THE EXTENDED TOUSCHEK LIFETIME

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Abstract

Scattering of particles within the bunch is called Touschek scattering. If large enough, such an energy transfer may eject the particle out of the bunch. If a particle is scattered in the dispersive region, it will induce a horizontal betatron oscillation which will be coupled into vertical motion when it passes through skew components. The amount of coupling is expressed in terms of the coupling coefficient, χ. If the coupling coefficient is large enough, the resulting vertical oscillations may exceed the normally small vertical admittance of the ring. Thus the particles may be lost even though the energy loss is within the momentum acceptance. The lifetime associated with this loss mechanism is called the extended Touschek lifetime. In the usual Touschek lifetime calculation, the lifetime increases as the coupling increases. Including the effect of the vertical oscillation results in a decrease of Touschek lifetime beyond some coupling value.

1 INTRODUCTION

Users of synchrotron radiation sources need hours to complete their experiments. Thus the beam of the Advanced Photon Source (APS) storage ring at Argonne National Laboratory is assumed to circulate stably for a minimum of 10 hours.

The main contributions to the total beam lifetime come from residual gas scattering and Touschek scattering. The residual gas scattering is comprised mostly of single-Coulomb scattering and bremsstrahlung. The single-Coulomb scattering or bremsstrahlung involves elastic or inelastic collisions between the bunch and the surrounding residual gas. Touschek scattering involves scattering of particles within the bunch. A particle may be ejected out of the bunch.

When introducing dispersion into the beam lifetime, we direct our attention to Touschek scattering. A particle passing through the dispersive region will induce a horizontal betatron oscillation which, when passing through skew components, will be coupled into vertical motion. If the coupling coefficient χ is large enough, the resulting vertical oscillations may exceed the small vertical admittance of the ring. The inclusion of this loss mechanism is called the extended Touschek lifetime. The residual-gas lifetime and the extended Touschek lifetime will reduce the total beam lifetime, causing it to slip below the minimum lifetime.

The program ZAP [1] has been altered to take into account the possible loss due to induced betatron oscillations. At each lattice position the energy loss required to produce, by coupling, a vertical oscillation that exceeds the vertical aperture is calculated. When this energy loss is less than the rf bucket half-height, it replaces the rf bucket half-height in the Touschek integral.

2 THE TOUSCHEK INTEGRAL

The normal Touschek lifetime, derived in Bruck [2], increases with increasing coupling coefficient. However, if horizontal oscillations in the dispersive region are included, an increased coupling coefficient can lead to vertical oscillation, which may exceed the normally small vertical apertures. When one includes this effect, one sees that increased coupling leads to decreased lifetime.

A bunch traveling in the dispersive region of the storage ring loses momentum Δp_1. This produces a horizontal betatron displacement η and its derivative η', given as:

$$x = \eta\left(\frac{\Delta p_1}{p_0}\right) \text{ and } x' = \eta'\left(\frac{\Delta p_1}{p_0}\right) \tag{1}$$

where p_0 is the design momentum. This displacement and slope leads to the Courant-Snyder invariant

$$W_x = \left(\frac{\Delta p_1}{p_0}\right)^2 \frac{1}{\beta_x}\left(\eta^2 + \left(\beta_x\eta' - \frac{1}{2}\beta_x'\eta\right)^2\right) = \left(\frac{\Delta p_1}{p_0}\right)^2 \boldsymbol{H}, \tag{2}$$

where $\boldsymbol{H}$ is the so-called Sands factor [3]; this is shown in Fig. 1 for the APS half-sector of length L = 13.8 m. Then the coupling coefficient χ is the ratio of the Courant-Snyder invariants $W_{y;max}/W_{x;max}$. The maximum W_y is determined by the vertical admittance, A_y

