A SELF-CONSISTENT BEAM LOADED TRAVELING WAVE ACCELERATOR MODEL FOR USE IN TRACE 3-D

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Abstract

An optics model of a constant gradient traveling wave (CGTW) accelerator structure has been implemented for TRACE 3-D. TRACE 3-D is an envelope code including space charge that is used to model bunched beams in magnetic transport systems and radio frequency (RF) accelerators when the effects of beam current might be significant. The new matrix model has been developed to allow incorporation of particle beam loading (current) effects on the accelerator gradient and the accelerator structure's beam focusing properties in a self-consistent manner. The beam loaded electric field for a CGTW accelerator structure is constant for only a particular design current (e.g. 0 current), otherwise it can be written as a function of accelerator attenuation and axial position along the structure. The variation of the electric field through the structure has been taken into account in the new model. CGTW structures differ substantially in focusing properties and beam loading properties from standing wave structures. Examples will be presented using the new TW model, propagating electron beams with different currents through the Stanford Linear Accelerator Center's 3 m structure. The results will be compared to the zero current TW structure model in TRANSPORT and the Tank model (a standing wave structure model) in TRACE 3-D. A computer demonstration of the new element will also be presented.

1 INTRODUCTION

TRACE 3-D [1] is an extremely popular first order optics code incorporating space charge effects in calculating the equivalent uniform beam envelope for particle beams. Implementation of a constant gradient traveling wave (CGTW) accelerator optics model into TRACE 3-D is useful to groups interested in generating models of entire CGTW beamlines using TRACE 3-D.

TRACE 3-D has several rf elements implemented into the standard version: rf gap, rfq cell, cavity, and the coupled cavity tank elements. These are all standing wave (SW) structures and possess focal properties differing from CGTW structures, due to the different fields seen by the particles as shown in Table 1.1. Note that for SW there is a transit time factor, T, and there appear backwards traveling wave components in the fields. In addition beam loading in CGTW structures is also different. Therefore, a model for this structure seemed desirable, particularly as many electron beam facilities rely on the SLAC 3 m structure [2] or a variation thereof.

The implementation of the model consisted of putting together two different pieces of the physics: 1) The R matrix elements for a CGTW; 2) The changing gradient as a function of accelerator structure parameters and beam parameters.

Field	CGTW Cavity	1'st Half SW Cavity	2'nd Half SW Cavity
E_z	$E_0\cos\phi$	$2E_0T[\cos\phi/2 + \sin\phi/\pi]$	$2E_0T[\cos\phi/2 - \sin\phi/\pi]$
E_r	$\frac{-\pi E_0 r\sin\phi}{2L}$	$-\pi E_0Tr[\cos\phi/\pi + \sin\phi/2]/2L$	$\pi E_0Tr[\cos\phi/\pi - \sin\phi/2]/2L$
B_θ	$\frac{-\pi E_0\beta r\sin\phi}{2cL}$	$\pi E_0T\beta r[\cos\phi/\pi - \sin\phi/2]/cL$	$\pi E_0T\beta r[\cos\phi/\pi + \sin\phi/2]/cL$

Table 1.1: Comparison of averaged electric and magnetic fields in a single SW or TW cavity. Note SW terms have backwards traveling wave components. E_0 is the applied field strength, ϕ the phase, β and γ the relativistic factors, L the cavity length, and T the transit time factor.

Sections 2 and 3 describe the R-matrix and gradient models. Section 4 compares TRACE 3-D results to TRANSPORT, numerical integration, and theory to demonstrate the accuracy of the model. Section 5 discusses an application of the new model. Section 6 presents a brief summary and some conclusions.

2 R-MATRIX ELEMENTS FOR THE CGTW

Previously, the TRANSPORT [1] code has been modified to achieve better first order simulation of a CGTW structure [4,5]. TRANSPORT now has R-matrix elements based on the WKB approximation. The R-matrix elements derived in this manner for TRANSPORT are used in the TRACE 3-D model. The non-zero R-matrix elements can be found in [1]. These matrix elements have been implemented in the current TRACE 3-D model.

3 THE ELECTRIC FIELD GRADIENT

The electric field gradient for a CGTW structure depends on the applied field and the field generated by the beam. Several derivations of the field developed [6,7] are in the literature. We use the field as expressed by [7]:

$$G = G_0\cos\phi - (I_0 r/2)\ln[1 - (z/L)(1 - e^{-2\tau})] \qquad \text{eq. 3.1}$$

where G is the net gradient, ϕ is the phase, G_0 is the applied gradient, I_0 is the average beam current, r the shunt impedance per unit length, L the structure length, τ the attenuation, and z the distance along the structure. The applied gradient is related to the input power, P, by:

$$G_0 = [rP/L)(1 - e^{-2\tau})]^{1/2} \qquad \text{eq. 3.2}$$

4 IMPLEMENTATION OF THE MODEL

The TRACE 3-D model was implemented by writing FORTRAN routines which first calculate the total gradient, G, across a step and then the R-matrix. The R-matrix elements are calculated assuming the gradient over the step is constant.

