LINEAR BEAM DYNAMICS EFFECTS OF THREE DIMENSIONAL STATIC MAGNETIC FIELDS OF INSERSION DEVICES AT SRRC

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Abstract

To understand the effects of the insertion devices on beam dynamics, Hamiltonian equations of charged particles in the presence of the insertion devices have been studied. Fields with insertion devices are considered to be periodic in the longitudinal direction with period length λ_s. The magnetic fields of the different types of magnet structure, namely AGID, APID and EPU, are derived and discussed. The linear beam dynamics effects for different kinds of insertion devices can be obtained directly from the canonical equations of the Hamiltonian mechanics.

1 INTRODUCTION

Various types of insertion devices (IDs) have been designed for generating specific light sources. In particular, adjustable gap insertion devices (AGIDs) are the most popular IDs. Special insertion devices such as adjustable phase insersion devices (APIDs) and elliptically polarizing undulators (EPUs) are being designed and realized for providing light sources of special properties. At SRRC, W20 wiggler and U10P, U5, U9 undulators are AGIDs, the testing purpose APU9P undulator belongs to the APID, and the EPU-5.6 is an EPU. Since these IDs are installed in the storage ring, the influence of the magnetic fields of IDs upon the stored electron beam should be concerned. Here, we study the linear beam dynamics effects of IDs by the Hamiltonian equations of a 3-dimensional periodic magnetic field.

2 TRANSVERSE EQUATIONS OF MOTION

We consider the general 3-dimensional magnetic field

$$\begin{aligned}
B_x &= \sum_n \Big[f_{1n}(x,y)\cos(nk_s s) + f_{2n}(x,y)\sin(nk_s s) \Big] \\
B_y &= \sum_n \Big[g_{1n}(x,y)\cos(nk_s s) + g_{2n}(x,y)\sin(nk_s s) \Big] \\
B_s &= \sum_n \Big[h_{1n}(x,y)\cos(nk_s s) + h_{2n}(x,y)\sin(nk_s s) \Big],
\end{aligned}$$

which is a periodic function in the longitudinal direction, i.e., $\hat{s}$ of the curvilinear coordinate system (x, y, s), with wave vector $k_s = \frac{2\pi}{\lambda_s}$. Using Maxwell's equations $\nabla\cdot\mathbf{B} = 0$ and $\nabla\times\mathbf{B} = 0$, we have four equations to solve for the f_{1n}, f_{2n}, g_{1n}, g_{2n}, h_{1n}, and h_{2n}. For convenient, we set $c_{mnu} \equiv \cosh(nk_{mnu}u)$ and $s_{mnu} \equiv \sinh(nk_{mnu}u)$, where $m = 1, 2$ and $u = x, y$. We obtain

$$\begin{aligned}
h_{1n} &= k_s\left(\alpha_{1n}c_{1nx} + \beta_{1n}s_{1nx}\right)\left(\gamma_{1n}c_{1ny} + \delta_{1n}s_{1ny}\right), \\
h_{2n} &= k_s\left(\alpha_{2n}c_{2nx} + \beta_{2n}s_{2nx}\right)\left(\gamma_{2n}c_{2ny} + \delta_{2n}s_{2ny}\right), \\
f_{1n} &= -k_{2nx}\left(\alpha_{2n}s_{2nx} + \beta_{2n}c_{2nx}\right)\left(\gamma_{2n}c_{2ny} + \delta_{2n}s_{2ny}\right), \\
f_{2n} &= k_{1nx}\left(\alpha_{1n}s_{1nx} + \beta_{1n}c_{1nx}\right)\left(\gamma_{1n}c_{1ny} + \delta_{1n}s_{1ny}\right), \\
g_{1n} &= -k_{2ny}\left(\alpha_{2n}c_{2nx} + \beta_{2n}s_{2nx}\right)\left(\gamma_{2n}s_{2ny} + \delta_{2n}c_{2ny}\right), \\
g_{2n} &= k_{1ny}\left(\alpha_{1n}c_{1nx} + \beta_{1n}s_{1nx}\right)\left(\gamma_{1n}s_{1ny} + \delta_{1n}c_{1ny}\right),
\end{aligned}$$

and

$$k_{mnx}^2 + k_{mny}^2 = k_s^2.$$

For charged-particles in such a magnetic field, the Hamiltonian in Frent-Serret coordinate system[1, 2] is given as