$$W_{y;max} = A_y = \frac{a^2}{\beta_y}, \tag{3}$$

where a is the vertical half-aperture, and β_y is the vertical β-function, at the center of the insertion region. β_y = 10 m at insertion of the APS storage ring, and a is assumed to be 2 mm, 3 mm, and 4 mm. Combining Eqs. (2) and (3) we obtain

$$\left(\frac{\Delta p_1}{p_0}\right) = \left(\frac{\Delta p_1}{\gamma}\right) = \sqrt{\frac{a^2}{\chi\beta_y\boldsymbol{H}}}, \tag{4}$$

$$\varepsilon_1 = \left(\frac{\Delta p_1}{\gamma\delta p_x}\right)^2. \tag{5}$$

The extended Touschek lifetime integral is

$$\frac{1}{\tau_{eT}} = \frac{\sqrt{\pi}r_0^2cN}{\gamma^2(\delta p_x)^3V}\int_{\varepsilon_0}^{\infty}\frac{1}{u^2}\left(\frac{u}{\varepsilon_1} - \frac{1}{2}\ln\left(\frac{u}{\varepsilon_1}\right) - 1\right)\exp(-u)du, \tag{6}$$

where r_0 is the classical electron radius, c is the speed of light, N is the number of positrons per bunch, γ is the design energy divided by the rest energy of the positrons, δp_x is related to the rms angular divergence, and V is the bunch volume [1]. In this equation ε_0 is the smaller of either ε_1 or ε_{rf}. The design momentum p_0 in Eqs. (4) and (5) is written in units of m_0c so that, for relativistic particles, $p_0 = m_0\gamma v/m_0c \approx \gamma$. The lower limit in Eq. (5) depends on the Twiss parameters, the dispersion and its derivative, the coupling coefficient χ, and the vertical half-aperture a.

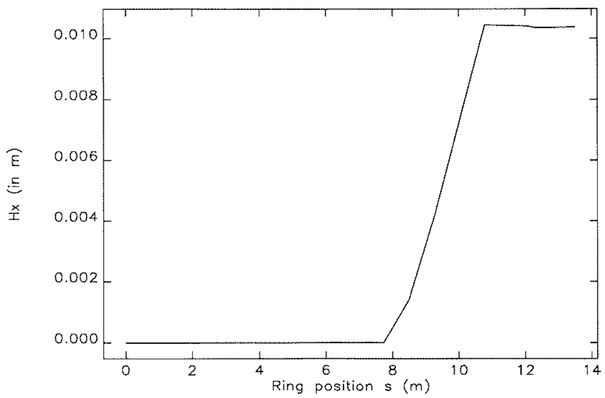

Figure 1: Sands factor.

3 THE METHOD OF CALCULATION

The extended Touschek integral in Eq. (6) and the lower limit in Eq. (5) are evaluated using the modified code ZAP. ZAP has the option of calculating the normal Touschek lifetime or the extended Touschek lifetime. If the latter option is used, the user inputs the vertical half-aperture a in mm, followed by β_y, the vertical β-function at the center of the insertion region in m.

The routine on the extended Touschek lifetime treats betatron oscillations produced in the dispersive region. The dispersive region is typified by the non-zero Sands factor $\boldsymbol{H}$. If $\boldsymbol{H} = 0$, the lower limit will be ε_{rf}.

If $\boldsymbol{H} \neq 0$, then ZAP calculates the lower limit due to betatron oscillations produced in the dispersive region via Eqs. (4) and (5). If $\varepsilon_1 < \varepsilon_{\mathrm{rf}}$, then ε_1 is used in Eq. (6) and the loss mechanism of extended Touschek lifetime is used; otherwise the loss mechanism of normal Touschek lifetime is used. Having decided what the lower limit will be, ZAP proceeds to evaluate the Touschek integral.

ZAP accepts horizontal and vertical emittances rather than the coupling coefficient. Those are calculated by

$$\varepsilon_x = \frac{\varepsilon_n}{1+\chi} \quad \text{and} \quad \varepsilon_y = \frac{\chi\varepsilon_n}{1+\chi}, \qquad (7)$$

where $\varepsilon_n = 8.2 \times 10^{-9}$ m-rad is the natural emittance of the APS storage ring.

