NOVEL STRUCTURE OF SHORT-PERIOD TWISTED UNDULATOR SPONTANEOUS EMISSION PROPERTIES

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Abstract

A novel short-period undulator scheme is developed. The geometry looks like helical transformation applied to an original linear undulator structure and provides a combination of fast and slow oscillations of the electron beam. The following properties of the radiation are considered: i) reduction of relative content of higher odd harmonics as the `slow` helical field increases; ii) radiation enhancement as the number of helical turns increases; iii) spectral line splitting and intensity adjustability along with circular polarization of the radiation for the short-period twisted structure.

1 INTRODUCTION

Undulators with two or more frequencies are attractable from many points of view. One of them is insertion devices with special radiation characteristics.

We are considering here a somewhat new kind of two-frequency motion that could combine useful radiation properties of both linear microundulator and helical undulator. It can be implemented in a real construction as depicted in fig. 1. As it was discussed in [1] such schemes can be regarded as belonging to a new class of `twisted linear undulator`.

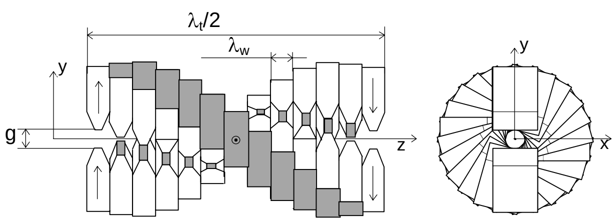

Fig. 1. Schematic drawing of the twister undulator structure.

2 UNDULATOR FIELDS

It was found in [2] the solution of Laplace`s equation for magnetic potential $\Delta\psi=0$ satisfying the condition of superimposed helical `slow` transformation to the original linear undulator structure:

$B_x=(B_w cosh_w z+B_t)sin(k_t z)$,

$B_y=(B_w cosh_w z+B_t)cos(k_t z)$,

where $k_t<<h_w$ and $h_w r<<1$, B_w and B_t are the amplitudes of the original linear (`microundulator` contribution) and `helical` undulator fields with periods λ_w and λ_t respectively, $h_w=2\pi/\lambda_w$ and $k_t=2\pi/\lambda_t$.

Under these assumptions one can derive:

$$\psi(r,j,z)=Re\sum_{m=-1}^{m=+1}\begin{pmatrix} C_m I_1(r(mh_w+k_t))\times \\ exp(i(j+(mh_w+k_t)z)) \end{pmatrix},$$

where $C_0=\dfrac{2iB_t}{k_t}, C_{\pm 1}=i\dfrac{2B_w}{k_t\pm h_w}$

3 ELECTRON TRAJECTORIES

Equations of motion for a single electron were solved assuming $K_t k_w r<<\gamma/2\pi$, where $K_t=eB_t/k_t mc$.

For the twisted undulator having zero first and second integrals and $\gamma>>1$ one can obtain

$$\vec{r}_\perp(z)=\frac{K_t}{\gamma k_t}\begin{pmatrix}\cos k_t z\\ -\sin k_t z\end{pmatrix}+\sum_{\pm}\frac{K_w^\pm}{\gamma\sqrt{2}}\begin{pmatrix}\cos(k_t\pm h_w)z\\ -\sin(k_t\pm h_w)z\end{pmatrix}\frac{1}{k_t\pm h_w}$$

$$r_z(t)=\beta_\parallel ct-\frac{1}{\gamma^2 h_w}\begin{bmatrix}\dfrac{K_w^+K_w^-}{\sqrt{2}}\sin 2h_w z(t)\\ +K_t\left(K_w^+ + K_w^-\right)\sinh_w z(t)\end{bmatrix}, \qquad (1)$$

where $\beta_\parallel=1-\dfrac{1}{2\gamma^2}\left(1+K_t^2+\dfrac{1}{2}\left(K_w^{+2}+K_w^{-2}\right)\right)$, and $K^{\pm}_w=eB_w/mc(k_t\pm h_w)\sqrt{2}$.

The last coupling term in (1) containing $\sin h_w z$ can be comparable with the term containing $\sin 2h_w z$ and does not appear for a conventional planar undulator. Hence one can expect appearance of even harmonics for on-axis view of spontaneous emission.

