FEASIBILITY STUDY OF A COMPACT XFEL∗

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Abstract

This paper discusses feasibility of a compact XFEL in the future. It gives theoretical argument how a compact XFEL can be realized. For that purpose, the energy dependence of parameters is discussed. It is shown that a much improved electron gun with an extremely low emittance and very small energy spread is an essential tool for the realization of a compact XFEL.

INTRODUCTION

X-ray free electron laser (XFEL) based on self amplified spontaneous emission (SASE) [1, 2] is considered the next generation light source. It is supposed to give highly bright photon beam in a sub-pico second pulse. Furthermore, the radiation is transversely coherent. However, it is highly doubtful that this magnificent tool of science will be as easily accessible as the third generation light sources, because the XFEL machine is so huge and generally costs very high. The Linac Coherent Light Source (LCLS) that is under construction consists of a long linear accelerator of 14.35 GeV electron beam energy and a long undulator of 112 m [3], while the European XFEL that is under proposal will be even bigger [4]. Therefore, it is a natural attempt to find the possibility of reducing the size of an XFEL machine to a reasonably modest size without degrading the radiation quality [5, 6].

This paper shows that the reduction of XFEL in size and energy is realizable only by an improved electron gun with lower emittance and smaller energy spread. This result may look obvious, but this paper shows it quantitatively and analytically. The needed improved gun does not seem to exist at the moment. However, recent development of technology makes it a realistic goal in the near future. There are a few schemes under intensive R&D. A well known example is the single crystal thermionic gun that is going to be used in the FEL physics, that is, the slice emittance. The needed improved electron gun does not seem to exist at the moment. However, recent development of technology makes it a realistic goal in the near future. There are a few schemes under intensive R&D. A well known example is the single crystal thermionic gun that is going to be used in the FEL physics, that is, the slice emittance. The needed improved electron gun does not seem to exist at the moment. However, recent development of technology makes it a realistic goal in the near future. There are a few schemes under intensive R&D. A well known example is the single crystal thermionic gun that is going to be used in the FEL physics, that is, the slice emittance.

ENERGY DEPENDENCE OF PARAMETERS

We want to find the energy dependence of an XFEL design. It is most clearly shown by ρ, the FEL parameter, defined by

\[ \rho = \frac{1}{2\gamma} \left[ \frac{I_{pk} \lambda u K^2 [JJ]^2}{I_A \pi^2 \sigma_x^2} \right]^{1/3}, \]

where \( I_A = 17.045 \) kA is the Alfen current, \( I_{pk} \) is the peak current after the bunch compressing, \( \sigma_x \) is the cross sectional beam size, and \( [JJ] \) is defined as

\[ [JJ] = J_0 \left( \frac{K^2}{4 + 2K^2} \right) - J_1 \left( \frac{K^2}{4 + 2K^2} \right). \]

\( K \) is the undulator parameter defined by

\[ K = 0.934B_0[\text{Tesla}]\lambda_u[\text{cm}], \]

where \( \lambda_u \) is the undulator period and \( B_0 \), the undulator peak magnetic field, depends not only on the undulator gap and period but also on the magnet material. If we consider a hybrid undulator with vanadium permendur, it is given by

\[ B_0 = 3.694 \exp \left[ -5.068 \frac{g}{\lambda_u} + 1.520 \left( \frac{g}{\lambda_u} \right)^2 \right]. \]

with \( g \) denoting the undulator gap. The undulator peak field is not in wide range but usually restricted to between 1 and 1.5 Tesla. Hence, in this paper, we will fix \( g/\lambda_u \) to keep \( B_0 \) unchanged.

Since \( \rho \) roughly describes the SASE FEL efficiency as in

\[ \rho \sim \frac{\text{generated field energy}}{\text{electron kinetic energy}}, \]

a high \( \rho \) is preferred in the XFEL design. The reason why XFEL needs high electron energy and low beam emittance is that they are necessary to make \( \rho \) high enough.