$$G = \frac{-e}{c}A_s - \left(1 + \frac{x}{\rho(s)}\right)p_0^{\mathrm{mech}}\left[\left(1 + \frac{\Delta p^{\mathrm{mech}}}{p_0^{\mathrm{mech}}}\right)^2 - \left(\frac{P_x}{p_0^{\mathrm{mech}}} - \frac{eA_x}{cp_0^{\mathrm{mech}}}\right)^2 - \left(\frac{P_y}{p_0^{\mathrm{mech}}} - \frac{eA_y}{cp_0^{\mathrm{mech}}}\right)^2\right]^{1/2},$$

where "P"s represent canonical momenta and p_0^{mech} means the (relativistic) mechanical/kinetic momentum. The radius of curvature at long straight sections is $\rho(s) \to \infty$. Furthermore, comparing with p_0^{mech}, the transverse mechanical momenta of the electron beams in a storage ring are small, i.e.,

$$\frac{p_u^{\mathrm{mech}}}{p_0^{\mathrm{mech}}} = \left(\frac{P_u}{p_0^{\mathrm{mech}}} - \frac{eA_u}{cp_0^{\mathrm{mech}}}\right) \ll 1$$

with $u = x, y$. Therefore we expand the Hamiltonian G in a Taylor series up to 2nd order of transverse mechanical momenta

$$G \simeq \frac{-e}{c}A_s - p_0^{\mathrm{mech}}\left[1 + \frac{\Delta p^{\mathrm{mech}}}{p_0^{\mathrm{mech}}} + \frac{1}{2}\left(\frac{\Delta p^{\mathrm{mech}}}{p_0^{\mathrm{mech}}}\right)^2 - \frac{1}{2}\left(\frac{P_x}{p_0^{\mathrm{mech}}} - \frac{eA_x}{cp_0^{\mathrm{mech}}}\right)^2 - \frac{1}{2}\left(\frac{P_y}{p_0^{\mathrm{mech}}} - \frac{eA_y}{cp_0^{\mathrm{mech}}}\right)^2\right].$$

The canonical equations of the Hamiltonian are

$$\left\{\begin{aligned}
\frac{dx}{ds} &= \frac{\partial G}{\partial P_x} \simeq \frac{P_x}{p_0^{\mathrm{mech}}} - \frac{eA_x}{cp_0^{\mathrm{mech}}}, \\
\frac{dy}{ds} &= \frac{\partial G}{\partial P_y} \simeq \frac{P_y}{p_0^{\mathrm{mech}}} - \frac{eA_y}{cp_0^{\mathrm{mech}}}.
\end{aligned}\right.$$

From the relation, $\mathbf{B} = \nabla\times\mathbf{A}$, we can select $A_s = 0$ and solve A_x and A_y to get the transverse equations of motion:

$$\left\{\begin{aligned}
\frac{d^2x}{ds^2} &\simeq \frac{d}{ds}\left(\frac{cP_x - eA_x}{cp_0^{\mathrm{mech}}}\right) \simeq \frac{e}{cp_0^{\mathrm{mech}}}\left(B_s\frac{dy}{ds} - B_y\right) \\
\frac{d^2y}{ds^2} &\simeq \frac{d}{ds}\left(\frac{cP_y - eA_y}{cp_0^{\mathrm{mech}}}\right) \simeq \frac{e}{cp_0^{\mathrm{mech}}}\left(B_x - B_s\frac{dx}{ds}\right).
\end{aligned}\right.$$

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Dividing u into two parts $u_0 + u_1$ with $u = x, y$, we set

$$\begin{cases} \dfrac{d^2x_0}{ds^2} = \dfrac{-e}{cp_0^{\mathrm{mech}}} B_y, & \dfrac{d^2x_1}{ds^2} = \dfrac{e}{cp_0^{\mathrm{mech}}} B_s \dfrac{dy}{ds}, \\ \dfrac{d^2y_0}{ds^2} = \dfrac{e}{cp_0^{\mathrm{mech}}} B_x, & \dfrac{d^2y_1}{ds^2} = \dfrac{-e}{cp_0^{\mathrm{mech}}} B_s \dfrac{dx}{ds}. \end{cases}$$

Since $\frac{du}{ds} \ll 1$ for $u = x, y$, we can apply an iteration/successive approximation[3]

$$\begin{cases} \dfrac{d^2x}{ds^2} \simeq \dfrac{d^2x_0}{ds^2} + \dfrac{e}{cp_0^{\mathrm{mech}}} B_s \dfrac{dy_0}{ds} \\ \dfrac{d^2y}{ds^2} \simeq \dfrac{d^2y_0}{ds^2} - \dfrac{e}{cp_0^{\mathrm{mech}}} B_s \dfrac{dx_0}{ds}. \end{cases}$$

The effects of different kinds of IDs on the electron beam can then be obtained after their periodic 3-dimensional magnetic fields are determined by writting down the parameters k's, α's, β's, γ's, and δ's.

