NONLINEAR LONGITUDINAL DYNAMICS STUDIES AT THE ALS *

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Abstract

We present an account of our efforts to understand unexpected observations in the course of performing measurements of the longitudinal beam transfer function. As the amplitude of excitation was increased, we observed a notch in the middle of the peak amplitude response of the transfer function. Our observations are explained by a bifurcation in the amplitude due to the nonlinearity of the oscillations, demonstrated by measurements of the longitudinal bunch profile using a streak camera.

1 INTRODUCTION

Beam transfer function (BTF) techniques are common in accelerators. In its simplest form, a swept frequency drive is used to excite beam oscillations in order to measure the synchrotron or betatron tunes. The BTF can also be used to measure the beam impedance and various other machine parameters. While engaged in such measurements at the Stanford Linear Collider damping ring (SLCDR), one of us observed an unusual phenomenon in the longitudinal transfer function[1]. As the level of excitation was increased, a deep notch appeared in the amplitude response of the BTF. In similar studies at the Advanced Light Source (ALS), we were able to reproduce the results observed at the SLCDR. In the course of further investigations at the ALS, we were led into the rich field of nonlinear dynamics and were treated to some stimulating observations of beam behavior. This paper describes our investigations and the physics which explains our observations.

2 MEASUREMENTS

In the fall of 1996, we began a program of machine studies using the longitudinal BTF to measure various machine parameters. In general, a network or FFT signal analyzer is used to excite longitudinal oscillations via phase modulation of the RF voltage. Beam oscillations are detected using a standard phase detection technique. A more complete description of the experimental setup is given in reference [2]. The BTF is defined as the relative amplitude and phase of the beam response to excitation as a function of frequency. Shown in Figure 1 is a plot of the amplitude and phase of single bunch longitudinal BTF measurements as a function of increasing excitation. For this case, the excitation frequency was swept from lower to higher frequency.

At small excitation amplitudes, the BTF agrees with theory[2]. However, we were surprised to see the distortion of the BTF as the excitation was increased, particularly the deep notch and sudden 180 degree phase shifts in both the amplitude and phase. We resolved to uncover its source. Because longitudinal oscillations are well known to be nonlinear, especially at larger amplitudes, we looked in the direction of a nonlinear effect.

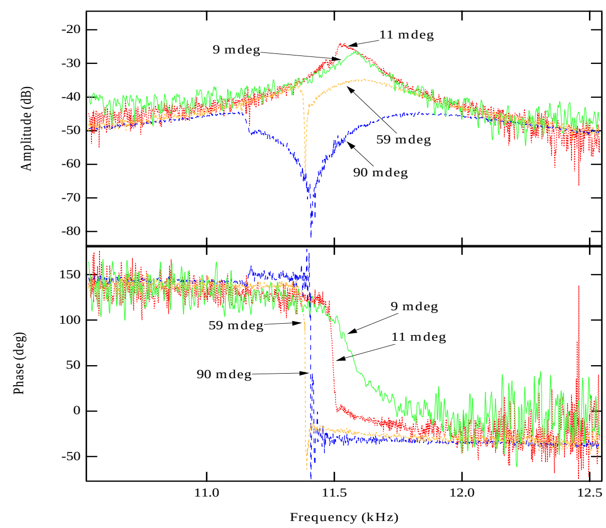

Figure 1: Amplitude and phase of longitudinal beam transfer functions for increasing frequency sweeps as a function of drive amplitude.

To make a more detailed study of the longitudinal dynamics near the notch in the transfer function, we used a streak camera (SC) to observe the evolution of the longitudinal distribution as the modulation frequency sweeps upward through the synchrotron frequency. A schematic diagram of the Hamamatsu C5680 SC is shown in Figure 2. The SC works by converting the time structure of an optical light pulse into vertical deflection at the CCD camera. In our experiments, the vertical deflection plates are driven by a 125 MHz sinusoidal voltage synchronized to the beam. In addition, there is an optional slow horizontal deflection which allows observation of the longitudinal profile as a function of time. The time scale of the horizontal sweep can be adjusted to observe several turns or thousands of turns. For sweep times longer than several hundred turns, individual turns can no longer be resolved and so the longitudinal profile appears as a continuous line across the image.

Shown in Figure 3 are images of the longitudinal profile vs. time for three modulation frequencies (f_m): below the frequency where the notch appears in the BTF, at the notch

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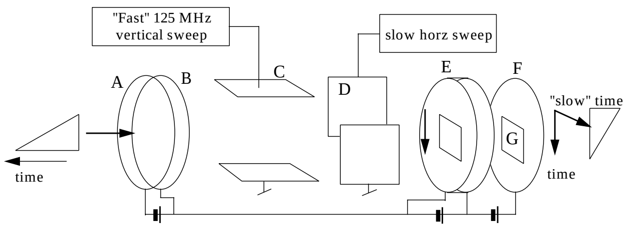

Figure 2: Schematic diagram of the streak camera in synchroscan mode with dual sweep. A: phothcathode, B: accel. mesh, C: vert. deflection electrode, D: horz. defl. electrode, E: microchannel plate F: phosphor screen, G: CCD camera.

