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Dynamics of Electron/Positron Bunch in Surko Trap of LEPTA Facility

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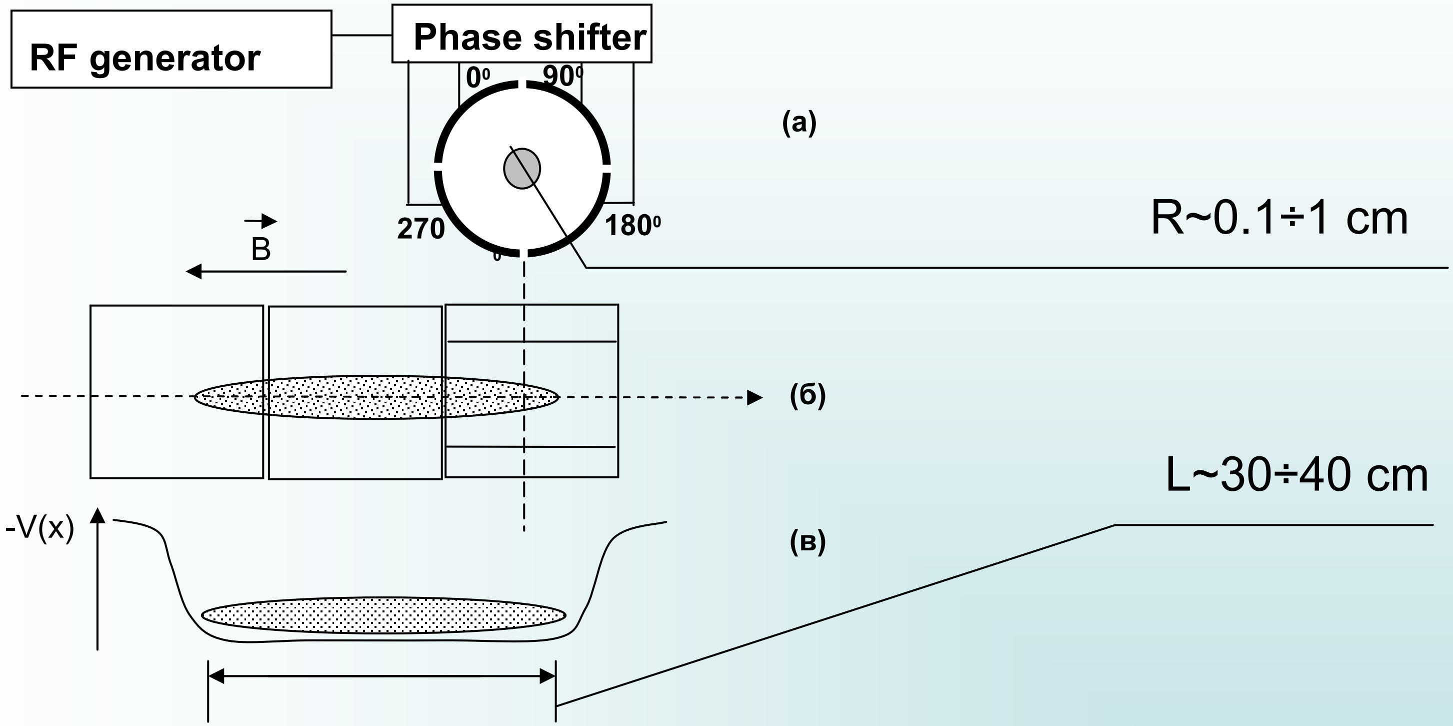
Positron trap of LEPTA facility



I. Meshkov, I. Seleznev, A. Sidorin, A. Smirnov, G. Trubnikov, S. Yakovenko, NIM B, **214**, 186 (2004)

Method of “rotating wall” (RW) of electric field

Compression of particle bunch with RW :

- Mg^+

  X-P. Huang et al., PRL, **78**, 875 (1997).
- e^-

  Anderegg, E. M. Hollmann, and C. F. Driscoll, PRL, 81, 4875 (1998).
- e^+

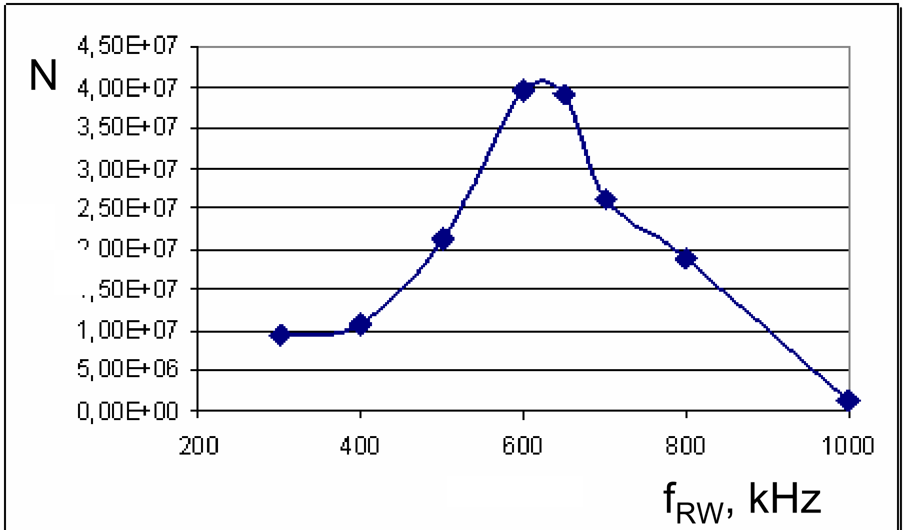
  R. G. Greaves and C. M. Surko , PRL, 85, 1883 (2000).

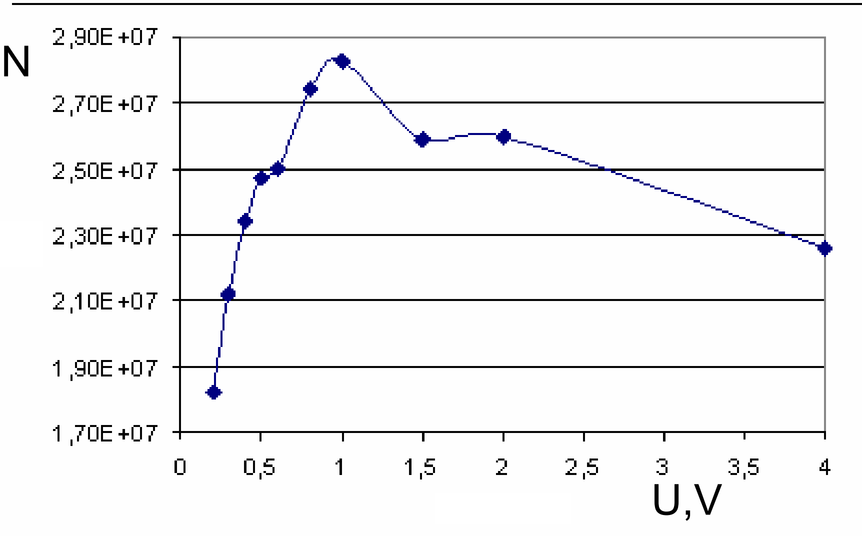
  T.J. Murphy and C.M. Shurko, Phys. Plasmas, **8**, 1878 (2001).

