, in particular, in the Optical Transition Radiation (OTR) is relevant to the general comprehension of the bunch collective effects in the radiative phenomena as well as to the beam diagnostics. In fact, beam diagnostics of fourth generation light source needs to monitor high energy and low emittance electron beams - normalized emittance of about 0.4π mm $mrad$ and energy up to almost 6 GeV are foreseen at the SwissFEL [1, 7, 8] - and has to face the problem to resolve beam transverse sizes even smaller than 10 μm by imaging onto a CCD camera an OTR photon flux that, in the space of the electron spatial coordinates, concentrates in few camera pixels. Thanks to the diffractive nature, beam-transverse-size effects in OTR would make possible to move the monitoring of the beam transverse size from the narrow-space of the electron transverse coordinates (x, y) to the broad-space of the conjugate Fourier wave-number coordinates (k_x, k_y), where the angular distribution of the TR can be observed [6].

VIRTUAL QUANTA AND TR

Diffractive modifications of the radiation energy spectrum due to the finite transverse size of an electron beam can be observed under conditions of temporal incoherence in a particle accelerator. The Luminosity of a positron-electron collider can be calibrated by measuring the counting rate of bremsstrahlung photons emitted by the colliding beams as a function of the beam transverse size. In an electron storage ring, the Brilliance of a synchrotron radiation source can be improved by optimizing the transverse emittance and, in particular, the transverse size of the beam. In an electron storage ring, the spectral components of the synchrotron radiation show a diffractive broadening of the angular distribution depending on the observed wavelength and scaling down with it. The transverse size of the electron beam in a storage ring can be monitored in the visible-UV by imaging onto a CCD camera the interference of a π-polarized spectral component of the synchrotron radiation field [9]. Taking into account the relativistic nature of TR emission and the phenomenological indications coming from the general context of the electromagnetic radiative mechanisms by relativistic electron beams, the study of the beam-transverse-size effects in TR emission and the experimental investigation of the beam density and energy conditions to appreciably observe them - in particular, in the optical visible range - appear promising and useful for the design of the SwissFEL diagnostics and for a correct interpretation of the experimental outcomes.

A relativistic charge in a rectilinear and uniform motion can induce a radiation emission in a medium as the polarization vector experiences a discontinuity, for instance, crossing the vacuum-metal interface. The fast dipolar oscillation of the charge induced on the dielectric interface by the incident relativistic charge generates indeed an instantaneous, broad wavelength band, radially polarized and double-directional radiation emission, the so called transition radiation [2, 3]. The TR photon flux is emitted from the metallic surface within a narrow cone whose aperture scales down with the inverse of the Lorentz γ factor of the charge ($\gamma = E/mc^2$). In the following, the case of a N electron bunch colliding at a normal angle of incidence with a flat metallic surface S, having arbitrary shape and size (either finite $S < \infty$ or infinite $S = \infty$) and being located in the plane $z = 0$ of the laboratory reference frame, will be described. Moreover, the metallic surface will be supposed to behave as an ideal conductor up to and beyond the visible optical range as in the case of a radiator surface made of a thin evaporated Aluminium film onto a dielectric substrate. Finally, the N electrons are supposed to move in vacuum in a rectilinear and uniform motion with a common velocity $\vec{w} = (0, 0, w)$ along the z-axis of the laboratory reference frame. This assumption sounds reasonable taking into account that the TR photon pulse is emitted on a temporal scale that is practically instantaneous with respect to the time scale of the phenomena characterizing the internal motion of the electrons in the bunch, such as the synchrotron motion. With reference to such an experimental scenario, the TR emission can be schematized as the collision of a relativistic beam with a double layer of charge at rest in the laboratory reference frame with respect to the direction of incidence of the beam. Furthermore, taking into account that the collisions between the two charged distribution is accompanied by the emission of backward radiation with respect to the direction of motion of the incident charge, from the point of view of the kinematics, the TR mechanism can be assimilated to a photon bremsstrahlung emission resulting from the head-on collision of two charged distribution as viewed in the rest reference frame of one of them. As the relativistic charge approaches the metallic surface, the induced conduction electrons displace in the transverse plane in order to maintain it equipotential so originating, in the double layer of charge, the fast dipolar oscillation that is responsible for the double-conical emission of backward and forward TR. Taking into account the boundary condition for the transverse component of the electric field on the radiator surface, the TR field from the radiator surface can be obtained as the wave propagation of the virtual quanta scattered by the radiator surface according to the Huygens-