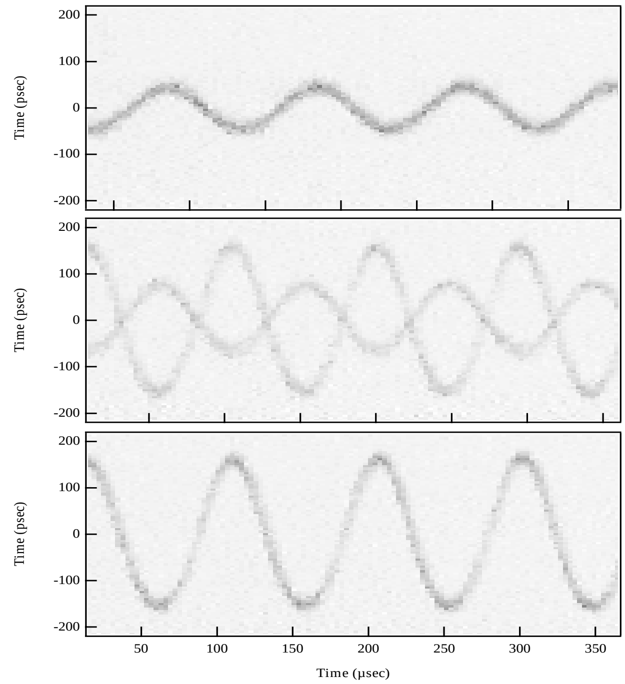

Figure 3: Images of the longitudinal profile (in psec) vs.time using the streak camera in dual scan mode: top) f_m below frequency of notch in BTF; middle) f_m at frequency of notch; bottom) f_m above notch frequency.

frequency, and above. The darker area represents higher intensity. For example, in Figure 3a, the sinusoidal pattern of the distribution is due to the phase modulations (the nominal RMS bunch length is 15-20 psec.) At this level of excitation, the bunch has oscillation amplitude of about 100 psec peak–peak. At the notch frequency, the bunch appears to split into two separate beamlets, oscillating with different amplitudes and out of phase by 180 degrees. Above the notch, the original bunch pattern disappears and only the second bunch pattern is evident.

Assuming that the streak camera profiles correctly indicate the presence of two beamlets in the same RF bucket, then an explanation for the notch in the BTF data presents itself. The phase detector for the BTF is sensitive only to *net* dipole phase oscillations of the bunch. As the modulation frequency passes through the notch, at some frequency the dipole moment of the two islands cancels.

Although the mystery of the notch in the BTF is solved, the question remains as to why the beamlets are formed. Fortunately, the theory of nonlinear longitudinal oscillations in the presence of phase modulation is well documented[3]. We provide here only a qualitative discussion of the theory.

Consider a driven harmonic oscillator (HO) which has a frequency dependence on the oscillation amplitude such that the frequency decreases at larger amplitudes. When the HO is driven near resonance, it reaches larger amplitudes which causing a detuning to lower frequencies. Synchrotron oscillations can be modeled in this form. If the longitudinal coordinates are expressed in a frame rotating with the modulation frequency, we can plot the phase space of the oscillations as shown in Figure 4. The two stable fixed points are marked A and B; C is unstable. The outer island corresponds to the point at 11 kHz on the upper branch of the response curve as shown in Figure 5. One characteristic feature of the response is that below some frequency, there are two stable amplitude responses while above this frequency there is a single stable response. The transition between these two is known as a bifurcation. Note that the outer island in the phase space representation moves to larger phase as the modulation frequency decreases below the bifurcation frequency. Radiation damping is ignored.

Consider a situation where both islands A and B are populated with electrons. In the lab frame, we observe the projection of the distribution on the horizontal axis in Figure 4 as the phase space diagram rotates about the origin. When the islands are situated as shown, the two beamlets are at their maximum separation in phase (or time). When the phase space has rotated by 90 degrees, the beamlets are coincident in phase but at their maximum energy separation. Figure 3a corresponds to the situation where there are two stable islands but only island B is populated. In Figure 3b, the modulation frequency has increased and islands A and B are populated but oscillating at different amplitudes. In Figure 3c, the modulation frequency is above the bifurcation frequency and only island A is stable.

Another issue is the mechanism for electrons to go from one stable island to another. We believe that the dominant diffusion mechanism is Touschek scattering. In this process, an elastic collision between electrons within a bunch has a finite probability of transferring enough energy to the longitudinal momentum such that the electrons are no longer within the RF bucket. Typically, the electrons scattered outside the momentum acceptance are lost. For the case of two stable islands as shown above, electrons are scattered outside the separatrix of island B whereupon the majority are damped into island A. To test this hypothesis,