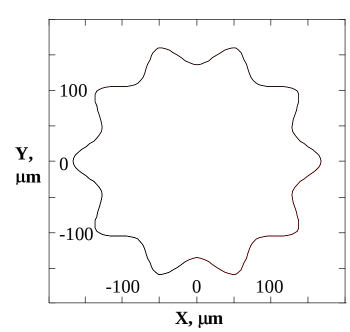

Figure 2. Electron trajectory view in transverse plane. E=500 MeV, B_w=10kG, B_t=1kG, λ_w=1cm, λ_t=10cm.

4 SPONTANEOUS EMISSION RADIATION RELATIONSHIPS

Energy radiated per unit solid angle ($d\Omega$) per unit frequency ($d\omega$) per electron was investigated numerically for both zero and non-zero imposed helical `slow` field B_t.

We separate two components of spectral angular density of radiation for spherical frame with θ and φ coordinates:

$$\frac{d^2 I}{d\omega d\Omega} = \frac{d^2 I_\theta}{d\omega d\Omega} + \frac{d^2 I_\varphi}{d\omega d\Omega}.$$

Substitution of the equations of motion (1) into the simplified formula for $d^2I/d\omega d\Omega$ expressed in [3] through the product $[\mathbf{n}[\mathbf{n}\boldsymbol{\beta}]]$ neglecting `near-field` effects gives the following general expression containing products of five sums of Bessel functions:

$$\frac{d^2 I_{\theta,\varphi}}{d\omega d\Omega} = \frac{(e\omega L)^2}{4\pi^2 c^3} \left| \sum_{j,k,l,m,n=-\infty}^{\infty} \begin{pmatrix} J_j(a_t) J_k(a_w^+) J_l(a_w^-) \times \\ J_m(a_w) J_n(a_{tw}) F^{\theta,\varphi}_{j,k,l,m,n} \end{pmatrix} \right|^2, \quad (2)$$

where:

$$\begin{pmatrix} F^{\theta}_{j,k,l,m,n} \\ F^{\varphi}_{j,k,l,m,n} \end{pmatrix} = (-1)^{m+n} i^{j-k-l} e^{i\varphi(j+l+k)} \times$$

$$\begin{pmatrix} \cos\theta(u_{j,k,l,m,n}\cos\varphi + v_{j,k,l,m,n}\sin\varphi) - \beta_{\|} Sn_{j,k,l,m,n}\sin\theta \\ u_{j,k,l,m,n}\sin\varphi - v_{j,k,l,m,n}\cos\varphi \end{pmatrix},$$

$$\begin{pmatrix} u_{j,k,l,m,n} \\ v_{j,k,l,m,n} \end{pmatrix} = -\frac{1}{\gamma} \begin{pmatrix} Ss_{j\pm1,k,l,m,n} & Ss_{j,k\pm1,l,m,n} & Ss_{j,k,l\pm1,m,n} \\ Sc_{j\pm1,k,l,m,n} & Sc_{j,k\pm1,l,m,n} & Sc_{j,k,l\pm1,m,n} \end{pmatrix} \begin{pmatrix} K_t \\ K_w^+/\sqrt{2} \\ K_w^-/\sqrt{2} \end{pmatrix},$$

$$Ss_{j\pm1,k,l,m,n} = \frac{1}{2i}(Sn_{j+1,k,l,m,n} - Sn_{j-1,k,l,m,n}), \quad Sn(x) = \frac{\sin x}{x},$$

$$Sc_{j\pm1,k,l,m,n} = \frac{1}{2}(Sn_{j+1,k,l,m,n} + Sn_{j-1,k,l,m,n}), \quad a_t = \frac{K_t}{\gamma} \nu \frac{k_s}{k_t} \sin\theta,$$

$$Sn_{j,k,l,m,n} = Sn\left(N_t \pi \left(\frac{h_w}{k_t}(\nu + k - l - 2m - n) - j - k - l \right) \right),$$

$$a_w^{\pm} = \frac{K_w^{\pm}}{\gamma} \frac{\nu k_s/\sqrt{2}}{k_t \pm h_w} \sin\theta, \quad a_w = \frac{K_w^+ K_w^-}{4\gamma^2} \frac{\nu k_s}{h_w} \cos\theta, \quad N_t = L/\lambda_t,$$

$$a_{tw} = \frac{K_w^+ + K_w^-}{\sqrt{2}\gamma^2} K_t \frac{\nu k_s}{h_w} \cos\theta, \quad k_s = \frac{h_w}{1 - \beta_{\|}\cos\theta}, \quad \nu = \omega / k_s c.$$