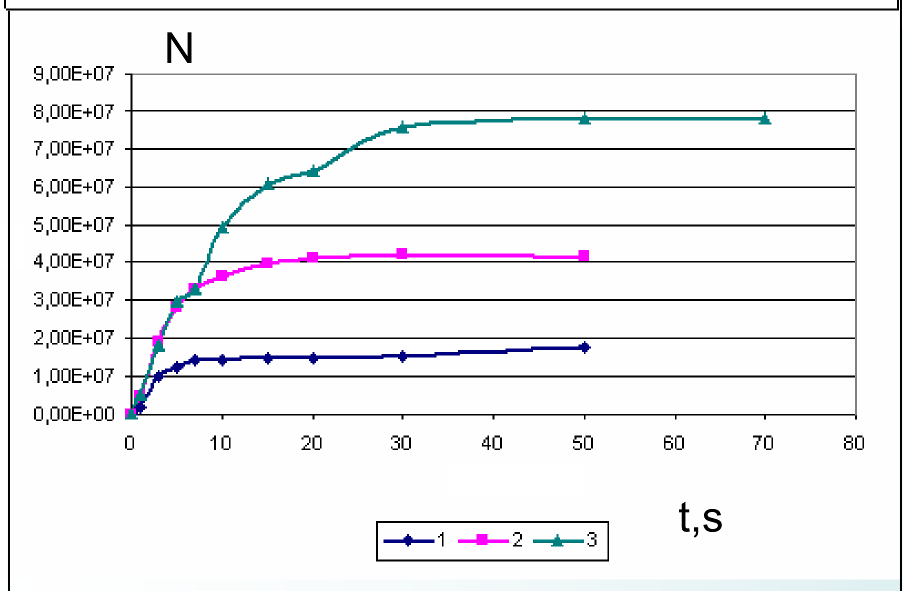
  J. R. Danielson, C. M. Surko, and T. M. O’Neil PRL., **99,** 135005 (2007).
- Pbar (Hbar production)

  J. R. Danielson , et al., PRL. **100**, 203401 (2008) .

Results of electron accumulation in the trap of LEPTA







Opposite Surko experiments!

