MULTIPLE INJECTIONS WITH BARRIER BUCKETS

K.Y. Ng
Fermi National Accelerator Laboratory,* P.O. Box 500, Batavia, IL 60510

Abstract
Multiple injections into the 50 GeV proton synchrotron, proposed by the Institute of Nuclear Study of Japan, from a 3 GeV booster using barrier buckets are simulated. For four successive injections of 4 bunches each time, having a half momentum spread of 0.5%, the final coasting beam in the synchrotron has a momentum spread of roughly $\pm 1.0\%$ in the core, with a tail extending up to $\pm 2.5\%$. The choice of debunching time, barrier velocity, barrier voltage, and barrier width is analyzed. Some beam kinematics relating to the barrier buckets are discussed.

1 INTRODUCTION

The Main Ring of the Japan Hadron Project (JHP) is a high-intensity fast-cycling synchrotron. The 16 bunches with a total of 2×10^{14} protons are injected from the booster in 4 batches cycling at the rate of 25 Hz. In order to minimize the high space charge, it has been suggested the use of rf barrier waves during the injection [1]. Such a simulation is described below. The detail and some beam kinematics relating to the barrier buckets are given in Ref. 2.

2 CHOICE OF DEBUNCHING TIME

The ring has an imaginary transition gamma of $\gamma_t = 27i$ giving a slip factor at injection $\eta = -0.05813$. For a bunch to debunch until the part with maximum momentum spread $\delta = +0.005$ meets the part with $\delta = -0.005$ longitudinally, $t_{\rm debunch} = T_0/(2|\eta\delta|) = 8.28$ ms is required, where $T_0 = 4.9526$ μs is the revolution time with the ring circumference taken as $C_0 = 1442$ m. We will use $t_{\rm debunch} = 10$ ms in the simulation.

3 CHOICE OF SQUEEZING TIME

In this simulation, we inject into the ring, as shown in Fig. 1, 4 bunches, each containing 1000 macro-particles distributed randomly in its elliptical envelope with maximum momentum spread $\delta = \pm 0.005$ and width $\Delta\tau = \frac{1}{17\times 8}T_0$; i.e, the full width is a quarter of the rf wavelength at revolution harmonic $h = 17$. After debunching for 10 ms, two square rf barrier waves are introduced at $\tau = 0$ on the phase axis. One barrier is fixed while the other moves slowly at the rate of $\dot{T}_2 = -3.884 \times 10^{-5}$ to the right until a space corresponding to four $h = 17$ rf wavelengths or 1.165 μs is opened. The time taken will be 30 ms, so that 40 ms has just elapsed and the next injection of 4 more bunches from the booster is just in time, as is illustrated in Fig. 2. The procedure then repeats. After debunching for another 10 ms and introducing rf barrier waves with squeezing for 30 ms, the third injection is ready as shown in Fig. 3. Another 10 ms of debunching and 30 ms of squeezing by barrier waves lead to the fourth injection in Fig. 4. Finally, we allow for another 10 ms of debunching before recapturing by the $h = 17$ rf system for acceleration. In the above, T_2 is the width of the rectangular part of the bucket. Since the moving barrier pulse squeezes the bucket, $\dot{T}_2 < 0$. In order that the longitudinal emittance of the bunch inside the barrier bucket is conserved, we must have [3] $|\dot{T}_2| \ll \frac{1}{2}|\eta\delta| = 1.45 \times 10^{-4}$. Therefore, the rate of barrier movement chosen in the simulation should be slow enough.

4 CHOICE OF BARRIER VOLTAGE AND WIDTH

The amount of momentum spread δ_b the pair of square barrier pulses can trap is given by [3]

$$\delta_b = \sqrt{\left(\frac{2}{\beta^2|\eta|}\right)\left(\frac{eV_0T_1}{E_0T_0}\right)}\,, \tag{1}$$