We consider below specific properties of the radiation compared to the conventional planar undulator, i.e. in the vicinity of $\nu \approx n$. We do not consider here the radiation features corresponding to helical undulator component (i.e. when $K_t \geq 1$ and $\nu \approx nk_t/h_w$) because in practice we have $K_w^2 << K_t^2$ and the influence of the `twisted undulator` component is negligible.

As it can be seen from (2) the main maxima of $d^2I/d\omega d\Omega$ spectral distribution correspond to the frequencies $\omega = \frac{(nh_w \pm k_t)/c}{1 - \beta_{\|}\cos\theta}$, where n is integer.

5 PERFORMANCE FEATURES OF THE TWISTED UNDULATOR

The results given below refer to on-axis radiation (φ=0, θ=0) emitted by a single electron having energy 500 MeV. Some results corresponding to non-axis radiation ($\theta \neq 0$) can be found in [1]. The first five harmonics are analyzed here at the frequencies $\nu = n + \lambda_w/\lambda_t$. The properties of radiation at the frequencies $\nu = n - \lambda_w/\lambda_t$ are qualitatively similar. The parameters of the undulator considered numerically (see the figs. 3-7) were the following: L=30cm, λ_w=1cm, B_w=6kG, N_t=3 with the exception of the figs. 3, 5, where N_t is a variable.

A number of features occurs in the twisted structure. The first is spectral line splitting mentioned above and demonstrated in ref. [1].

The second general feature is circular polarization of the on-axis radiation at $N_t = L/\lambda_t > 1$, where L is the undulator length. To obtain pure circular polarization it is enough to provide $N_t = 0.5$ in `half-turn twisted` structure (see fig. 3). The degree of circular polarization is very close to unity at $N_t = n/2$ and almost does not depend on harmonic number or helical field B_t (see fig. 4). The on-axis radiation of the first harmonic corresponds to pure circular polarization (deviation equals zero).

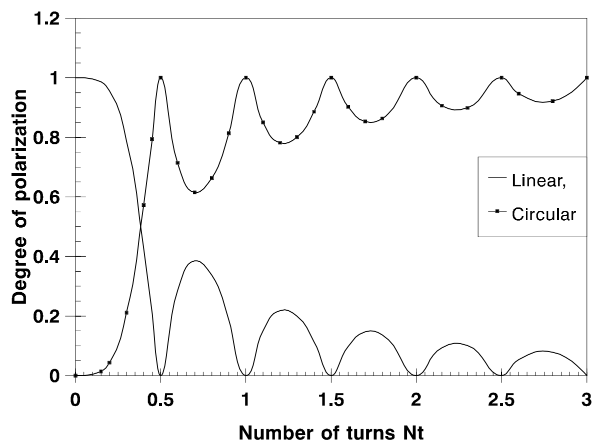

Figure. 3. Degree of linear and circular polarization as a function of number of turns $N_t = L/\lambda_t$. B_t=0, ν=1.1.

The third feature is possibility of spectral-angular intensity increase compared to the source (untwisted) linear undulator. One can see from fig. 5, that when λ_t/λ_w is less then 5.9 (or number of turns per undulator length exceeds 5.1) the first harmonic density exceeds that value for equivalent linear undulator.

Relative change of odd harmonics is depicted in fig. 6 as a function of helical field B_t. The density for each harmonic is normalized to its value at B_t=0. These dependencies demonstrate decrease of higher harmonic relative content as the helical field increases.

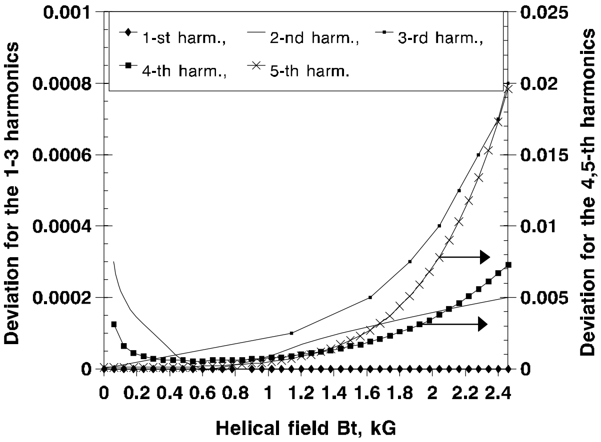

Figure. 4. Deviation of the degree of polarization from unity as a function of B_t for different harmonics. N_t=3.