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In Eq. (1), note that $\sigma_x^2 = \beta \epsilon_n / \gamma$ where $\beta$ is the betatron function. $\beta$ is an independent parameter we can choose freely. It is usual to choose the optimal $\beta$ that gives the shortest saturation length. The optimal $\beta$ was evaluated in [12] and is given by

$$\beta_{opt} = 11.2 \left( \frac{I_A}{I_{pk}} \right)^{1/2} \frac{{\epsilon}_n^{3/2} \lambda_u^{1/2}}{\lambda_r K[JJ]}$$

(6)

Using $\beta_{opt}$, $\rho$ is completely described by the known parameters as in

$$\rho = \frac{1}{2} K[JJ] \left( \frac{I_{pk} \lambda_u}{{I_A} \epsilon_n} \right)^{1/2} \left( \frac{\lambda_r}{89.6 \pi^2 \epsilon_n \gamma^2} \right)^{1/3},$$

(7)

Apparently, $\rho$ has the $E$ dependence of $E^{-2/3}$. But, this is wrong because $\lambda_u$ is also dependent on $E$. We find it from the undulator resonant condition,

$$\lambda_r = \frac{\lambda_u}{2 \pi^2} \left( 1 + \frac{K^2}{2} \right),$$

(8)

Note that Eq. (8) is a cubic equation for $\lambda_u$ for given $\lambda_r$ and $B_0$. Arranging Eq. (8) for $\lambda_u$ gives

$$\lambda_u^3 + \frac{2}{a^2} \lambda_u - \frac{4 \lambda_r \gamma^2}{a^2} = 0,$$

(9)

where $a = 0.934 B_0$. Solving this cubic equation, we obtain $\lambda_u$ as a function of $\gamma$ (or $E$). To obtain a rough dependence of $\lambda_u$ on $E$, we see in Eq. (8) that when $K > \sqrt{2}$, which is usually the case, Eq. (9) is roughly approximated to

$$\lambda_u^3 \approx \frac{4 \lambda_r \gamma^2}{a^2}.$$

(10)

Then, we can derive the rough dependence of $\lambda_u$ on $E$ as

$$\lambda_u(E) \propto E^{2/3}.$$  

(11)

The graph of $\lambda_u$ versus $E$ for $\lambda_r = 1.5$ Å is shown in Fig. 1 for later use. As a boundary condition, we used the LCLS values, $E = 14.35$ GeV, $\lambda_u = 3$ cm, $B_0 = 1.32$ T, which means that $g/\lambda_u$ is fixed to 0.217.

Since $K$ has the same $E$ dependence as $\lambda_u$, the rough $E$ dependence of $\beta_{opt}$ becomes

$$\beta_{opt} \propto E^{-1/3} \left( \frac{\epsilon_u}{I_{pk}} \right)^{1/2} \epsilon_n.$$

(12)

It is straightforward to see the rough dependence of $\rho$ as

$$\rho \propto \left( \frac{E}{\epsilon_n} \right)^{1/3} \left( \frac{I_{pk}}{\epsilon_n} \right)^{1/2} \epsilon_n.$$

(13)

LOW ENERGY

Equation (13) shows that $\rho$ decreases as $E$ decreases as $E^{1/3}$. This means the degraded machine performance and radiation quality. Fortunately, $\rho$ depends on the ratio $E/\epsilon_n$.

FEL Theory

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XFEL is realizable only by a much improved electron gun. SHORT UNDULATOR

To build a compact XFEL, not only the linac size but also the undulator size should be reduced. Actually, the needed undulator length is reduced when the electron energy is lowered. This is easily seen by the behavior of the one dimensional gain length defined by

$$L_G = \frac{\lambda_u}{\sqrt{3\rho}}$$

or more accurately of the saturation length given by

$$L_{sat} = L_G(1 + \Lambda) \ln \left( \frac{P_{sat}\lambda_r}{2\rho^2 Ec} \right),$$

where $\Lambda$ is the famous fitting formula by Ming Xie [13] and $P_{sat} = 1.6pE/e(1 + \Lambda)^2$ is the saturated peak power. Using Eqs. (11) and (13), we obtain the rough dependence

$$L_G \propto (Ee_n)^{1/3} \left( \frac{\epsilon_n}{I_{pk}} \right)^{1/2}.$$  \hspace{1cm} (16)