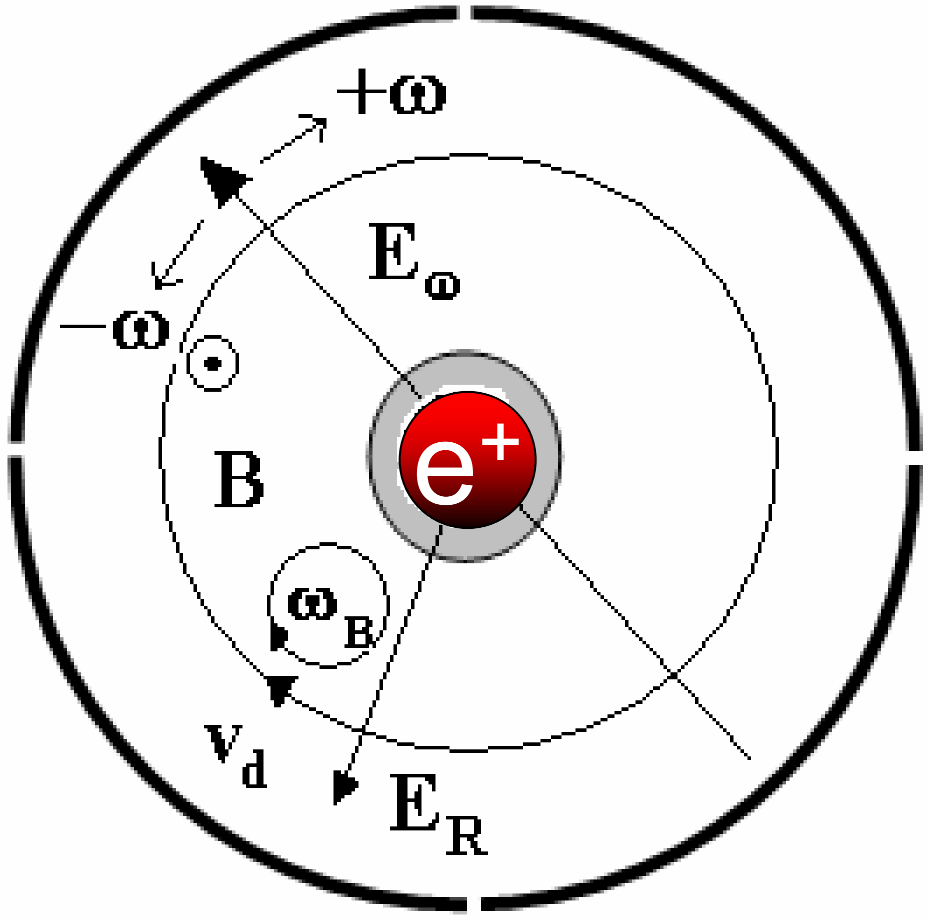
$$f^{res}_{RW} \simeq -650\ kHz$$

$$U_{RW} \simeq 1\ V$$

S. Yakovenko, Pulsed injector of low energy positrons, Ph.D. thesis, JINR, Dubna, 2007.

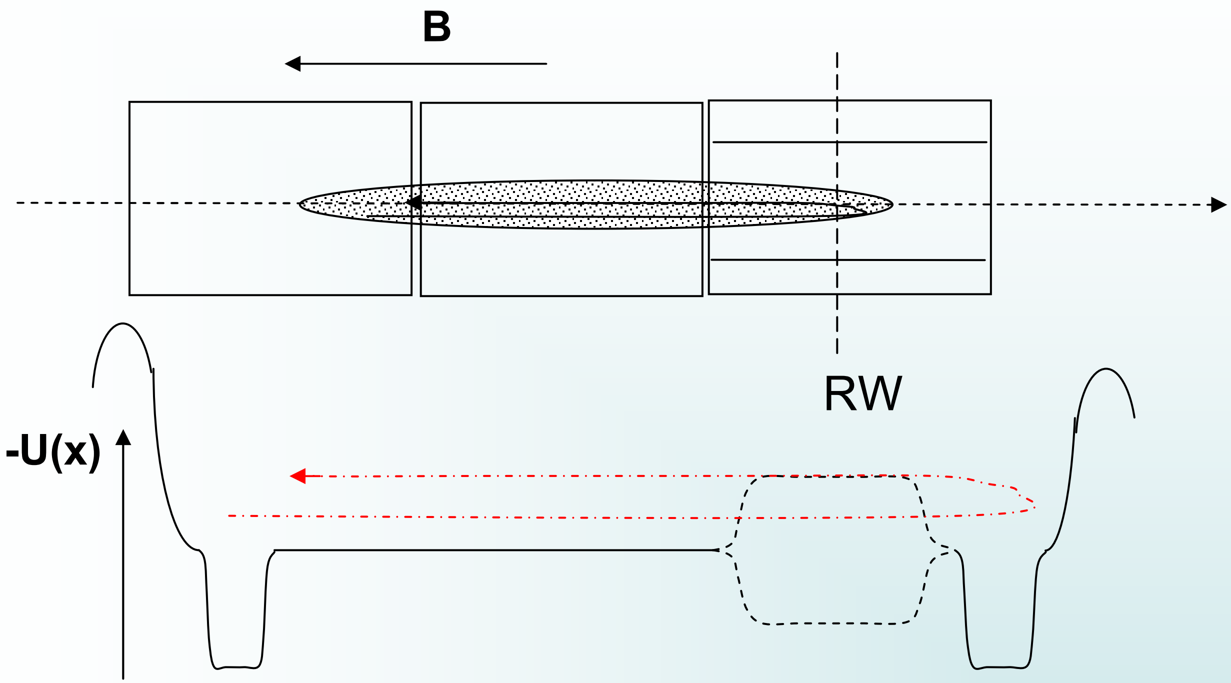
Transverse motion of the trapped particle

Solution of positron dynamics equations in the bunch



E_ω, V/cm	0.05
f_{RW}, kHz	-650
n_e, cm^{-3}	$10^7 \div 10^8$
ω_p, c^{-1}	$3.5 \cdot 10^7 \div 2 \cdot 10^8$
B, Gauss	1200
ω_B, c^{-1}	$2.1 \cdot 10^{10}$
p_{N2} , pascal	$(2.4 \div 3.4) \cdot 10^{-4}$
R , cm	0.1÷2
L , cm	30÷40

Longitudinal motion of the trapped particle



$B_{long} \parallel E_{long}$

$$L \sim 30 \div 40 \text{ cm} \qquad E_{part} \sim 1\ eV \qquad f^{res}_{RW} \simeq T^{-1}_{long}$$

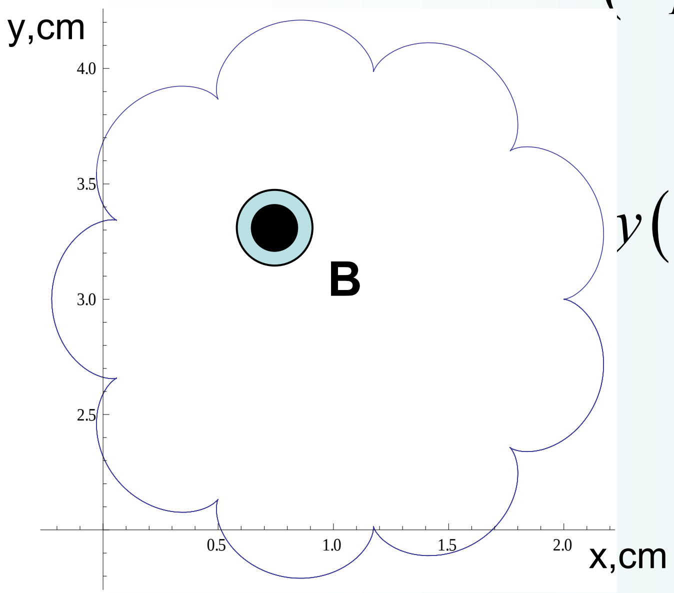
Optimal buffer gas pressure $2.25 \cdot 10^{-6}$ Torr

Transverse positron motion in the crossed B-field and RW E-field

$$\xi'' + i\omega_B \xi' = \varepsilon e^{-i\omega t} \qquad \varepsilon = \frac{e}{m}E \qquad \xi = x + iy$$

$$x(t) = \frac{\varepsilon}{\omega(\omega_B - \omega)}\cos\omega t - \frac{\varepsilon}{\omega_B(\omega_B - \omega)}\cos\omega_B t + x_0 - \frac{\varepsilon}{\omega_B \omega}$$

$$y(t) = -\frac{\varepsilon}{\omega(\omega_B - \omega)}\sin\omega t + \frac{\varepsilon}{\omega_B(\omega_B - \omega)}\sin\omega_B t + y_0$$



Transverse positron motion in the crossed B-field and RW E-field and E-field of the bunch space charge

$$\ddot{\xi} + i\omega_B\dot{\xi} - \frac{e}{m}E_R^0\xi = \frac{e}{m}E_\omega^0 e^{-i\omega t} \qquad E_R = 2\pi ner \qquad \omega_p = \sqrt{\frac{4\pi e^2 n}{m}}$$

$$\ddot{\xi} + i\omega_B\dot{\xi} - \eta\xi = \varepsilon e^{-i\omega t} \qquad \varepsilon = \frac{e}{m}E_\omega^0 \qquad \omega_B = \frac{eB}{mc}$$

$$\eta = \frac{e}{m}E_R^0 = 2\pi\frac{e^2 n}{m} = \frac{\omega_p^2}{2}$$

$$x(t) = \frac{\varepsilon}{\omega(\omega_B-\omega)-\eta}\cos\omega t + \left(x_0\frac{\omega_B+\omega'}{2\omega'} - \frac{\varepsilon\left(\omega_B+\omega'-2\omega\right)}{2\omega'\left(\omega(\omega_B-\omega)-\eta\right)}\right)\cos\frac{\omega_B-\omega'}{2}t + \left(-x_0\frac{\omega_B-\omega'}{2\omega'} + \frac{\varepsilon\left(\omega_B-\omega'-2\omega\right)}{2\omega'\left(\omega(\omega_B-\omega)-\eta\right)}\right)\cos\frac{\omega_B+\omega'}{2}t + \left(y_0\frac{\omega_B+\omega'}{2\omega'}\right)\sin\frac{\omega_B-\omega'}{2}t + \left(-y_0\frac{\omega_B-\omega'}{2\omega'}\right)\sin\frac{\omega_B+\omega'}{2}t$$

RW resonant positron motion

$$\omega'^2 = \omega_B{}^2 - 2\omega_p{}^2$$

$$\omega_B \pm \omega' - 2\omega = 0$$

$$\omega(\omega_B - \omega) - 2\omega_p{}^2 = 0$$

$$\omega = \frac{\omega_B}{2}\left(1 \pm \sqrt{1 - \frac{2\omega_p{}^2}{\omega_B{}^2}}\right)$$

$$\omega^+{}_1 = \omega_B$$

$$\omega'^2 \gg 0$$

$$\omega^-{}_2 = \frac{\omega_B}{2}\frac{\omega_p{}^2}{\omega_B{}^2} = 2\pi\frac{nec}{B}$$

