

# THE SHORT-RANGE WAKEFIELDS IN THE SBLC LINAC

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*Abstract*

The short-range longitudinal and transverse wakefields of a point charge in the SBLC linac are obtained using a modal summation technique. Simple functional fits to these wakes are given, which can be used as Green functions in beam dynamics simulations of bunches. These results, however, are valid only after the beam has traversed a critical number of cells  $N_{crit}$ . Using time domain computations with Gaussian bunches we have obtained results that are consistent with  $N_{crit}$  varying as  $\alpha a^2/(L\sigma_z)$ , with  $a$  the iris radius and  $L$  the period length of the structure,  $\sigma_z$  the bunch length and  $\alpha$  a constant on the order of 1. For the loss per cell to reach to within a few per cent of the asymptotic value we find that  $\alpha \sim 0.5 - 1.0$ .

## 1 INTRODUCTION

In the S-Band Linear Collider (SBLC) project[1] long trains of short, intense bunches of electrons and positrons are each accelerated through 16 km of linac to 250 GeV before colliding at the interaction point. The accelerating structure is a disk-loaded, constant gradient structure consisting of 180 cells. Each bunch is Gaussian, with rms length  $\sigma_z = 300 \mu\text{m}$ , which is short compared to the iris radius  $a$ :  $\sigma_z/a \sim .025$  (for FEL operation it is even shorter:  $\sigma_z/a \sim .0021$ ). For a single cavity with beam tubes the longitudinal high frequency impedance varies with frequency as  $\omega^{-1/2}$ , implying that the longitudinal wakefield of a short bunch varies as  $\sigma_z^{-1/2}$  [2]. For a periodic structure the longitudinal high frequency impedance (real part) varies as  $\omega^{-3/2}$ , and the wakefield per cell of a bunch approaches a constant value as  $\sigma_z \rightarrow 0$ [3, 4]. For a structure consisting of a finite number of repeating cells with beam tubes the wake of the first cell is the same as the single cell result and the wake per cell asymptotically approaches that of the periodic result as the beam progresses down the structure. For a Gaussian bunch the number of cells needed for the periodic solution to be valid,  $N_{crit}$ , is given by[3, 4]

$$N_{crit} = \frac{\alpha a^2}{L\sigma_z}, \quad (1)$$

with  $L$  the structure period length and  $\alpha$  a constant of proportionality on the order of one.

In this report first a frequency domain approach is used to obtain the short-range longitudinal (monopole) and transverse (dipole) wakefields in a periodic, SBLC-like structure. Then time domain calculations are performed to test the validity of Eq. 1, and to find  $\alpha$ . More details can be found in Ref. [5].

## 2 THE ASYMPTOTIC WAKEFIELDS

In the SBLC linac the cell dimensions vary within a structure, but only gradually. We first find the wakes for 5 purely periodic models, with cell dimensions similar to 5 representative cells of the actual SBLC structure. For our 5 representative cells we take cells 1, 45, 90, 135, and 180 for which  $a$  is 1.1 cm, 1.225 cm, 1.35 cm, 1.475 cm, and 1.6 cm, respectively. Note that the cell length  $L = 3.33$  cm, iris thickness  $t = 5.3$  mm, and cavity radius  $b \approx 4$  cm. The wakes of the 5 models are then averaged to give the wakefields representing an entire structure.

To obtain the longitudinal wakefield of a periodic structure we use the computer program KN7C[6] to obtain the synchronous frequencies and loss factors of lower monopole modes. We approximate the high frequency contribution to the impedance using the so-called Sessler-Vainsteyn optical resonator model[6], a model that has also been applied, for example, to the SLC[7] and the NLC[8]. The real part of the impedance (assuming  $t/L$  is small) becomes[8]

$$R_z(\omega) = \sum_{n=1}^N \pi k_n \delta(\omega - \omega_n) + \frac{2Z_0 j_{01}^2}{\pi L \zeta^2} \times \quad (2)$$

$$\times \frac{\sqrt{\nu} + 1}{(\nu + 2\sqrt{\nu} + 2)^2} \Theta(\omega - \omega_N) \quad \omega > 0,$$