Several test calculations have been performed to demonstrate the accuracy of the model. First for the 0 current case a comparison to previous TRANSPORT and Runge-Kutta integration results obtained by Hurd and McGill [4] and Carey [5] was performed, as shown in Table 4.1. The initial energy is 100 MeV, the maximum energy gain is 3.19 MeV over a length of 290 cm (1.1 MV/m) and the phase lag is 30 degrees.

Element	Runge-Kutta	TRANSPORT	TRACE 3-D
$R_{11} = R_{33}$	1.197	1.197	1.198
$R_{12} = R_{34}$	0.307	0.307	0.307
$R_{21} = R_{43}$	1.386	1.386	1.395
$R_{22} = R_{44}$	1.179	1.179	1.180
R_{55}	0.653	0.653	0.651
R_{56}	2.059	2.060	2.058
R_{65}	-0.281	-0.281	-0.282
R_{66}	0.624	0.624	0.622

Table 4.1: Comparison of R-matrix Elements

The zero current comparison of the TRACE 3-D R-matrix to the TRANSPORT and numerically integrated values of the R-matrix elements is quite good. The largest discrepancy, for the $R_{21} = R_{43}$ elements, is 0.6%. This is within the accuracy of the input data for this model.

Next, as shown in Table 4.2, two models with SLAC accelerator structure parameters were built check beamloading [6,7]. The first case is a single 3 meter SLAC section. The second case is for two 3 meter SLAC sections. For both, the beam gains 70 MeV (from 5 MeV to 75 MeV) under the loaded conditions. The SLAC structure parameters used for these tests are: L = 3.01 meters; $\tau = 0.572$ nepers; $\lambda = 10.5$ cm; r = 57 Mohms/m. The phase lag is 0, the currents and energy gains are given in Table 4.2:

No. of CGTW Structures	Current (mA)	Beam Energy Theory (MeV)	Beam Energy TRACE 3-D (MeV)
1	0	80.2	80.2
1	255	70.0	70.1
2	0	113.4	113.4
2	544	70.0	70.2

Table 4.2: Beam Loading Calculations, Theory and TRACE 3-D

The two cases studied for beam loading effects also show good agreement between theory and TRACE 3-D. The largest difference of 0.3% for a beam loading of 544 mA occurs with a step size of 5 mm or less through the structure. This is within the accuracy expected.

5 BEAMLINE DESIGN APPLICATIONS

As an example of a beamline design application where use of a CGTW model makes a difference, take the model of a single 3 m SLAC section accelerating a 255 mA beam from 5 to 75 MeV. Input beam parameters, which are the same for both the CGTW and SW (TRACE 3-D Tank element) models are given in Table 5.1.

The two structures differ slightly in length because for the CGTW structure 3.01 m is exactly 86 cells long, each with phase advance of $2\pi/3$, while the SW structure is 3.0396 m long with 58 cavities whose phase advance is π. Because of this the accelerating gradient in the SW case also differs slightly from the CGTW case, but more importantly the input gradients differ because the CGTW model starts with an applied gradient for 0 current and then calculates the loaded gradient (see section 3). Table 5.2 gives a complete list of the structure parameters.

Input Parameter	Initial Value
Initial Energy	5 MeV
I_0	255 mA
$\varepsilon_x = \varepsilon_y$	4 π mm-mrad
$\alpha_x = \alpha_y$	0.1640
$\beta_x = \beta_y$	1.5907 mm/mrad
ε_z	21.22 π mm-mrad
α_z	-4.397
β_z	0.1584 mm/mrad

Table 5.1: Initial beam characteristics for CGTW and SW structure comparison. Emittances are 5 times r.m.s.

Other differences in how the structures are defined are the use of shunt impedance and attenuation (of the rf power through the structure) for the CGTW structure. Through the use of the shunt impedance, r, the attenuation, α, and the other input parameters, many possible traveling wave structures can be modeled with this new element.

Structure Parameter	CGTW	SW
Length	3.0100 m	3.0396 m
Phase	0	0
Applied Gradient	26.64 MV/m	23.03
No. of Cavities	86	58
Phase Adv./cav.	$2\pi/3$	π
Attenuation	0.572 nepers	n/a
Shunt Impedance	57 MΩ/m	n/a

Table 5.2: Structure parameters for CGTW and SW comparison.