Effect of particle collisions with buffer gas molecules

$$m\ddot{x} = \frac{e}{c}\dot{y}B + eE_x^{\omega} + eE_x^R - \mu\dot{x}$$

$$m\ddot{y} = -eE_y^{\omega} - \frac{e}{c}\dot{x}B + eE_y^R - \mu\dot{y}$$

$$\ddot{\xi} + \left(i\omega_B + 2\gamma\right)\dot{\xi} - \eta\xi = \varepsilon e^{-i\omega t}$$

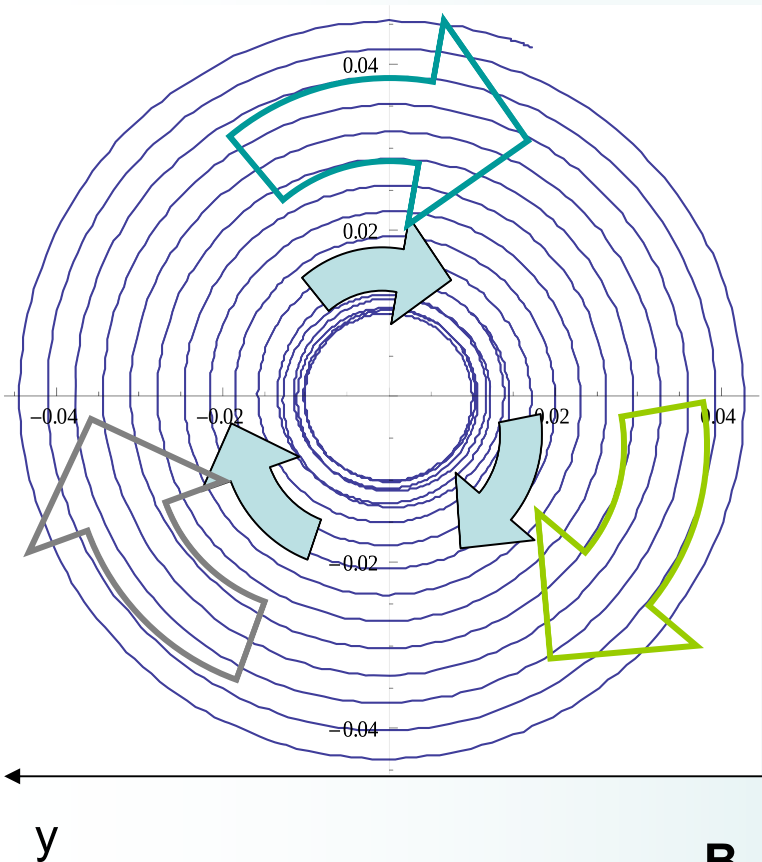
$$\xi\left(t\right) = e^{-\frac{i\omega_B t}{2}} e^{-\gamma t}\left(C_1 e^{\frac{\sqrt{4\eta+4\gamma^2+4i\omega_B\gamma-\omega_B^2}}{2}t} + C_2 e^{-\frac{\sqrt{4\eta+4\gamma^2+4i\omega_B\gamma-\omega_B^2}}{2}t}\right) + \frac{\varepsilon}{\omega(\omega_B-\omega-2i\gamma)-\eta} e^{-i\omega t}$$

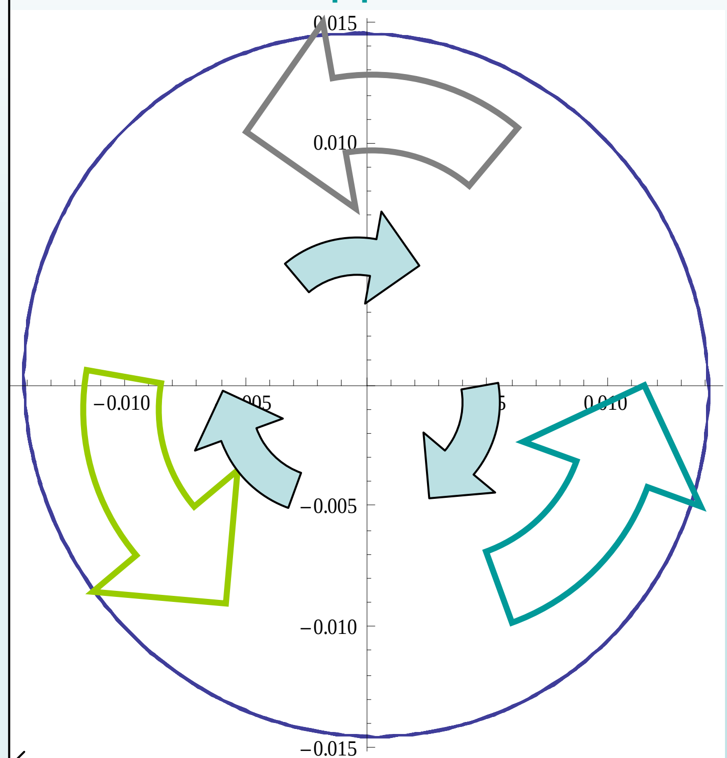
C M Surko at.al. J. Phys. B: At. Mol. Opt. Phys. 38 (2005) R57-R126
J Sabin Del Valle at.al. J. Phys. B: At. Mol. Opt. Phys. 38 (2005) 2069

Particle trajectories (in transverse plane) depending of RW direction

The particle trajectories have a spiral form when directions of RW and particle drift coincide

The particle trajectories have a circle form of constant radius when directions of RW and particle drift are opposite