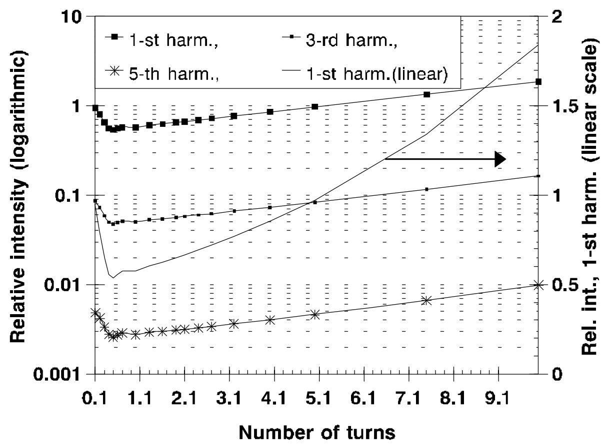

Figure 5. Spectral angular on-axis density for odd harmonics related to that for the first harmonic of linear undulator versus $N_t=L/\lambda_t$. B_t=0.

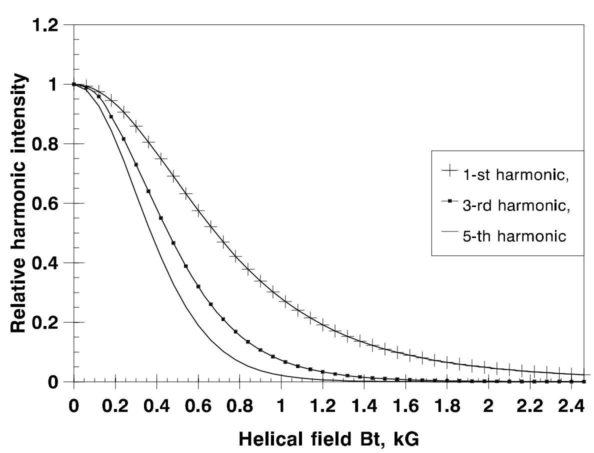

Figure 6. Spectral angular on-axis density for odd harmonics related to that values of corresponding harmonics for conventional linear undulator versus helical field B_t.

On-axis radiation of the even harmonics predicted in the previous section has the maxima close to $B_t \sim B_w\lambda_w/\lambda_t$, however its intensity is negligible compared to the odd harmonics (see fig. 7).

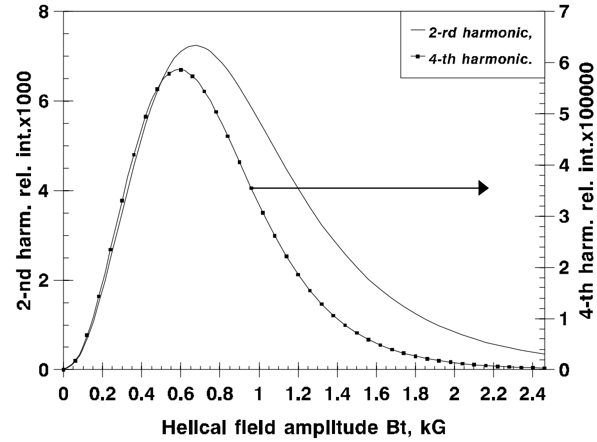

Figure 7. Spectral angular on-axis density for even harmonics related to that for the first harmonic of linear undulator versus helical field.

6 DISCUSSION

The twisted undulator structure combines some properties of both helical and linear undulator. The main features are circular polarization, spectral line splitting, and continuous adjustability of the radiation intensity by the helical field variation. The structure can be considered also as a \`twisted microundulator\` providing radiation with circular polarization.

Although the insertion device proposed can not be used in FELs, it is of potential interest for applied research of plasma beat waves and in solid state physics, e.g. circular dichroism.

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