where E_0 is the total energy of the particle and β is velocity relative to the velocity of light. Note that the barrier voltage V_0 and barrier width T_1 in Eq. (1) become $V_0T_1 \to \int_{\rm barrier} V(\tau)d\tau$, when the barrier wave is of arbitrary shape than square. To confine $\delta_b = 0.018$ say, we need $V_0T_1 = 173.26$ kV-μs . It is not good to use too small a barrier voltage, because this will make the width of the barrier too wide. Remember that the stable bucket consists of a rectangular part where particles do not see the barrier pulses and two curved parts where the particles are exposed to the barrier voltage. A large barrier width increases the curved parts of the bucket and more particles will be left outside the bucket when the barrier waves are switched on. Too narrow a barrier width is also not desired. This will boost the barrier voltage to too high a value, making it more difficult to generate. Also, whenever a particle drifts towards the barrier, it will gain or lose energy by an amount equal to V_0 per turn, independent of whether the barriers are moving or not. If this change in energy is too large, some particles may be thrown outside the maximum momentum offset $\delta_{\rm max}$ of the bunch even when the barriers are not moving. In order to preserve the conservation of the bunch area, the barrier voltage must be limited to $eV_0/(\beta^2E_0) \ll \delta_{\rm max}$. This constraint gives $V_0 \ll 18600$ kV using $\delta_{\rm max} = 0.005$. We actually choose $V_0 = 625$ kV and $T_1 = 0.30$ μs in our simulation. Then, a particle with $\delta = 0.005$ will lose its extra energy in approximately $E_0\delta/(eV_0) = 31.4$ turns and penetrate the barrier by an amount approximately equal to $\tau_{\rm penetrate} = |\eta|\beta^2E_0T_0\delta^2/(2eV_0) = 0.0314$ μs. These barrier waves can produce a bucket height of $\delta_b = 0.0187$ when $\tau_{\rm penetrate} = T_1$, the barrier width.

5 MOMENTUM-OFFSET DISTRIBUTION

To get an estimate of the momentum spread of most of the particles after each squeezing by the barrier pulse, we ne-

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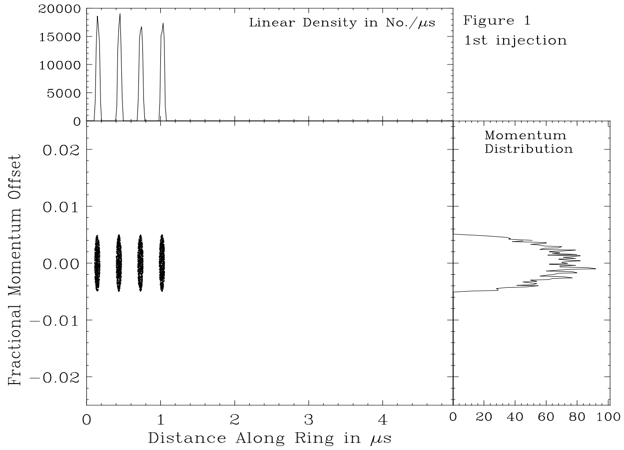