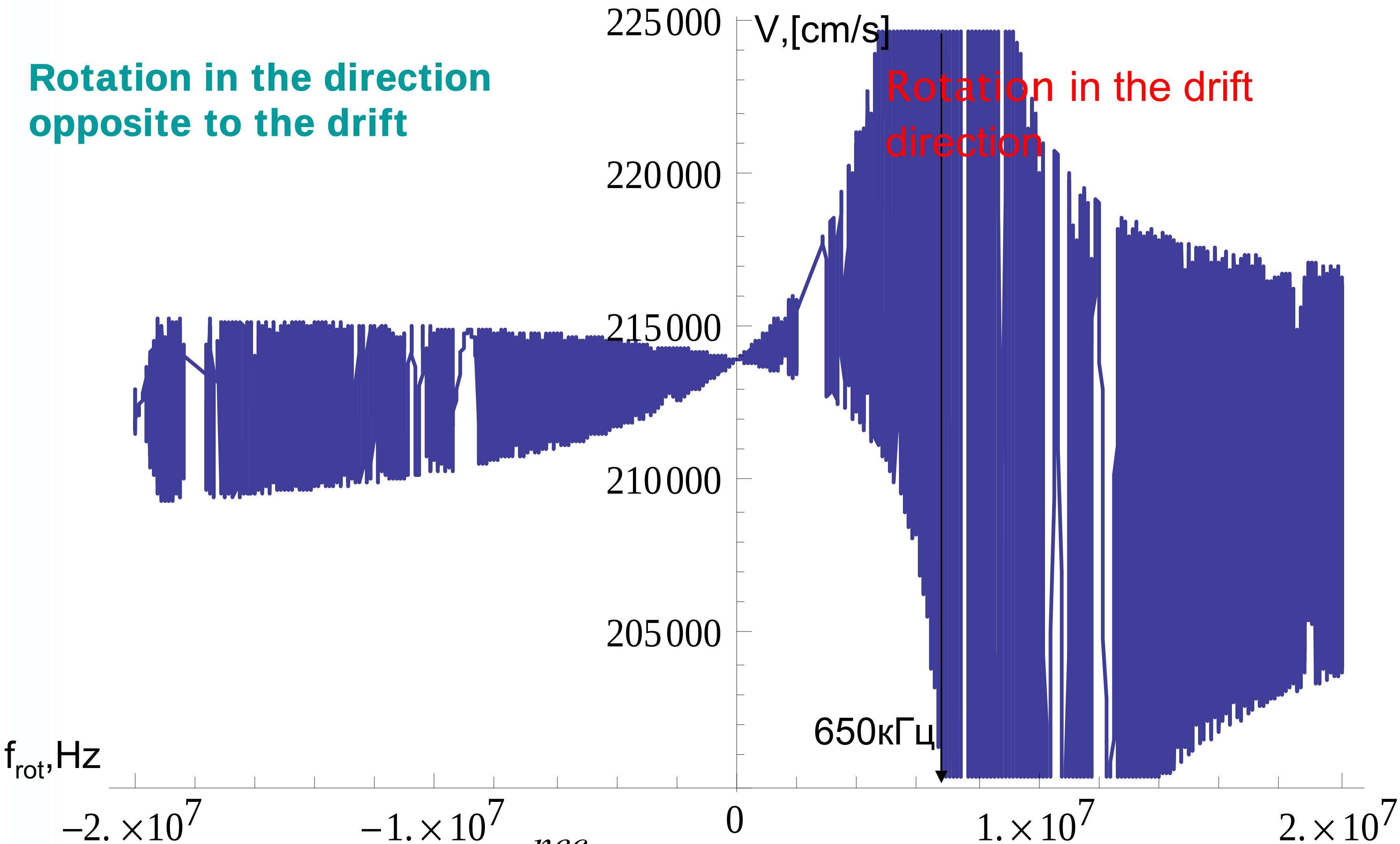




Wall rotation in particle drift direction

Wall rotation in the direction opposite to particle drift

Dependence of particle rotation velocity on RW frequency



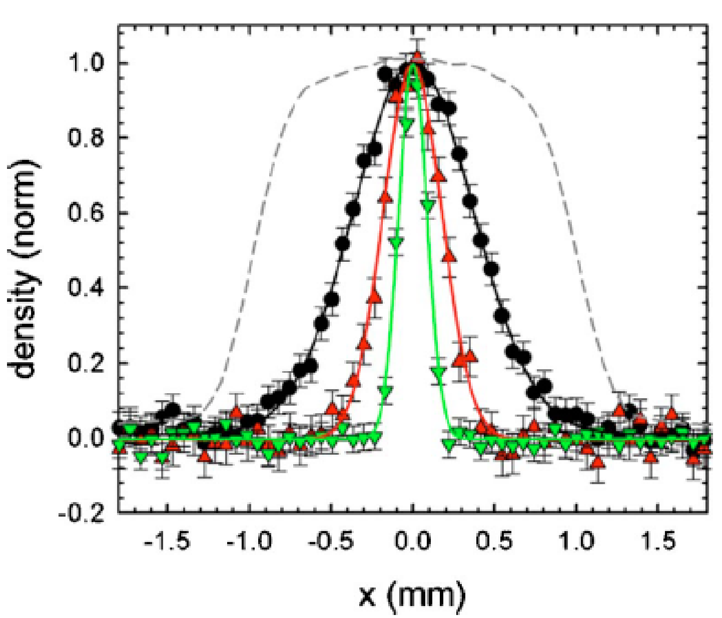
$$f_{res} = \frac{nec}{B} = /5.8 \cdot 10^{7}\, cm^{-3}/ = 650kHz$$

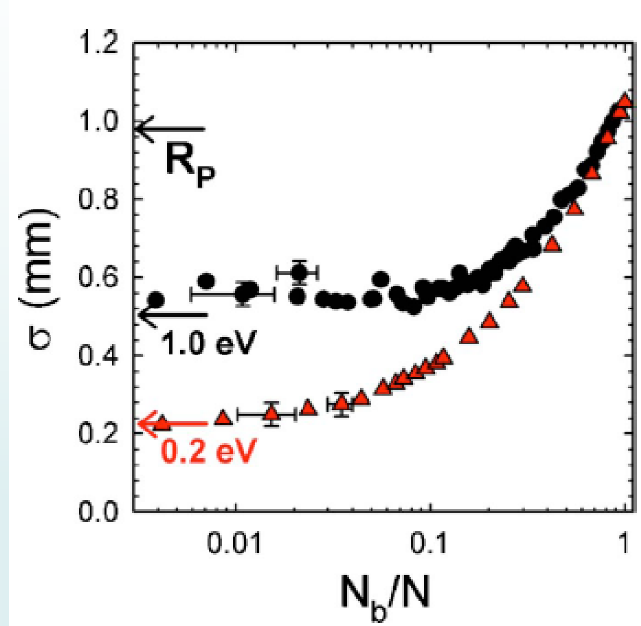
Numerical simulation of particle motion in the trap

- Collective motion of the particles
- Gaussian distribution of the particle density
- Longitudinal motion of the particle in the trap
- “The overstep” method

C.K. Birdsall, A. B. Langdon Plasma physics, via computer simulation McGraw-Hill Book Company 1985