From the synchrotron radiation loss one obtains the synchrotron frequency. In the present calculation if we assume the synchrotron radiation loss is due to dipoles only, then the synchrotron frequency is 1.96 kHz. The energy spread is assumed to be 1.8×10^{-3} and 3.6×10^{-3} for bunch lengths of 1 cm and 2 cm, respectively.

4 ANALYSIS OF RESULTS

In general, the total beam lifetime is the inverse of the sum of the inverses of residual gas and Touschek lifetimes. If the limits due to oscillations in the dispersive region are included, the Touschek lifetime is replaced by extended Touschek lifetime. We thus have

$$\frac{1}{\tau_{tot-nT}} = \frac{1}{\tau_{gas}} + \frac{1}{\tau_{nT}} \quad \text{total with normal Touschek} \quad (8)$$

$$\frac{1}{\tau_{tot-eT}} = \frac{1}{\tau_{gas}} + \frac{1}{\tau_{eT}} \quad \text{total with extended Touschek} \quad (9)$$

where τ_{gas} is the residual gas lifetime, τ_{nT} is the normal Touschek lifetime, τ_{eT} is the extended Touschek lifetime, $\tau_{tot\text{-}nT}$ is the total beam lifetime with normal Touschek lifetime and $\tau_{tot\text{-}eT}$ is the total beam lifetime with extended Touschek lifetime. The total beam lifetimes are displayed in Figs. 2 through 5 for vertical half-apertures of 2 mm, 3 mm, and 4 mm, as shown in the legends to those figures. In the legends "normal" stands for $\tau_{tot\text{-}nT}$, calculated via Eq. (9), and "extended" stands for $\tau_{tot\text{-}eT}$, calculated via Eq. (10).

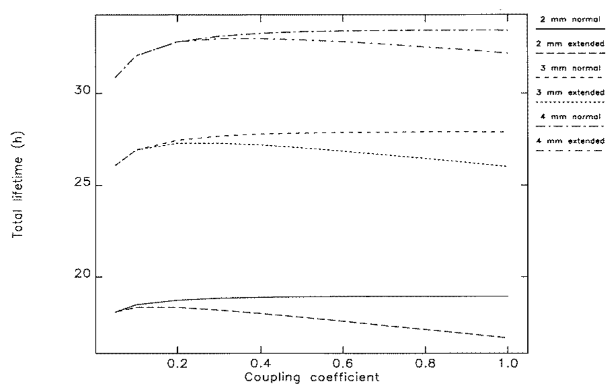

Figure 2: Total beam lifetime with and without extended Touschek lifetime for 5.22-mA bunch current, 0.028 bucket half-height and 2-cm rms bunch length.

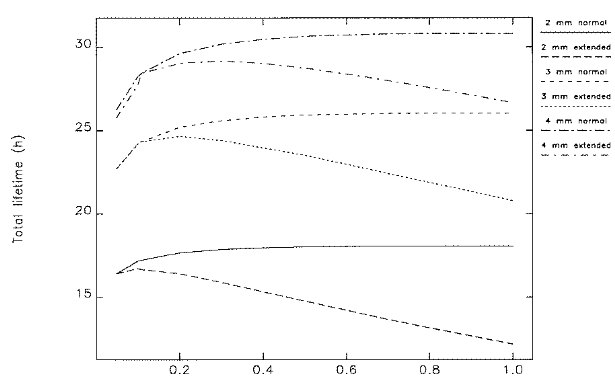

Figure 3: Total beam lifetime with and without extended Touschek lifetime for 5.22-mA bunch current, 0.028 bucket half-height and 1-cm rms bunch length.