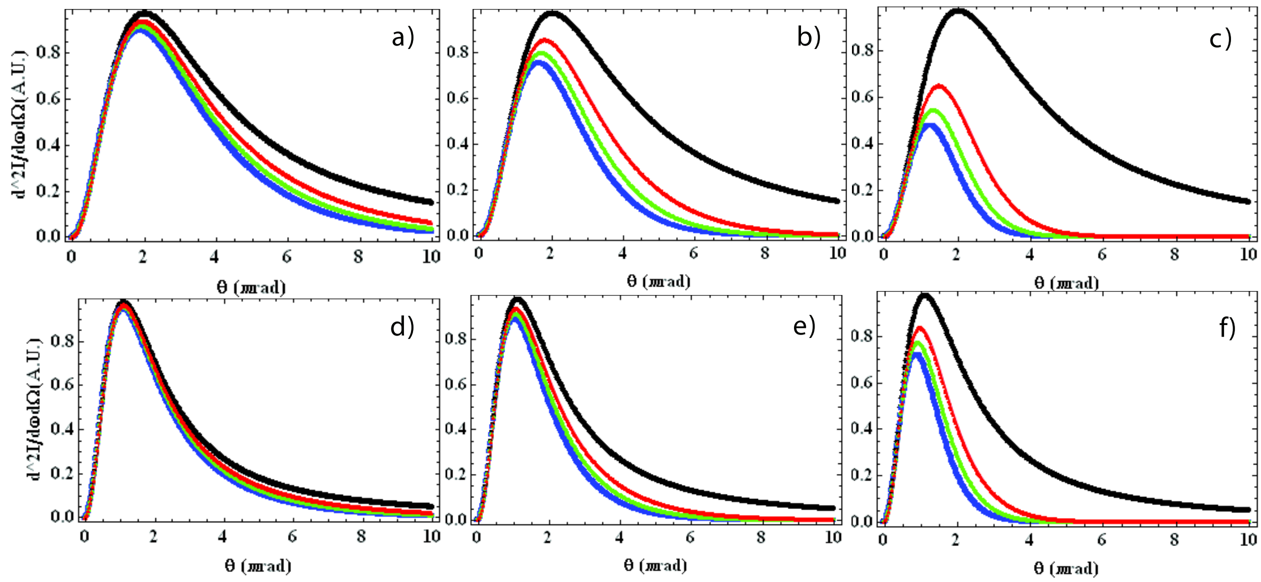

Figure 1: Normalized angular distribution of TR spectral intensity at wavelength 460 nm (Blue curve), 530 nm (Green curve) and 680 nm (Red curve), calculated for beam energy 250 MeV (a,b,c) and 450 MeV (d,e,f) and beam transverse size $\sigma = 10\ \mu m$ (a,d), $\sigma = 20\ \mu m$ (b,e), $\sigma = 40\ \mu m$ (c,f), are compared to the case $\sigma = 0$ (Black curve).

Fresnel principle [3, 10]. At a distance R from the radiator surface $z = 0$, the harmonic component $E^{tr}_{x,y}(\vec{\kappa},\omega)$ of the TR field can be expressed in terms of the corresponding harmonic component of the virtual quanta field $E^{vq}_{x,y}(x, y, z = 0,\omega)$ by solving the Helmholtz-Kirchhoff integral theorem. Under far field approximation, this integral reads as the Fourier transformation of the virtual quanta field with respect to spatial coordinates $\vec{\rho} = (x, y)$ of the radiator surface S [3, 4, 5]:

$$E^{tr}_{x,y}(\vec{\kappa},\omega) = \frac{k}{2\pi R}\int_S d\vec{\rho}\, E^{vq}_{x,y}(\vec{\rho},\omega)e^{-i\vec{\kappa}\cdot\vec{\rho}} \quad (1)$$