Gaussian distribution of the positron density in "The Surko trap"





$$n(r) = n_0 G(A) \exp\left[-(r/\sigma)^2\right]$$

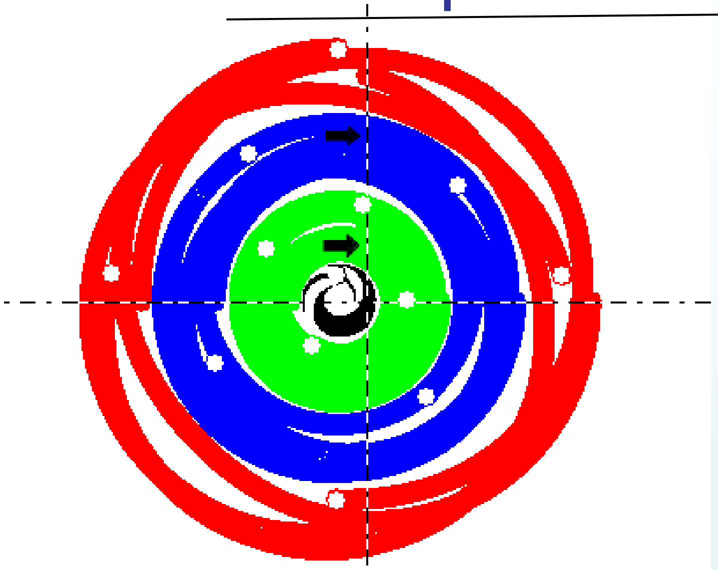
$$\sigma = 2\lambda_D$$

$$G(A) = (\pi A)^{-1/2} \exp[-A]$$

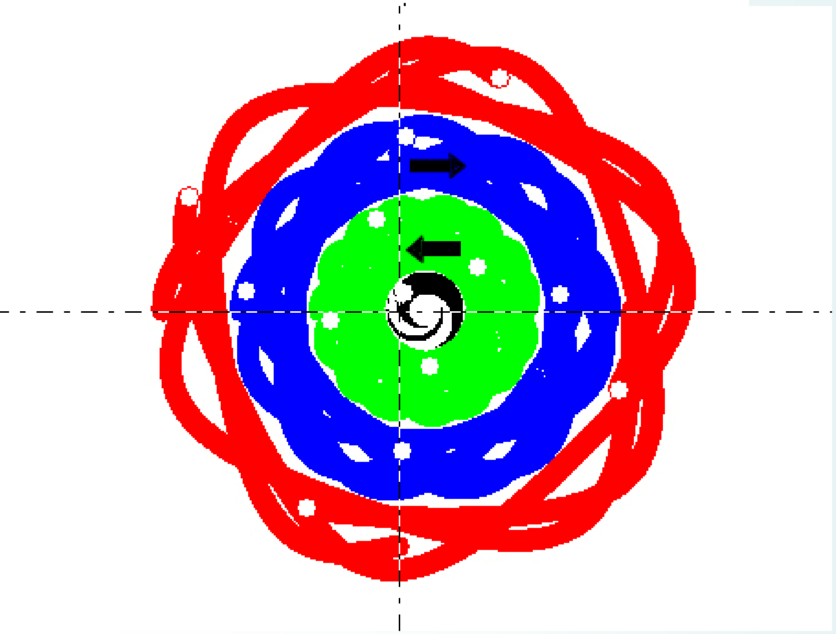
$$A \equiv -(e/T)\left[V_c - V_{ex} - \varphi(r)\right]$$

J. R. Danielson, T. R. Weber and C. M. Surko Appl. Phys. Lett. **90**, 081503 (2007)

Test particle motion in “the gaussian bunch”



Wall rotation in particle drift direction



Wall rotation in the direction opposite to particle drift

Energy balance of the particle bunch

Energy Losses:

1. Inelastic collisions with molecules of buffer gas

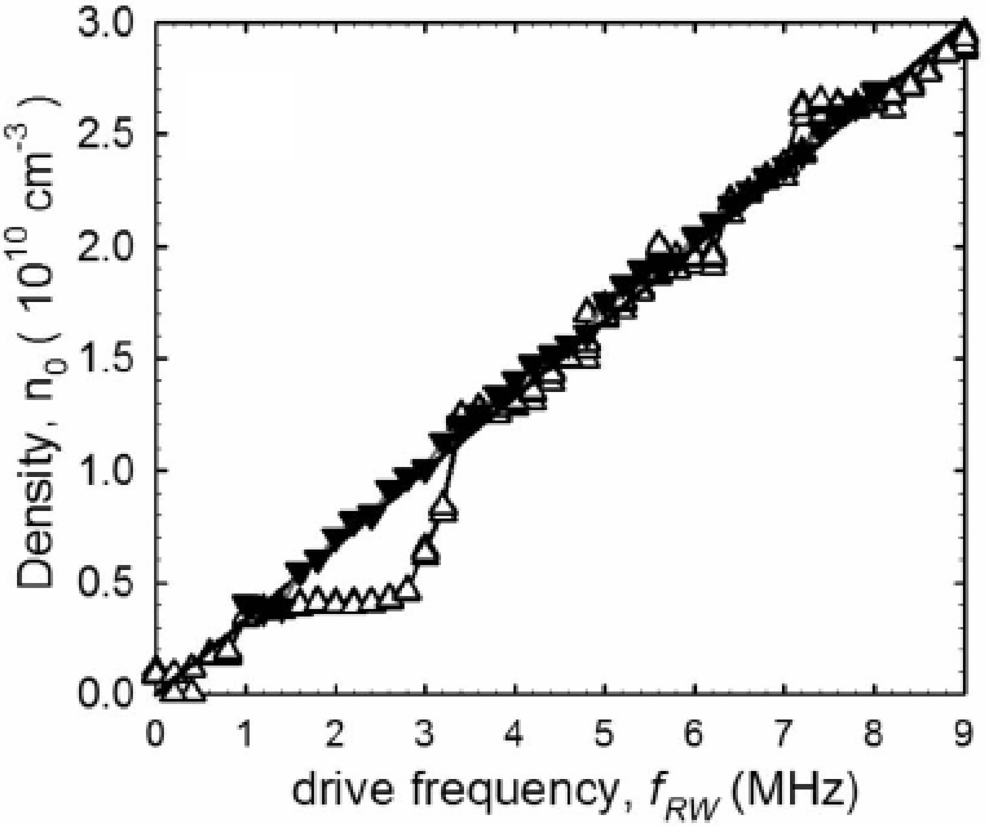
2. Synchrotron ("cyclotron") radiation

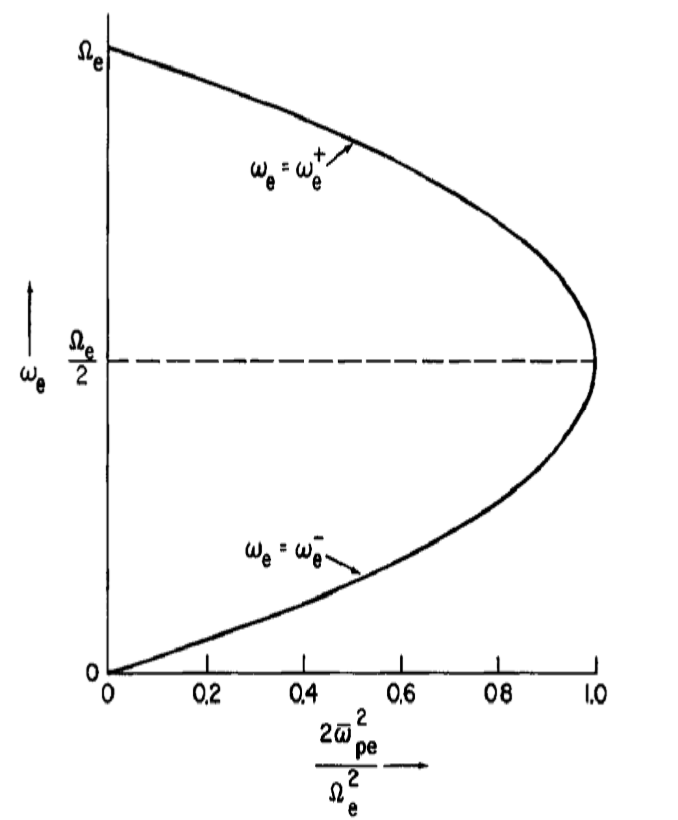
$$\lambda_c = \frac{1}{\tau_c} = -\frac{\dot{T}}{T} \sim B$$