Figures 5.1 and 5.2 show the different beam envelopes calculated for CGTW and SW cases. The SW structure is strongly focusing and brings the beam to a waist near the beginning of the structure, while the CGTW structure slightly defocuses the beam. Final values for the Courant-Snyder (Twiss) parameters are quite different as shown in the figures and as is apparent from the very different phase space ellipses in the upper right corners.

The R-matrices (including the first order effects of space charge) which TRACE 3-D calculates are shown in tables 5.3 and 5.4.

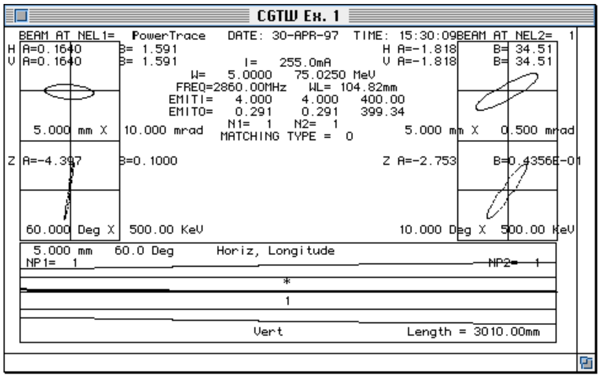

Figure 5.1: TRACE 3-D graphics showing beam envelope for CGTW case. The beam is slightly defocusing throughout the structure.

The differences between the two cases demonstrate the desirability to have both a traveling wave element available in TRACE 3-D as well as the existing standing wave elements. Many accelerator facilities around the world presently use traveling wave structures, of which the SLAC 3 m structure discussed in this paper is the most prevalent. The ability to make use of TRACE 3-D for modeling these facilities, with a reasonably accurate CGTW element, will improve the capacity of scientists, accelerator operators, and designers to explore different configurations and operating points quickly and efficiently.

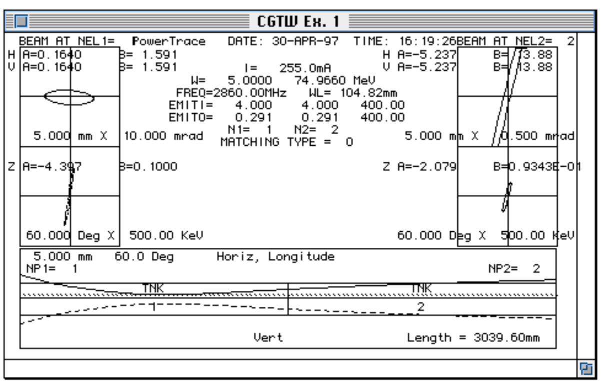

Figure 5.2: TRACE 3-D graphics showing beam envelope for SW case. Note that the scales for 5.1 and 5.2 are the same, and the phase ellipses on the right of both figures are quite different. The beam is focused to a waist near the beginning of the structure.

-0.6901	0.4728	0	0	0	0
-0.2905	0.0937	0	0	0	0
0	0	-0.6901	0.4728	0	0
0	0	-0.2905	0.0937	0	0
0	0	0	0	0.9992	0.0010
0	0	0	0	1.0640	0.0738

Table 5.3: R-matrix for SW element, with first order space charge effects folded in.

1.2564	0.6279	0	0	0	0
0.0604	0.0880	0	0	0	0
0	0	1.2564	0.6279	0	0
0	0	0.0604	0.0880	0	0
0	0	0	0	1.0059	0.0125
0	0	0	0	0.0444	0.0726

Table 5.4: R-matrix for the CGTW element, with first order space charge effects folded in.

6 CONCLUSIONS

This paper has presented a new traveling wave element for TRACE 3-D. It uses the same R-matrix as TRANSPORT but with the electric field recalculated at each step to account for beam loading effects. Electric field variation due to beam loading effects are calculated from theory. Several tests have been performed to compare the accuracy of this element to TRANSPORT and Runge-Kutta integration in the calculation of 0 current R-matrix elements, and to theory in the calculation of beam loading effects on energy gain. Good agreement has been found.

An example demonstrating the quite different behavior of CGTW and SW rf elements shows the utility of this new element in TRACE 3-D. Many different CGTW structure configurations can be modeled through different values of the accelerator structure parameters: Length, applied gradient, shunt impedance, and attenuation. Although not discussed in this paper, an extension of this work to model a constant impedance traveling wave structure would be quite simple.

The beam calculations presented were performed with a version of TRACE 3-D code that works in the Shell for Particle Accelerator Related Codes (S.P.A.R.C.) software environment[8].

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