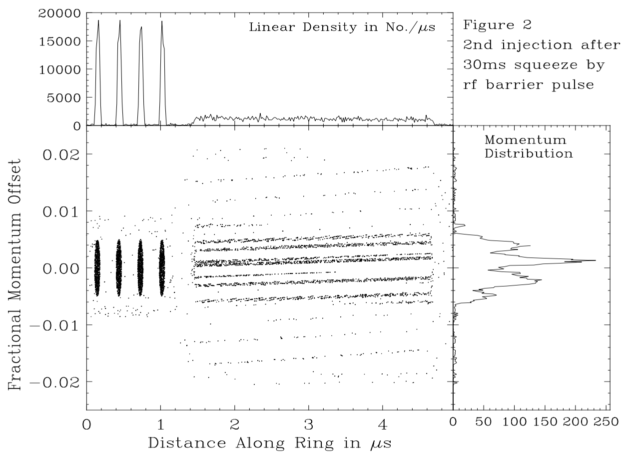

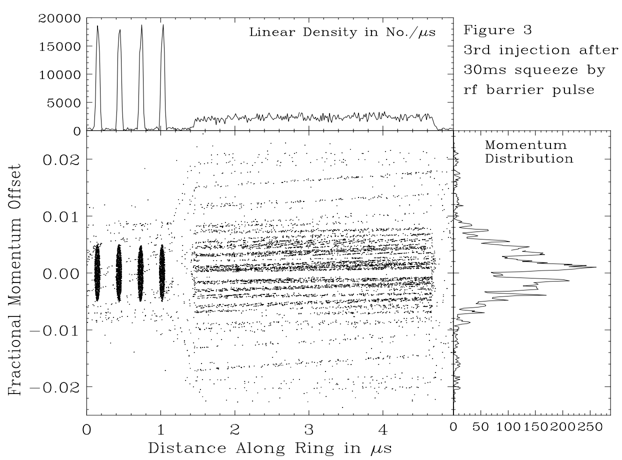

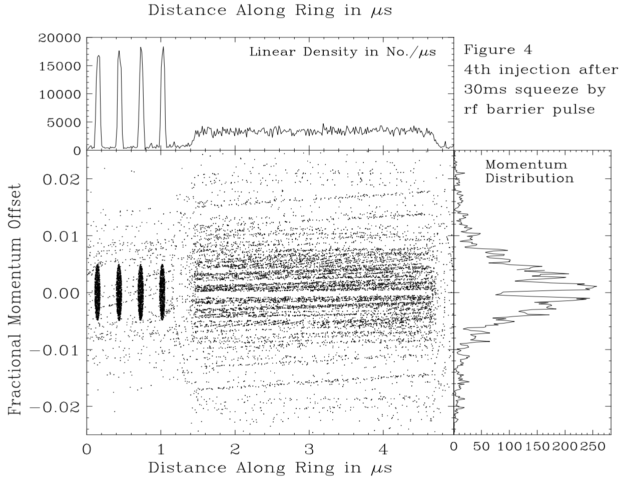

glect the curved part of the barrier bucket. The rectangular part of the bucket has a width of $T_{2\,\rm init} = T_0 - 2T_1$ at the time when the barrier waves are introduced, and becomes $T_{2\,\rm final} = \frac{13}{17}T_0 - 2T_1$ at the end of the squeeze. The momentum spread will be increased by the factor $F = (T_0 - 2T_1)/(\frac{13}{17}T_0 - 2T_1) = 1.356$. Ideally, in the fourth injection after the third squeezing by the rf barrier, the momentum spread should increase only by the factor $F^3 = 2.547$ to $\delta = \pm 0.0127$. We see in Fig. 4a that for most part of the beam, the momentum spread actually increases by such a ratio after 3 barrier squeezes. However, there is a small part of the beam having momentum spread as large as $\delta = \pm 0.025$ or even ± 0.030. This is because the above consideration is correct only for a bunch that is initially at equilibrium inside the barrier bucket. Here, the beam particles are *captured* into the barrier bucket when the barrier pulses are turned on. Since we have a debunching before capturing into the barrier bucket, particles can be anywhere along the phase axis at the time of capture. For those particles that are captured into the curved parts of the bucket and are very close to the boundaries of the bucket, they can acquire large amount of energy through the barrier pulses and leave the barrier pulse with much larger momentum offset than the estimate given above. It can be seen in Fig. 1 that there are particles with momentum offsets much larger than $1.356 \times 0.005 = 0.0068$ after the first squeeze by the moving barrier pulse. There are also particles that have not been captured into the barrier bucket at all. For a particle with initial momentum offset $\delta_{i0} > 0$ outside the stable barrier bucket, it will first drift across the moving barrier pulse as in Fig. 5, and result in a momentum offset of δ_{f1} given by

$$\left(\delta_{f1} + \frac{\dot{T}_2}{|\eta|}\right)^2 = \left(\delta_{i0} + \frac{\dot{T}_2}{|\eta|}\right)^2 + \delta_b^2 \ , \qquad (2)$$

where $\dot{T}_2$ is negative. The particle then drifts across the stationary barrier pulse to the space opened up by the moving barrier, after making synchrotron drifting once around the ring. The momentum offset will be reduced to δ_{i1} with $\delta_{f1}^2 = \delta_{i1}^2 + \delta_b^2$. Since the initial momentum offset is at most $\delta_{i0} = 0.005$, during the first synchrotron *rotation* (not oscillation or libration) outside the barrier bucket, we have therefore $\delta_{f1} = 0.0194$ and $\delta_{i1} = 0.0067$. On the average, this particle will encounter the moving barrier pulse 4 times during the 30 ms squeeze time. At the end of the first squeeze, we have $\delta_{f4} = 0.0206$. After that there is another 10 ms of debunching and some of these large-momentum-offset particles can land outside the barrier bucket again when the next barrier pulses are turned on. Thus, for the second squeezing, there may be particles having $\delta_{i0} = 0.0206$ to start with. At the end of the second squeeze after another 4 encounters with the moving barrier pulse, we obtain $\delta_{f4} = 0.0282$ by solving again Eq. (2). Continuing on in this way until the end of the third squeeze, we will have some particles with the largest momentum offset of $\delta_{f4} = 0.0340$. When we analyze the momentum distribution in Fig. 4 more carefully, we do find

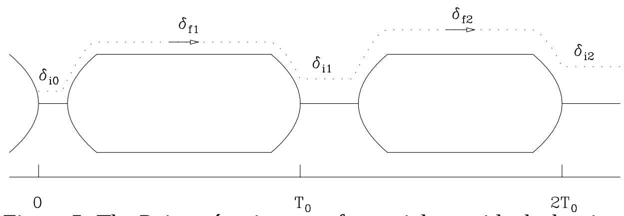

Figure 5: The Poincaré trajectory of a particle outside the barrier bucket with the left barrier pulse moving to the right.

18 macro-particles out of 16,000 in the momentum-offset range of 0.025 to 0.030, and 1 particle in the range of 0.030 to 0.035.