3 AJUSTABLE GAP INSERTION DEVICES

Common properties of AGID structure are

$$B_y(-y) = B_y(y) \Longrightarrow \gamma_{1n} = \gamma_{2n} = 0,$$

$$\left.\begin{array}{l} B_x(-x) = -B_x(x) \\ B_y(-x) = B_y(x) \\ B_s(-x) = B_s(x) \end{array}\right\} \Longrightarrow \beta_{1n} = \beta_{2n} = 0.$$

3.1 B_y is symmetric in $\hat{s}$ direction

The W20 wiggler and U10P undulator of SRRC have the property $B_y(-s) = B_y(s)$, which implies $k_{1ny}\alpha_{1n}\delta_{1n} = 0$. The magnetic field becomes

$$\begin{aligned} B_x &= \sum_n -k_{2nx}\alpha_{2n}\delta_{2n}s_{2nx}s_{2ny}\cos(nk_s s) \\ B_y &= \sum_n -k_{2ny}\alpha_{2n}\delta_{2n}c_{2nx}c_{2ny}\cos(nk_s s) \\ B_s &= \sum_n k_s\alpha_{2n}\delta_{2n}c_{2nx}s_{2ny}\sin(nk_s s). \end{aligned}$$

The second order differential equations of x and y expaned near the s-axis become

$$\begin{aligned} \frac{d^2x}{ds^2} &\simeq \frac{e}{cp_0^{\mathrm{mech}}} \sum_n k_{2ny}\alpha_{2n}\delta_{2n}\cos(nk_s s) + \mathcal{O}(2), \\ \frac{d^2y}{ds^2} &\simeq -\left(\frac{e}{cp_0^{\mathrm{mech}}}\right)^2 \left[\sum_n nk_{2ny}\alpha_{2n}\delta_{2n}\sin(nk_s s)\right] \\ &\quad \cdot \left[\sum_n \frac{k_{2ny}}{n}\alpha_{2n}\delta_{2n}\sin(nk_s s)\right] \cdot y + \mathcal{O}(2), \end{aligned}$$

where $\mathcal{O}(2)$ are terms of orders equal to or higher than second order in x and y. The effective focusing strengthes are $K_x = 0$ and

$$\overline{K}_y = \frac{-1}{2}\left(\frac{e}{cp_0^{\mathrm{mech}}}\right)^2 \sum_n (\alpha_{2n}\delta_{2n}k_{2ny})^2.$$

3.2 B_y is anti-symmetric in $\hat{s}$ direction

On the other hand, the U5 and U9 undulators of SRRC are the types with $B_y(-s) = -B_y(s)$. We get $k_{2y}\alpha_2\delta_2 = 0$ and

$$\begin{aligned} B_x &= \sum_n k_{1nx}\alpha_{1n}\delta_{1n}s_{1nx}s_{1ny}\sin(nk_s s) \\ B_y &= \sum_n k_{1ny}\alpha_{1n}\delta_{1n}c_{1nx}c_{1ny}\sin(nk_s s) \\ B_s &= \sum_n k_s\alpha_{1n}\delta_{1n}c_{1nx}s_{1ny}\cos(nk_s s). \end{aligned}$$

The differential equations of x and y near the s-axis are

$$\begin{aligned} \frac{d^2x}{ds^2} &\simeq \frac{-e}{cp_0^{\mathrm{mech}}} \sum_n k_{1ny}\alpha_{1n}\delta_{1n}\cos(nk_s s) + \mathcal{O}(2), \\ \frac{d^2y}{ds^2} &\simeq -\left(\frac{e}{cp_0^{\mathrm{mech}}}\right)^2 \left[\sum_n nk_{1ny}\alpha_{1n}\delta_{1n}\cos(nk_s s)\right] \\ &\quad \cdot \left[\sum_n \frac{k_{1ny}}{n}\alpha_{1n}\delta_{1n}\cos(nk_s s)\right] \cdot y + \mathcal{O}(2). \end{aligned}$$