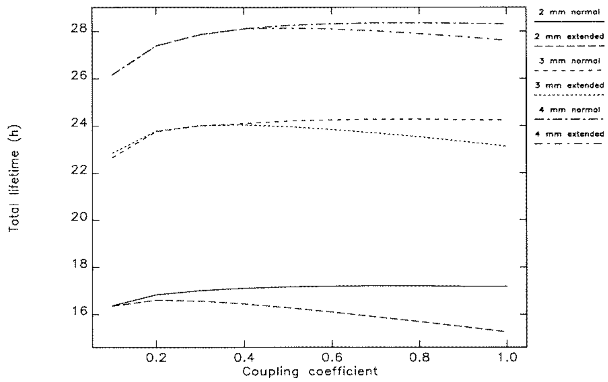

Figure 4: Total beam lifetime with and without extended Touschek lifetime for 5.22-mA bunch current, 0.02 bucket half-height and 2-cm rms bunch length.

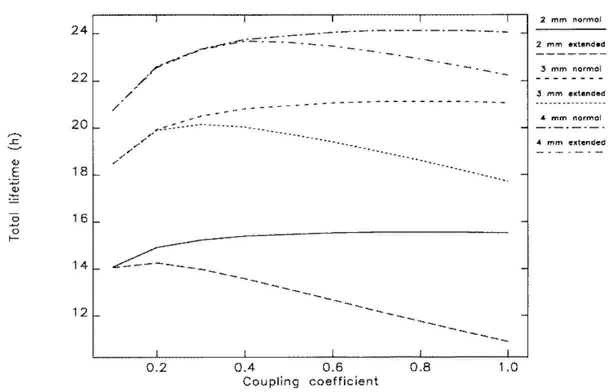

Figure 5: Total beam lifetime with and without extended Touschek lifetime for 5.22-mA bunch current, 0.02 bucket half-height and 1-cm rms bunch length.

For a given coupling coefficient, $\tau_{tot\text{-}eT}$ is always smaller than $\tau_{tot\text{-}nT}$. For a fixed coupling coefficient χ, the difference between $\tau_{tot\text{-}nT}$ and $\tau_{tot\text{-}eT}$ quantifies the influence of oscillations in the dispersive region. The difference increases with increasing coupling and decreasing vertical half-aperture, for fixed beam current, fixed rms bunch length, and fixed rf bucket half-height. The difference also increases with decreasing rms bunch length if all other parameters are kept fixed. This is seen from Figs. 2 and 3 on one hand, and Figs. 4 and 5 on the other.

The influence of betatron oscillations in the dispersive region may cause the total lifetime to slip below the minimum lifetime for optimum storage ring operation, or come perilously close to doing so. This can happen even for pressures as low as 1 nTorr if the rf bucket half-height, the rms bunch length, and the vertical half-aperture are small enough and the coupling coefficient is large enough. In Fig. 5 we have such a case. For 2-mm vertical half-aperture and 100% coupling, $\tau_{tot\text{-}eT}$ is 10.9 hours, while $\tau_{tot\text{-}nT}$ is 15.52 hours. (The minimum lifetime for optimum operation of the APS storage ring is 10 hours.) For a pressure of just 1.2 nTorr, the residual-gas lifetime is about 15.66 hours and the extended Touschek lifetime is about 26 hours, which means that the total beam lifetime, from Eq. (10), is 9.51 hours. The inescapable conclusion is: *under certain conditions, inclusion of betatron oscillations in the dispersive region may cause the total beam lifetime to become quite small, even at low pressures*. With higher pressures the situation can only get worse, because the residual-gas lifetime will be correspondingly lower. In such a case, not only bunches with very high coupling, but even bunches of lower coupling may be affected.

Under normal operating conditions the contribution to the lifetime resulting from oscillations in the dispersive region is not seen, because the coupling is too low. For example, in the APS storage ring the coupling is about 3%. The extended Touschek lifetime is not really different from the normal Touschek lifetime in this case.

5 ACKNOWLEDGEMENTS

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6 REFERENCES

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