where $k = \omega/c$ is the wave-number and $\vec{\kappa} = (k_x, k_y)$ is the transverse component of it. In the case of a N electron bunch with spatial coordinates $\vec{r}_{0j} = (x_{0j}, y_{0j}, z_{0j})$ at a given time t_0 ($j = 1,..,N$) in a rectilinear and uniform motion with velocity $\vec{w} = (0, 0, w)$, the expression of the virtual quanta field [4, 5] at the radiator surface ($z = 0$) reads

$$E^{vq}_{x,y}(x, y, z = 0,\omega) = \frac{ie}{w\pi}\sum_{j=1}^{N} e^{-i(\omega/w)z_{0j}} \times \times \int d\vec{\tau}\, e^{i\vec{\tau}\cdot\vec{\rho}}\,\frac{\tau_{x,y}\, e^{-i\vec{\tau}\cdot\vec{\rho}_{0j}}}{\tau^2+\alpha^2} \quad (2)$$

where $\alpha = \frac{\omega}{w\gamma}$, $\vec{\rho}_{0j} = (x_{0j}, y_{0j})$ and $\vec{\tau} = (\tau_x, \tau_y)$ are the conjugate Fourier coordinates of the radiator surface $\vec{\rho} = (x, y)$. Eq.(2) follows from the Fourier transform of the electric field by a N electron bunch [4, 5, 6]:

$$\vec{E}(\vec{k},\omega) = -i(8\pi^2 e)\frac{[\vec{k} - (\omega\vec{w}/c^2)]}{[k^2 - (\omega/c)^2]} \times \times \left(\sum_{j=1}^{N} e^{-i\vec{k}\cdot\vec{r}_{0j}}\right)\delta(\omega - \vec{w}\cdot\vec{k}). \quad (3)$$

With reference to Eqs.(1,2), the formal expression of the TR field by a N electron bunch reads:

$$E^{tr}_{x,y}(\vec{\kappa},\omega) = \sum_{j=1}^{N} H_{x,y}(\vec{\kappa},\omega,\vec{\rho}_{0j})\, e^{-i(\omega/w)z_{0j}} \quad (4)$$

where

$$H_{\mu,j} = \frac{iek}{2\pi^2 Rw}\int_S d\vec{\rho}\int d\vec{\tau}\frac{\tau_\mu e^{-i\vec{\tau}\cdot\vec{\rho}_{0j}}}{\tau^2+\alpha^2}e^{i(\vec{\tau}-\vec{\kappa})\cdot\vec{\rho}} \quad (5)$$

with $H_{\mu,j} = H_\mu(\vec{\kappa},\omega,\vec{\rho}_{0j})$ and $\mu = x, y$. Finally, with reference to Eq.(4,5), from the flux of the Poynting vector the formal expression of the TR energy spectrum in the most general case of a radiator surface S having arbitrary size and shape follows:

$$\frac{d^2 I}{d\Omega d\omega} = \frac{cR^2}{4\pi^2}\sum_{\mu=x,y}\Big(\sum_{j=1}^{N}|H_{\mu,j}|^2 + + \sum_{j,l(j\neq l)=1}^{N} e^{-i(\omega/w)z_{0j}}e^{i(\omega/w)z_{0l}}H_{\mu,j}H^*_{\mu,l}\Big) \quad (6)$$