3. Bremsstrahlung

Particles loss due to transverse diffusion across magnetic field

Flip RW frequency





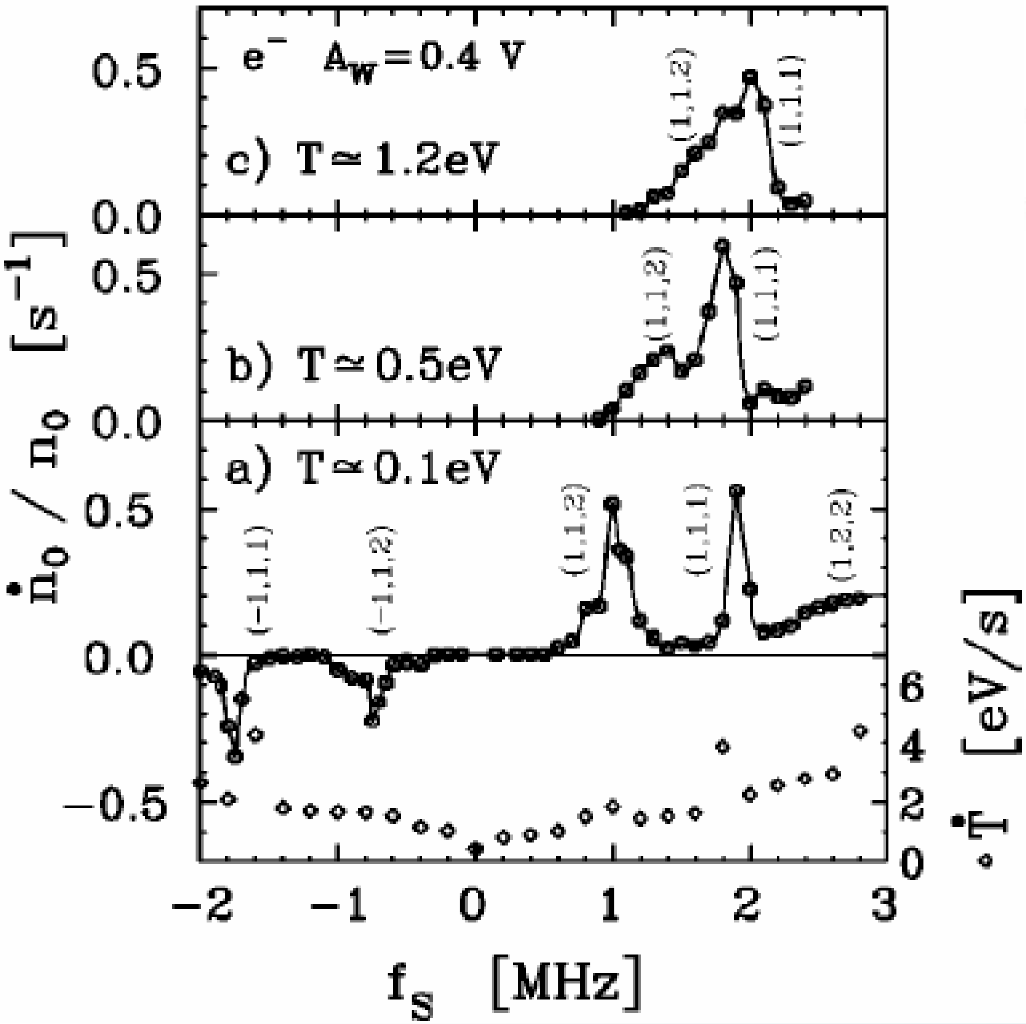
$$\Delta\omega = \omega_E - \omega_{RW} \qquad \Delta\omega \sim \upsilon_{\|} / L \sim \sqrt{T} \qquad \omega'^2 \to 0$$

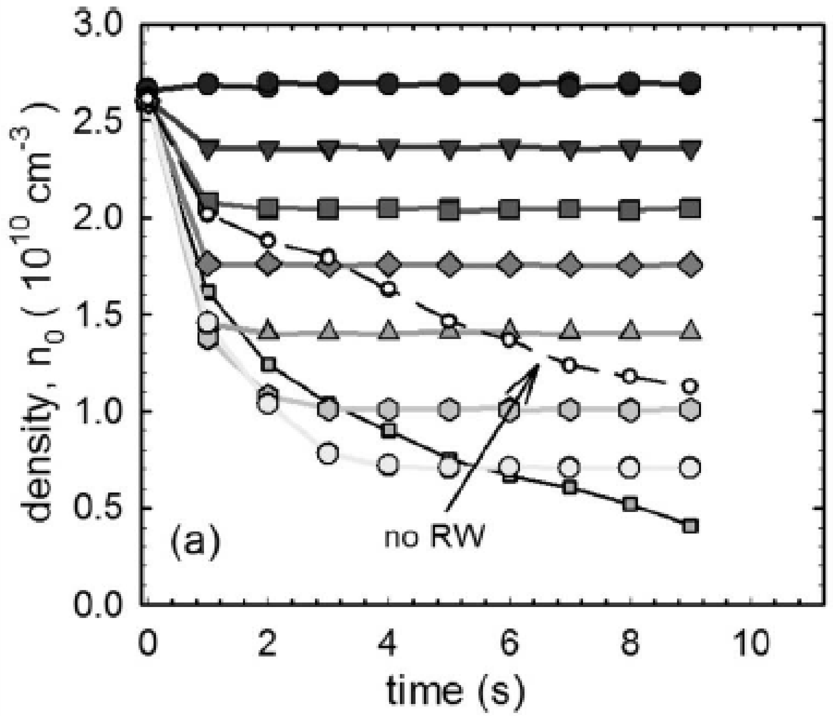
$$n_B = \frac{B^2}{8\pi mc^2}$$

$$\omega_s = m_\theta \omega_{RW} \qquad \omega = \frac{\omega_B}{2}\left(1 \pm \sqrt{1 - \frac{2\omega_p^2}{\omega_B^2}}\right) \qquad \omega^-_2 \neq 2\pi \frac{nec}{B}$$

R.C. Davidson Theory of nonneutral plasmas (Benjamin, 1974)