We obtain the effective focusing strengthes $K_x = 0$ and

$$\overline{K}_y = \frac{-1}{2}\left(\frac{e}{cp_0^{\mathrm{mech}}}\right)^2 \sum_n (\alpha_{1n}\delta_{1n}k_{1ny})^2.$$

For AGIDs, the linear beam dynamics effects are clear and only non-linear effects should be controlled carefully. In the horizontal plane, the oscillation term comes from the B_y field. The effective focusing strength in the vertical plane is caused by the interaction of the horizontal oscillation motion and the B_s field.

4 AJUSTABLE PHASE INSERTION DEVICES

With the APID structure, we have the condition

$$\left.\begin{array}{l} B_x(-x) = -B_x(x) \\ B_y(-x) = B_y(x) \\ B_s(-x) = B_s(x) \end{array}\right\} \Longrightarrow \beta_1 = \beta_2 = 0.$$

The linear properties of beam dynamics of APID are more complex than the case with AGID. There exist oscillations and coupling/mixing of motions in the horizontal and the vertical planes. If the phase of APID defined as $\phi \equiv 2\pi\frac{\Delta s}{\lambda_s}$ is zero, then $\gamma_1 = \gamma_2 = 0$, and APID becomes AGID disscussed in section 3.

By setting $\delta_1 = \gamma_2 = 0$ and $B_x = 0$, our 3-dimensional magnetic field can be reduced to the 2-dimensional field discussed by Roger Carr [4, 5]. The equations of motion become

$$\begin{aligned} \frac{d^2x}{ds^2} &\simeq \frac{e}{cp_0^{\mathrm{mech}}}\left[\sum_n k_s\alpha_{2n}\delta_{2n}\cos(nk_s s)\right] - \\ &\quad \frac{e}{cp_0^{\mathrm{mech}}}\left[\sum_n nk_s^2\alpha_{1n}\gamma_{1n}\sin(nk_s s)\right] \times y + \cdots \end{aligned}$$

and

$$\frac{d^2y}{ds^2} \simeq -\left(\frac{e}{cp_0^{\mathrm{mech}}}\right)^2 \left[\sum_n \alpha_{1n}\gamma_{1n}\cos(nk_s s)\right] \times \left[\sum_n \frac{k_s}{n}\alpha_{2n}\delta_{2n}\sin(nk_s s)\right] - \left(\frac{e}{cp_0^{\mathrm{mech}}}\right)^2 k_s^2 \times \Bigg\{ \Bigg[\sum_n \alpha_{1n}\gamma_{1n}\cos(nk_s s)\Bigg]^2 + \left[\sum_n n\alpha_{2n}\delta_{2n}\sin(nk_s s)\right] \times \left[\sum_n \frac{1}{n}\alpha_{2n}\delta_{2n}\sin(nk_s s)\right]\Bigg\} \times y + \cdots.$$

There is an intrinsic oscillation in the y plane, though the quantity of the oscillation amplitude is quite small. A particle tracking of $(\frac{1}{4}, \frac{-3}{4}, 1, -1, \cdots, 1, \frac{-3}{4}, \frac{1}{4})$ structure also shows a small vertical drift besides the tiny oscillation. The drift is proportional to the period number of APID. If we take the average of the x and y terms in the differential equations of motion, the horizontal linear coupling term is cancelled and the vertical average effective focusing strength is

$$\overline{K}_y = \frac{-1}{2}\left(\frac{e}{cp_0^{\mathrm{mech}}}\right)^2 \left(B_{y0}^2 + B_{s0}^2\right) = \frac{-k_s^2}{2}\left(\frac{e}{cp_0^{\mathrm{mech}}}\right)^2 \left[\sum_n (\alpha_{1n}\gamma_{1n})^2 + \sum_n (\alpha_{2n}\delta_{2n})^2\right],$$

where B_{y0} and B_{s0} are the amplitudes of B_y and B_s. For SRRC's APU9P, $B_{y0}^2 + B_{s0}^2$ does not keep constant such that the vertical tune shift appears when phase is changed. Experiment has confirmed that the vertical tune shift is proportional to $B_{y0}^2 + B_{s0}^2$.

