# A NOVEL DESIGN FOR A HIGH POWER SUPERCONDUCTING DELAY LINE

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# Abstract

Potential designs for a high power superconducting delay line of approximately 10ms duration are described. The transmitted signal should have low dispersion and little attenuation to recapture the original signal. Such demands cannot be met using conventional metal conductors. This paper outlines a proposal for a new transmission line design using low temperature superconducting material which meets system specifications. The 25W line is designed to carry pulsed signals with an approximate rise time of 8nsec and a maximum voltage magnitude of 25kV. Predicted electrical design and performance of the line will be presented.

# **1 APPLICATION OF THE LINE**

The line will be used to provide transit time isolation of the reflections expected to be generated in the kicker of an AHF accelerator [1]. The kicker will operate with beam currents of 3 to 6 kA and will launch beam-induced voltages up the transmissions lines connecting it to its pulsers. The most straightforward way to eliminate the effects of these reflections is to transit time isolate the kicker from the pulser. Thus the 10  $\mu$ sec line will actually allow a 20  $\mu$ sec operational interval.

# 2 WHY USE SUPERCONDUCTORS?

# 2.1 Attenuation

The general expression for attenuation is

$$\alpha = \frac{1}{2} \frac{R_s}{Z_0} \sqrt{\frac{\omega}{2}}$$

where  $R_s$  is the skin-effect impedance dependent on the geometry and resistivity of the structure [2].

For a coax where *a* and *b* are the inner and outer diameter,  $R_s$  is derived to be

$$R_{s} = \frac{\sqrt{j\omega}\mu\Gamma_{r}^{2}}{2\pi} \left[ \frac{1}{a^{2}} - \frac{1}{b^{2}} + \frac{1}{2\Gamma_{r}a} \frac{I_{0}(a/\Gamma_{r}) + I_{2}(a/\Gamma_{r})}{I_{1}(a/\Gamma_{r})} + \frac{1}{2\Gamma_{r}b} \frac{K_{0}(b/\Gamma_{r}) + K_{2}(b/\Gamma_{r})}{K_{1}(b/\Gamma_{r})} \right]$$

where  $I_n$  and  $K_n$  are the modified Bessel functions of the first and second kind.  $\Gamma_r$  is associated with the skin depth and is

$$\Gamma_r = \begin{cases} 1 / \sqrt{j\varpi\mu\sigma} & \text{normal conductor} \\ 1 / \sqrt{j\varpi\mu\sigma + 1/\lambda^2} & \text{Type I superconductor} \end{cases}$$

where  $\lambda$  is the penetration depth of the superconductor. At high frequencies the expression simplifies to a more familiar expression [2][3].

$$R_s = \frac{\sqrt{j\omega}\mu}{2\pi} \Gamma_r \left(\frac{1}{a} + \frac{1}{b}\right).$$

Suppose the line was made of copper ( $\sigma = 5.92e7(\Omega \text{-m})^{-1}$ ) and the inner and outer diameters were 2cm and 3cm respectively. With an 8nsec. rise time, we used 125MHz to be the highest critical frequency, which is about 3 times that of the roll-off frequency. For 125MHz,  $\alpha = 7.658e\text{-}4/\text{m}$ . For a 3km line, this corresponds to a power attenuation of  $(1\text{-}e^{-2\alpha\cdot3000\text{m}})=98.9\%$ . The signal has virtually disappeared!

On the other hand, for the same set of dimensions, a niobium  $(\lambda = 39 \text{nm}^{\text{o}}, \sigma = 8.03 \text{e}6(\Omega \text{-m})^{-1})$  line yields an  $\alpha$  of approximately 5.1025 e-6(1 + j)/m. The imaginary term contributes to the inductance of the line. The power attenuation is now 3% at 125MHz which is an upper limit. This is the most compelling reason to use a superconducting line.

# **3 DESIGN OF A SUPERCONDUCTING LINE**

The size of the transmission line is fundamentally limited by the critical field of the superconducting material. Of the type I conductors, niobium (Nb) is the most generous material since it has the highest critical *B*-field of .206T [2] at T=0K.

Assuming the distribution for the critical field to be  $B_c(T) = B_c(0)[1 - (T/T_c)^2]$ , the critical field at 4.2K (the usual operating point of liquid helium) is .1635T. The *B*-field of a structure can be written as B=gI where *g* is a geometry factor. The condition on *g* is therefore  $g \le B_c/I$ .

<sup>(1)</sup> see back cover of [3].

# 3.1 Coaxial line

For a concentric coax,  $g = \mu/2\pi r$  where *r* is either the inner or outer radius of the line. This yields the condition that  $r > \mu I/2\pi B_c$ . If one wants to operate at 20% of maximum capacity, then  $a = .2\mu I/2\pi B_c$  where *a* is the inner radius. The current on the line can be as high as 1kA. This requires that *a*=6.12mm.

Since the layer of niobium only needs to be much greater than the penetration depth  $\lambda$ , a thin coat of approximately .5um will be deposited on a copper substrate as shown in Fig. 1. The need for a metal substrate will be discussed in section 4. The area between the two conductors will be filled with a material that may or may not have dielectric properties. The inner conductor should be hollow to allow helium to flow through the tube.