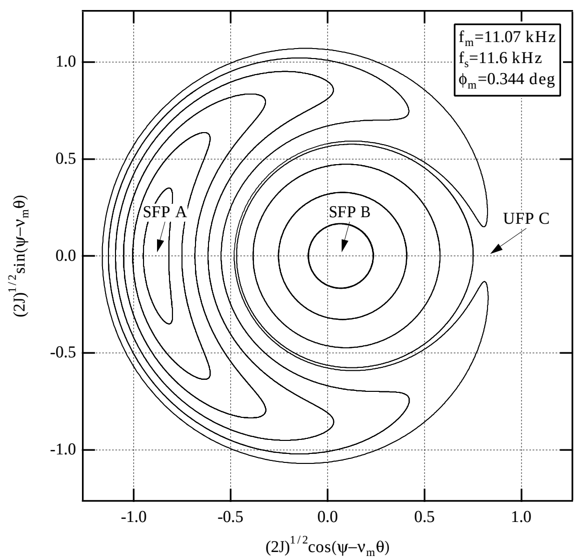

Figure 4: Phase space diagram in a frame rotating at the modulation frequency. The two stable fixed points are shown.

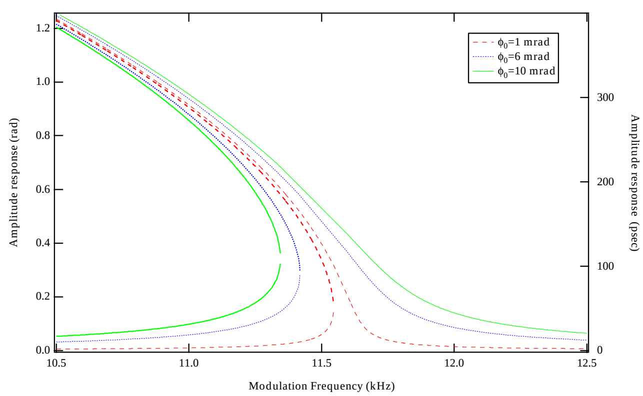

Figure 5: Response of driven longitudinal oscillations for 3 excitation amplitudes. The nominal synchrotron frequency is 11.6 kHz.

we modulated at a fixed frequency and recorded the longitudinal profile vs. time. An example is shown in Figure 6. The diffusion into the outer island is evident as well as a small decrease in the inner island. We extracted the diffusion rate by comparing the relative rate of increase of the peak signal in the outer island compared to the inner island.

A plot of the diffusion rates as a function of modulation frequency for several beam currents is shown in Figure 7. Calculations of the scattering rate using the separatrix of island B as the momentum acceptance show approximate agreement with the measurements. We are currently analyzing this in more detail.

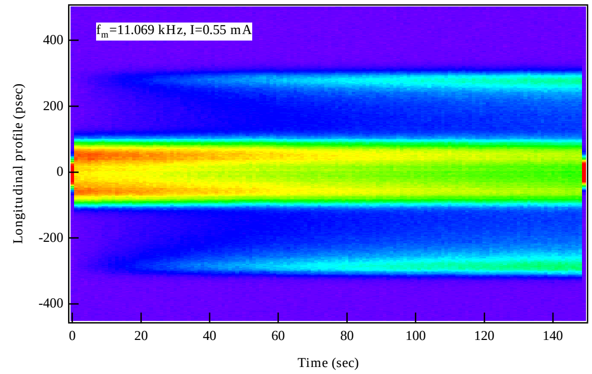

Figure 6: Longitudinal profile vs time at a fixed modulation frequency and amplitude showing diffusion from the inner to the outer island.

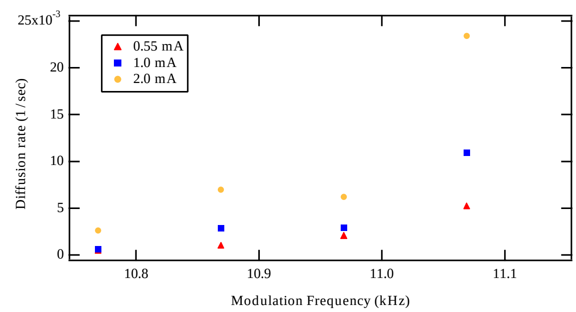

Figure 7: Diffusion rates as a function of modulation frequency for 3 different bunch currents.

3 CONCLUSIONS

The behavior of the BTF for upward swept frequency excitation at large amplitudes can be understood through the nonlinear dynamics of synchrotron oscillations. The notch in the BTF is explained by the presence of two beamlets formed in the two stable islands below the bifurcation frequency. Similar effects are observed for downward swept excitation. We hope that the diagnostics of nonlinear longitudinal dynamics can be extended further to be useful in understanding topics such as higher order momentum compaction, collective effects, and Touschek and intrabeam scattering. We are grateful to Prof. S.Y. Lee for helpful discussions.

4 REFERENCES

[1] M. Minty, F. Zimmermann, *Longitudinal beam transfer function measurements at the SLC damping rings*, these proceedings.

[2] J. Byrd, *Longitudinal beam transfer function diagnostics in the ALS*, these proceedings.

[3] H. Huang, et. al.,*Experimental determination of the Hamiltonian for synchrotron motion with rf phase modulation* Phys. Rev. E **48**, 4678, 1993. G. Tsironis, S. Peggs, T. Chen, *Tune modulation, Mathieu stability, and the driven pendulum*1990 EPAC, p 1753.