5 ELLIPTICALLY POLARIZING UNDULATOR

The condition $B_s(s, x = 0, y = 0) = 0$ of EPU gives $\alpha_{1n}\gamma_{1n} = \alpha_{2n}\gamma_{2n} = 0$. Furthermore, from the structure of SRRC's Sasaki-type EPU-5.6, we have

$$\left.\begin{array}{l} B_x(x,y) = B_x(-x,-y) \\ B_y(x,y) = B_y(-x,-y) \\ B_x(-x,y) = B_x(x,-y) \\ B_y(-x,y) = B_y(x,-y) \end{array}\right\} \Longrightarrow \beta_{1n}\delta_{1n} = \beta_{2n}\delta_{2n} = 0.$$

Comparing with the on-axis magnetic field of Sasaki type EPU[6]

$$B_x = \left(\hat{B}_x \sin^2\frac{\phi}{2}\right)\sin k_s s - \left(\hat{B}_x \sin\frac{\phi}{2}\cos\frac{\phi}{2}\right)\cos k_s s$$
$$B_y = \left(\hat{B}_y \sin\frac{\phi}{2}\cos\frac{\phi}{2}\right)\cos k_s s + \left(\hat{B}_y \cos^2\frac{\phi}{2}\right)\sin k_s s,$$

we have $n = 1$ and

$$k_{11x}\beta_{11}\gamma_{11} = \hat{B}_x \sin^2\frac{\phi}{2},\ k_{21x}\beta_{21}\gamma_{21} = \hat{B}_x \sin\frac{\phi}{2}\cos\frac{\phi}{2},$$
$$k_{11y}\alpha_{11}\delta_{11} = \hat{B}_y \cos^2\frac{\phi}{2},\ k_{21y}\alpha_{21}\delta_{21} = -\hat{B}_y \sin\frac{\phi}{2}\cos\frac{\phi}{2}.$$

The equations of motion become

$$\frac{d^2x}{ds^2} \simeq \frac{e}{cp_0^{\mathrm{mech}}}\left(k_{2y}\alpha_2\delta_2\cos k_s s - k_{1y}\alpha_1\delta_1\sin k_s s\right) - \left(\frac{e}{cp_0^{\mathrm{mech}}}\right)^2 \left(k_{1x}\beta_1\gamma_1\cos k_s s + k_{2x}\beta_2\gamma_2\sin k_s s\right) \cdot \left(k_{1y}\alpha_1\delta_1\cos k_s s + k_{2y}\alpha_2\delta_2\sin k_s s\right) \cdot y - \left(\frac{e}{cp_0^{\mathrm{mech}}}\right)^2 \cdot \left(k_{1x}\beta_1\gamma_1\cos k_s s + k_{2x}\beta_2\gamma_2\sin k_s s\right)^2 \cdot x + \mathcal{O}(2)$$

and

$$\frac{d^2y}{ds^2} \simeq \frac{e}{cp_0^{\mathrm{mech}}}\left(-k_{2x}\beta_2\gamma_2\cos k_s s + k_{1x}\beta_1\gamma_1\sin k_s s\right) - \left(\frac{e}{cp_0^{\mathrm{mech}}}\right)^2 \left(k_{1x}\beta_1\gamma_1\cos k_s s + k_{2x}\beta_2\gamma_2\sin k_s s\right) \cdot \left(k_{1y}\alpha_1\delta_1\cos k_s s + k_{2y}\alpha_2\delta_2\sin k_s s\right) \cdot x - \left(\frac{e}{cp_0^{\mathrm{mech}}}\right)^2 \cdot \left(k_{1y}\alpha_1\delta_1\cos k_s s + k_{2y}\alpha_2\delta_2\sin k_s s\right)^2 \cdot y + \mathcal{O}(2).$$

The equations of motion contain oscillation terms and linear coupling terms and make the linear beam dynamics effects becomes complex. The oscillation terms come from the interaction of v_s and the transverse magnetic fields B_x and B_y. The longitudinal magnetic field B_s couples the transverse oscillations in both planes.

6 CONCLUSION

The 3-dimensional periodic magnetic field discussed here can cover the AGID, APID, and EPU in a unified way by adding suitable structure conditions. AGID is the most popular type of IDs and its influence on the stored beam is well known and clear. A detail linear beam dynamic effects of APU with 2-dimensional field form have been achieved and discussed in section 4.

The analytical solutions of the differential equations of motion for EPU can not be written down in a simple form. Numerical calculation/simulation is by far the easiest way to study the EPU's effects on beam dynamics.

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