Since the logarithm is insensitive to the variation of its variable, the behavior of $L_{sat}$ under the energy scaling is mostly given by the behavior of $L_G$. The only possible correction comes from $\Lambda$, which is given by

$$\Lambda = a_1\eta_d^{a_2} + a_3\eta_c^{a_4} + a_5\eta_b^{a_6} + a_7\eta^{a_8} + a_9\eta^{a_9} + a_{10}\eta^{a_{11}} + a_{12}\eta^{a_{14}} + a_{13}\eta^{a_{15}} + a_{16}\eta^{a_{17}} + a_{18}\eta^{a_{18}},$$

where the scaled parameters are defined by

$$\eta_d = \frac{\lambda_u}{4\pi\beta_{opt}\epsilon_n}, \quad \eta_c = \frac{\lambda_u}{4\pi\beta_{opt}\epsilon_n}, \quad \eta_b = \frac{4}{\sqrt{3\rho}} \frac{\sigma}{E},$$

and $a_1, \cdots, a_{19}$ are determined numbers. Although this correction to $\Lambda$ is not critical because $\Lambda$ often is smaller than 1, it can sometimes give non-negligible change to $\Lambda$. Note that $\Lambda$ includes $\delta E/E$ in $\eta_c$, which gets larger when $E$ is lower than usual. This correction is easily removed if we use an improved gun with reduced $\delta E$.

Equation (16) shows that both working at a low energy and using the improved gun reduces the needed undulator length. Again, we see that $\epsilon_n$ and $\delta E/E$ are key parameters to realize a compact XFEL.

UNDULATOR GAP

The discussion so far may give the impression that the energy and size of an XFEL can be reduced to as small scale as the gun emittance and energy spread allow. However, this may not be true. A limitation for how compact an XFEL machine can be may come from the narrow undulator gap, no matter how low $\epsilon_n$ and $\delta E/E$. Recall that $g/\lambda_u$ is fixed in this discussion to maintain the same undulator peak field. As $\lambda_u$ decreases, $g$ also decreases. Figure 1 shows that $\lambda_u$ is only 1 cm at around $E = 4$ GeV, which means $g$ is only around 2 mm. Since the electron beam size is so small (<100μm), there is practically no lower limit of the undulator gap as far as beam passing is concerned. However, the narrower the undulator gap is, the tighter the alignment and beam control tolerances are.

An adequate $g$ should be chosen by taking into consideration the above factors. In many cases, this chosen $g$ may determine $E$ and $\lambda_u$.

UNDULATOR WAKEFIELD

A potential problem in realizing a compact XFEL is the undulator wakefield, which is inversely proportional to the undulator gap. We saw above that the undulator gap can be very narrow in a compact XFEL. The undulator wakefield creates relative energy spread between the slices, the rms of which is given by [3]

$$\sigma_w = -\frac{2NL(W_z)_{rms}}{E},$$

where $L$ is the undulator length, and $(W_z)_{rms}$ is the rms of the wakefield over a bunch. For a Gaussian bunch, we have [3]

$$(W_z)_{rms} \approx 1.02 \frac{\Gamma(3/4)}{2\sqrt{\pi^2}} \frac{1}{\sigma_g^{3/2}} \frac{e}{g} \left( \frac{Z_0}{\sigma} \right)^{1/2},$$

where $\sigma$ is the conductivity of the metal. It would be no problem to replace $L$ by $L_{sat}$ in Eq. (20). Since $\sigma_w$ spreads energy between slices not within a slice, it does not prevent the FEL process from occurring but causes slices with large energy deviation radiate out of resonance. The final result would be simply the radiation power reduction.

Since $\sigma_w$ is inversely proportional to $E$, it is supposed to grow and give more power reduction for lower $E$. Using Eq. (16) and the fact that $eN$ is proportional to $I_{pk}$, we can find the rough dependence of $\sigma_w$ as

$$\sigma_w \propto \left( \frac{\epsilon_n}{E} \right)^{2/3} \left( \frac{I_{pk}}{\epsilon_n} \right)^{1/2} \left( \frac{\epsilon_n}{g} \right)^{2/3}.$$  \hspace{1cm} (22)

We see that the growth of $\sigma_w$ by lowering $E$ and $g$ can be canceled by using a low emittance electron beam. There is no undulator wakefield problem in a compact XFEL.

CONCLUSION

It would be really great, if it is possible to build a LCLS-quality XFEL in a compact size at a lower energy. This paper has shown that a compact hard X-ray FEL can be constructed only by using a much improved electron gun.
with extremely low emittance and energy spread. The necessary technology for the improved guns is not at hand but under development. If these guns are realized in the future, a practical limit for how compact an XFEL can be may come from the undulator gap, which should not be too small considering the alignment and beam control difficulties. Finally, there will be no undulator wakefield problem in the compact XFEL.

REFERENCES