Some comments about the results above. (1) The formulation of the Fourier transform of the electromagnetic field by a N electron bunch - Eq.(3) - is covariant as well as covariant-consistent are the formal steps - Eqs.(1,2,4,5,6) - leading from the virtual quanta field to the radiation energy spectrum, more details in [11]. (2) The virtual quanta field - Eq.(2) - shows a field structure as a train of N transverse traveling waves hitting the metallic surface with a relative phase delay only dependent on the difference between the longitudinal coordinates of the N electrons z_{0j} ($j = 1,..,N$). The TR field - Eqs.(4,5) - results from the linear addition of the N single electron field amplitudes $H_{\mu,j}$ - complex quantities, in principle - whose relative phase $e^{-i(\omega/w)z_{0j}}$ only depends on the electron longitudinal coordinates z_{0j} $(j = 1, .., N)$. The formal structure of the virtual quanta and TR fields - Eqs.(2,4,5) - and, consequently, of the TR energy spectrum - Eq.(6) - are temporal-causality consistent. In fact, in the considered experimental context, where all the electrons move with the same velocity along the z-axis of the reference frame, the temporal sequence of the particle collisions with the metallic surface S - located in the plane $z = 0$ - is only ruled by the distribution of the N longitudinal coordinates z_{0j} $(j = 1, .., N)$. Consequently, on the basis of the temporal causality principle, the N single electron radiation field amplitudes - Eqs.(2,4,5) - composing the TR field are expected to be emitted by the metallic surface according to a relative phase delay only dependent on the difference of the electron coordinates z_{0j} $(j = 1, .., N)$. (3) The relative phase structure characterizing the N single electron field amplitudes $H_{\mu,j}$ in the radiation field - Eqs.(4,5) - has to satisfy the temporal causality constraint whether the radiator surface is finite ($S < \infty$) or infinite ($S = \infty$). For such a purpose, the single electron field amplitude $H_{\mu,j}$ - Eq.(5) - is represented according to an implicit integral form as a sort of special function depending on the radiator surface S that is not fixed *a priori*. Consequently, the formal expression of the resultant TR energy - Eq.(6) - is invariant and temporal-causality-consistent whatever is the size and shape of the radiator surface S.

Eq.(6) represents the TR energy spectrum by a N electron bunch whose spatial coordinates (x_j, y_j, z_j) $(j = 1, .., N)$ follows the primitive discrete distribution function $\rho(\vec{r}, t) = \sum_{j=1}^{N} \delta(\vec{r} - \vec{r}_j)$. Under the hypothesis that the beam particle density and energy are sufficiently high so that, in the wavelength range of interest, the single particle virtual quanta field shows an appreciable spatial overlapping in the transverse plane with the corresponding field of an adjacent electron, it can be reasonable to guess that the electron beam can be described in terms of the continuous limit of the primitive discrete distribution function of the N electron coordinates [5]. Provided that the results and the physical implications of Eq.(6) have to remain valid and invariant passing from the primitive spatial distribution of the electron coordinates to the continuous limit approximation of it, the continuous limit extension of the TR energy spectrum by a N electron beam can be obtained by averaging Eq.(6) with respect to the continuous distribution function of the particle coordinates. The passage from the discrete to the continuous limit representation of the radiation energy spectrum passes through the *ansatz* of the average of the square module of the single particle field amplitude, see Eq.(4,5,6). This *ansatz* is justified in [5] where it is argued that the continuous limit representation of the radiation energy spectrum has to remain invariant whether the continuous limit is applied to the flux of the Poynting vector - Eq.(6) - or directly to the radiation field - Eqs.(4,5) - prior to the calculation of the flux of the Poynting vector [5, 6]. With reference to [4, 5, 11] for more details on the derivation of the final formulation of the radiation energy spectrum under the continuous limit approximation, in the present context the attention will be addressed to the temporal incoherent part of the continuous limit of such a formula that, in the case of a infinite radiator surface ($S = \infty$), reads for a gaussian beam as

$$\frac{d^2 I}{d\Omega d\omega} = N \frac{(e\beta)^2}{\pi^2 c} \frac{sin^2\theta \, e^{-k^2 sin^2\theta(\sigma_x^2 cos^2\phi + \sigma_y^2 sin^2\phi)}}{(1 - \beta^2 cos^2\theta)^2}. \quad (7)$$

Angular distributions of the spectral radiation intensity for a beam energy of 250 MeV - relevant to the SwissFEL Injector Test Facility - and 450 MeV are shown in Fig.1, where diffractive beam-finite-transverse-size effects on the OTR are numerically calculated according to the continuous limit approximation for the distribution function of the electron spatial coordinates, which is here supposed to be valid under high energy and density conditions of the beam.

CONCLUSION

Compared to the case of a single and individually radiating electron, short wavelength diffractive modifications of the spectral and angular distribution of the TR intensity due to the finite transverse size of the electron bunch are worthy to be investigated for understanding the role played by the bunch collective effects in the radiation emission and, possibly, for exploiting such effects to monitor electron beams with μm-order transverse size. The experimental investigation of the electron beam density and energy conditions allowing such bunch collective effects to be appreciably observed in the OTR are underway at the SLS linac and in preparation at the SwissFEL Injector Test Facility. The theoretical model predicting them is covariance and temporal-causality consistent.

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