Fig. 1. Coaxial line with Nb layers deposited on copper tubes

Design 1: The outer conductor radius, for an impedance of  $25\Omega$ , is 9.28mm. Design 2: If one puts in a dielectric material of  $\varepsilon_r=10$ , the outer radius goes up to 22.85mm. The advantage of the dielectric is that the length of the line is reduced by a factor of  $1/\sqrt{10}$  which yields a length of 948m.

# 3.2 Stripline

For manufacturing considerations, it may be easier to fabricate striplines instead of coaxial lines. The solution for the stripline problem, both open and closed, has been derived by Primozich [4]. As it turns out, a stripline with a dielectric filler is too dispersive (see section 5). Therefore, the design cannot have a dielectric material. Design 3: The stripline dimensions are shown in Fig. 2.



Fig. 2. Dimensions of stripline design

## 3.3 Sheath helix

Sheath helix lines are like coaxial lines, only the inner conductor is wrapped at an angle. These are slow wave structures and ideal for a delay line. However, they are more dispersive than regular coax. Fig. 3 shows two sets of two overlayed 10kV pulses. The solid line is the actual signal at the output of the line and the dotted is the output had there been no dispersion. These plots are derivations based on Lund [5] and are for uncurved lines only.

Design 4: Fig. 3(a) corresponds to a line with  $\varepsilon_r = 1$ , b/a = 1.15, p(pitch)/a = 2.1, and v/c = .336. The line length is reduced to about 1/3 of the original length. Part (b) corresponds to  $\varepsilon_r = 3$ , b/a = 1.4, p/a = 2.75, and v/c = .264. The second line yields a slightly slower wave but at the cost of increased dispersion as evident by the wiggles at the rising part of the pulse.



Fig. 3. a) — 10 kV pulse at output of line for sheath helix without dielectric, – – same pulse if line were non-dispersive

b) same as a) except line has dielectric. Wiggles indicate more dispersion

The sheath helix line is presented here as a backup design in case the stripline is not easy to manufacture because of it's length or the dielectric used for the coaxial line proves to be lossy or difficult to work with.

#### **4 THE NEED FOR A SUBSTRATE**

Niobium can be deposited on copper as illustrated in Fig. 1 where the fields and currents flow. The copper acts as a stabilizer in case the superconducting portion exceeds critical current and quenches. Quenches can distort the original signal by introducing transient high resistance to the line.

Copper is also a much less expensive conductor than niobium so the minimal use of niobium is advantageous from a bulk material cost point of view. In addition, if the helix line is chosen as the final design, a substrate for the inner conductor will need to be implemented.

## **5 DISPERSION**

The previous section has already shown that dispersion from skin depth is negligible for a superconducting line. The dispersion on the superconducting transmission line, instead, is caused by wrapping the long line onto either a drum or into a pancake shape. The electromagnetic problem for a curved coax has been solved in [7].

The propagation constant for a curved coax is as follows

$$\gamma^{2} = \gamma_{0}^{2} \left[ 1 + \frac{A_{2}}{R^{2}} + O(\frac{1}{R^{4}}) \right]$$

where  $\gamma_0 = \omega \sqrt{\mu \varepsilon}$ ,  $A_2 = A \omega^2 + B$ , and R = R(s). The constants *A* and *B* can be found in [7]. For a pancake design, the outer cable radius is, as a function of *s*, the total length, and  $R_0$ , the inner radius,

$$R(s) = \sqrt{R_o^2 + \frac{2bs}{\pi}}$$

If one wants to find the total dispersion at the end of the line, the propagation constant would need to be integrated over the length of the line since it varies with *s*. In other words,

$$e^{-j\gamma s} \rightarrow e^{-j\int_{0}^{s}\gamma ds}$$

Therefore, the phase shift of the signal at the end of the line is

$$\Delta \phi = -\frac{A_2(\omega)}{2} \gamma_0 \int_0^s \frac{ds'}{R^2(s')} = -\gamma_0 \frac{\pi A_2}{4b} \ln \left( 1 + \frac{2bs}{\pi R_0^2} \right)$$

The phase velocity and  $\Delta \phi$  for design 2 is plotted

in Fig. 4. As one can see in Fig. 4b, the phase distortion is less than  $1^{\circ}$  at the end of the line. The striplines were treated as equivalent coax lines to estimate the dispersion due to bending.



Fig. 4. a) phase velocity as function of position in a pancake-wound coax line

b) it's corresponding phase shift at 125 MHz

The parameters for designs 1-3 were run through a kicker dynamic code to examine the acceptability of the output signal. All 3 designs produced clean output signals. An output plot of design 2 is shown in Fig. 5 where  $R_0$ =1m.



Fig. 5. Design 2 placed in kicker dynamics program produced almost exact replication of original pulse (figure actually has 2 overlapping curves)

# **6 CONCLUSION**

Attenuation in long transmission lines made use of superconductors a necessity. The attenuation in the superconducting line is very small, with joints connecting sections of the line contributing to most of it. Dispersion, on the other hand, can arise simply from the geometry of the line. Curved lines and helix lines are subject to dispersion. Two coaxial lines, one stripline, and a sheath helix design were presented. The challenge of fabricating the line will determine which design is ultimately adopted.

## **7 ACKNOWLEDGEMENTS**

The authors would like to acknowledge the technical support of Michael Krogh of AlliedSignals and Steve Sampayan. This work was performed under the auspices of the U.S. Department of Energy by the Lawrence Livermore National Laboratory under Contract No. W-7405-Eng-48.

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