The above analysis depends on the time-integrated barrier voltage only. However, if we use a higher barrier voltage while keeping V_0T_1 constant, more particles will be captured into the larger stable barrier bucket, although the bucket height remains the same. Thus, the probability for particles to attain large momentum offsets outside the bucket becomes smaller. Moreover, because the larger barrier voltage increases only the rectangular area of the bucket but not the bucket height, the bunch area that has momentum offset within $\delta = \pm 0.005$ (for the first injection) will be relatively larger. Thus not so many beam particles will attain higher momentum offsets via synchrotron oscillations. When one of the barrier pulse moves, more particles will follow the momentum-offset increase. A simulation by doubling the barrier voltage to $V_0 = 1250$ kV while halving the barrier width to $T_1 = 0.15$ μs actually shows less particles landing at larger momentum-offsets. However, as was pointed out in the previous section, too high a barrier voltage is not desired.

6 DISCUSSIONS

6.1 Bunch width at injection

The simulation results depend very strongly on the momentum spread of the bunches at injection, but are very insensitive to the initial bunch length, since there is always a debunching period before every squeeze. In practice, however, the initial bunch length cannot be too long, because some gaps must be provided for the kicker rise and fall times. In the above simulations, the total bunch length is $\frac{1}{4}$ of a $h = 17$ rf wavelength. Thus the space between the end of the squeezed barrier bunch and the first bunch in the next injection is $\frac{3}{8}$ of a $h = 17$ rf wavelength, or 109 ns. There will be a gap of similar length between the fourth bunch and the front of the squeezed barrier bunch. These gaps will be long enough for the injection.

6.2 Double barrier pulses

Instead of using one negative pulse and one positive pulse to set up the barrier bucket and perform the bunch squeezing, we may utilize instead a pair of identical double pulses. Each double pulse consists of a positive voltage V_0 of duration T_1 followed by a negative voltage $-V_0$ of duration T_1 similar to one sinusoidal period of an rf wave. At switch-on, the two pulses overlap each other. As one pulse moves to the right while the other one remains stationary, the space opened up by the moving pulse also forms a stable barrier bucket between the negative half of the moving pulse and the positive half of the stationary pulse. Thus some particles will be trapped there and they will have their momentum offsets decreased gradually, because this bucket is becoming wider. However, there are disadvantages also. The particles that are trapped in the space opened by the moving barrier can be lost when the kicker is fired for the next injection. Also, stable barrier bucket only starts to form after the moving barrier moves a distance of T_1. Before that, the two barrier pulses overlap at least partially. At switch-on, the two barrier pulses overlap completely; i.e., an equivalent pulse height of $2V_0$ width T_1 followed by pulse height of $-2V_0$ width T_1. The barrier bucket forms at this moment will have a bucket height $\sqrt{2}\delta_b = 0.0257$ instead, where δ_b is the bucket height when single barrier pulses are used. Thus particles will bound off from the barriers having much larger momentum offsets. A simulation with the double barrier pulses using $V_0 = 625$ kV and $T_1 = 0.30$ μs shows that the momentum distribution spreads out wider.

6.3 Pros and cons of the method

There are pros and cons for using the barrier pulses in multiple injections. The advantage is obviously the much shorter exposure of bunches of very high linear intensity to the vacuum chamber, and we hope that no collective instabilities would develop during this shorter duration. For example, the linear linear density in Figs. 1 to 4 has been reduced by a factor of $\sim$ 6, and the reduction will be more significant if the bunch width at injection becomes narrower. The disadvantage is that microwave instability can develop during debunching when the local momentum spreads of the debunched bunches become small enough. Also, because of the introduction of the barrier pulses and the movement of one of them sends quite a number of beam particles to large momentum offsets, the momentum spread of the final beam will become much larger. Finally, there must be another recapturing of the beam particles into the $h = 17$ rf buckets for acceleration. Beam loss will become inevitable during the recapturing. Thus, there will be beam loss as well as emittance blowup during the whole procedure, which may or may not be tolerable.

Another method is to lengthen the bunches in the booster by bunch rotation and perform simple bucket-to-bucket injection into the main ring. For example, the local linear density will be reduced by a factor of 3.2 already, if each bunch is lengthened to occupy 80% of the $h = 17$ bucket. The gap between two consecutive bunches becomes 58 ns and is still long enough to accommodate the kicker rise or fall time. Since no recapturing will be necessary, the beam loss during injection can be kept to a minimum.

7 REFERENCES

[1] Some preliminary simulations have been performed at the Institute of Nuclear Study of Japan.

[2] K.Y. Ng, *Injection of JHP Main Ring Using Barrier Buckets*, Fermilab Report FN-654, 1997.

[3] S.Y. Lee and K.Y. Ng, *Particle dynamics in storage rings with barrier rf systems*, 1996, to appear in Phys. Rev. **E**.