Electro-mechanical mode and resonances column plasma





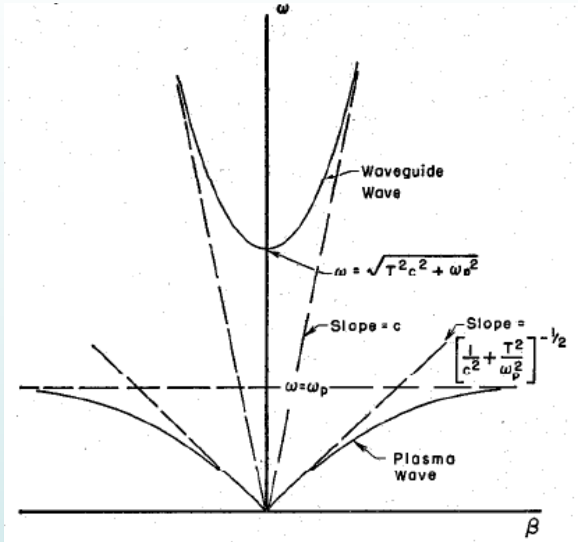
$$m_r = 1, 2, 3...$$

$$m_\theta = \pm 1, \pm 2$$

$$m_Z = 0, 1, 2, 3...$$

Electro-mechanical wave TG in cylinder column plasma

$$\varphi(r,\theta,z,t) = A \cdot J_{m_\theta}(b \cdot r)\exp\left[i\left(\omega t - m_\theta \theta - k_Z z\right)\right]$$



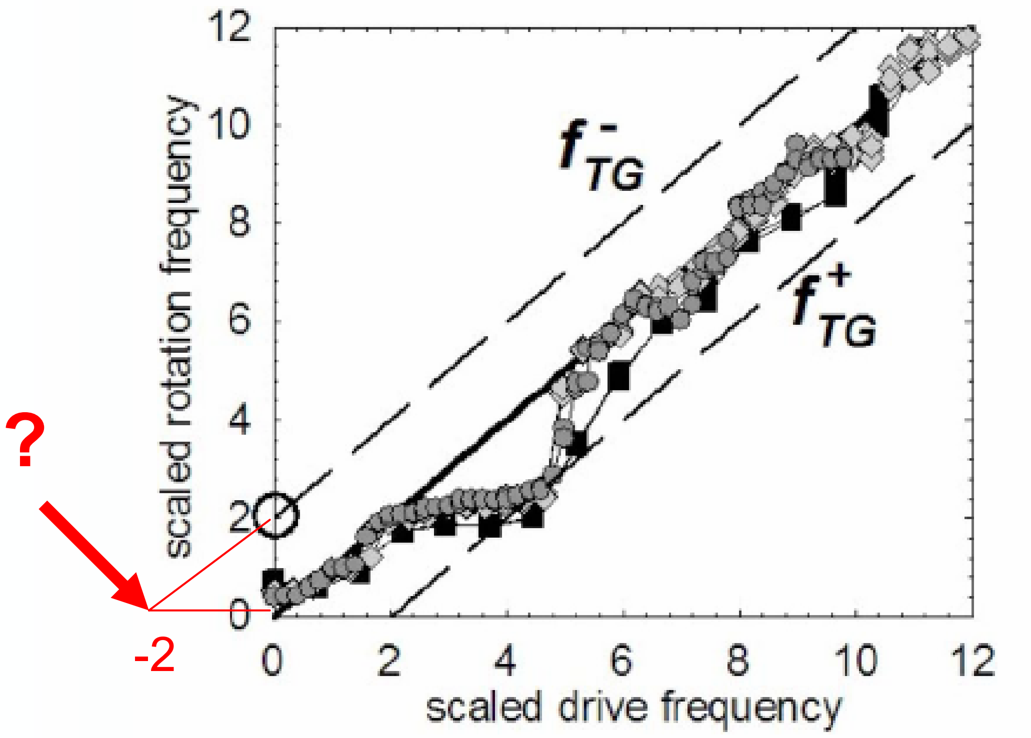
$$b^2 = -\beta^2 \left[\frac{\left(\omega^2 - \omega_p^2\right)\left(\omega^2 - \omega_B^2\right)}{\omega^2\left(\omega^2 - \omega_p^2 - \omega_B^2\right)} \right]$$

$$\beta R = \pm p_{m_\theta m_r} \left[\frac{\omega^2\left(\omega^2 - \omega_p^2 - \omega_B^2\right)}{\left(\omega^2 - \omega_p^2\right)\left(\omega^2 - \omega_B^2\right)} \right]$$

$$m_Z = k_z L / \pi$$

W. Trivelpiece and R. W. Gould, J. Appl. Phys. **30**, 1784 (1959).
F. Anderegg, E. M. Hollmann, and C. F. Driscoll, PRL, **81**, 4875 (1998)

Trivelpiece-Gould wave and frequency RW



$$f_{RW}^{TG} = m_\theta f_E \pm \frac{1}{p_{m_\theta m_z}} \frac{\omega_p}{2\pi} \frac{R}{L} \pi m_Z$$

$$J\left(p_{m_\theta m_r}\right) = 0$$

$$f_{RW}^{TG} \ll \frac{\omega_p}{2\pi}$$

$$m_\theta f_E \simeq \frac{1}{p_{m_\theta m_z}} \frac{\omega_p}{2\pi} \frac{R}{L} \pi m_Z$$

$$m_\theta = -1$$

$$f_{RW}^{TG+} = 0$$

$$f_{RW}^{TG-} = -2 f_E$$

$$f_{RW}^{TG-} = -2\frac{nec}{B} = -2\frac{2.7 \cdot 10^7 ec}{1200} \approx -650 \ kHz \,!$$

$$m_\theta = 1$$

$$f_{RW}^{TG-} = 0 \qquad f_{RW}^{TG+} = 2 f_E$$

Proposal for new experiments in Surko trap of LEPTA facility

- Define dynamics of the dispersion in distribution of the positron density in the accumulation process:

$$n(r),\ n(t),\ n(\omega),\ \Delta n/\Delta t(\omega).$$

- Realize searching for other resonance RW frequencies on different TG modes.
- Optimization working parameters of the trap of the LEPTA facility.

Conclusions

1. Solutions of the positron dynamics equations in the bunch are in the following conditions:
   - Longitudinal magnetic field;
   - Rotating electric field;
   - Electric field of space charge positronic bunch;
   - Collisions with molecules of buffer gas.
2. Tracks (in transverse plane) and velocities of positrons in the trap were calculated for parameters of the trap of the LEPTA facility.
3. RW rotation resonance of the frequency was defined.
4. Numerical simulation of particle motion in the trap was realized.
5. Electro-mechanical TG modes and resonances in the positron bunch were defined.
5. Proposal for new experiments in the Surko trap for positrons was suggested.

Thank you for attention!