with  $k_n$  the loss factor and  $\omega_n$  the frequency of the  $n^{\text{th}}$  mode,  $Z_0 \cong 377 \Omega$ ,  $j_{01} \cong 2.41$  the first zero of the Bessel function  $J_0$ ,  $\zeta \cong 0.824$ ,  $\nu = 4a^2\omega/(c\bar{L}\zeta^2)$ , with  $c$  the speed of light and  $\bar{L} = \sqrt{L(L-t)}$ ;  $\Theta(x) = 0$  for  $x < 0$ , 1 for  $x > 0$ . Fourier transforming  $R_z(\omega)$  we obtain the longitudinal wakefield:

$$W_z(s) = \sum_{n=1}^N 2k_n \cos \frac{\omega_n s}{c} + \frac{Z_0 c j_{01}^2 \bar{L}}{\pi^2 a^2 L} \times \quad (3)$$

$$\times \int_{\nu_N}^{\infty} \frac{\sqrt{\nu} + 1}{(\nu + 2\sqrt{\nu} + 2)^2} \cos\left(\frac{\zeta^2 \bar{L} s \nu}{4a^2}\right) d\nu,$$

with  $\nu_N = 4a^2\omega_N/(c\bar{L}\zeta^2)$ . The transverse (dipole) wakefield is obtained in the analogous manner, but using the computer program TRANSVRS [9] to obtain the dipole mode frequencies and loss factors.

For the longitudinal case we have found, for each of the 5 representative structures,  $\omega_n$  and  $k_n$  for all modes up to 75 GHz (about 250 modes) using KN7C. Comparing, at the higher frequencies, the binned modal contribution to the Sessler-Vainsteyn part of Eq. 3 we find good agreement, to within 10%. Then substituting into Eq. 4 we obtain the wakefields shown in Fig. 1 (the solid curves). The values

at the origin, which should equal [4]  $W_z(0) = Z_0 c / (\pi a^2)$  ( $= 198 \text{ V/pC/m}$  for cell 90 dimensions), are 4–5% low, indicating some calculation error. The average of the 5 representative wakes (with the wakes of cell 1 and 180 weighted by half) is given by the dashed curve in Fig. 1. Finally, a fit to the average wake, given by

$$W_z = 200 \cdot (\text{V/pC/m}) \cdot \exp \left[ -0.77 (s/\text{mm})^{\frac{1}{2}} \right], \quad (4)$$

is shown by the dots in Fig. 1.

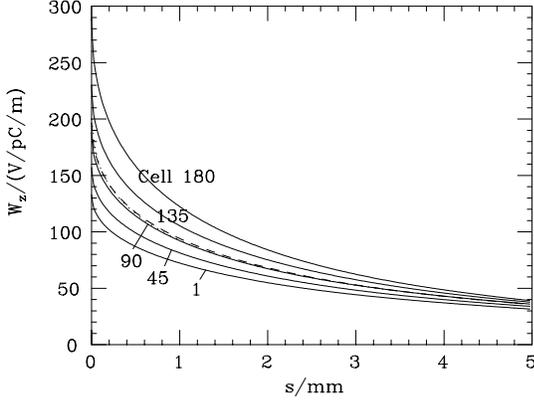


Figure 1: The longitudinal wakefield of representative cells in the SBLC structure (solid curves). The dashes represent the average, the dots the model fit, Eq. 4.

For the transverse case modes up to 68 GHz (about 350 modes) were calculated. Comparing, at the higher frequencies, the binned modal contribution to the Sessler-Vainsteyn part of the impedance we again find good agreement. The transverse wakefields for the 5 geometries are shown in Fig. 2 (the solid curves). In this case the slope at the origin should equal [8]  $W'_x(0) = 2Z_0 c / (\pi a^4)$  ( $= 2.17 \text{ V/pC/mm}^2/\text{m}$  for cell 90); our numerical results agree to within 2% in all cases. The average wake is given by the dashed curve in Fig. 2. A fit to the average wake, given by

$$W_x = 4.10 (\text{V/pC/mm/m}) \cdot \left[ 1 - \left( 1 + 1.15 [s/\text{mm}]^{\frac{1}{2}} \right) \exp \left( -1.15 [s/\text{mm}]^{\frac{1}{2}} \right) \right], \quad (5)$$

is indicated by the dots.

### 3 TRANSITION TO THE ASYMPTOTE

To test the validity of Eq. 1 for  $N_{crit}$  (and the applicability of the asymptotic solutions) and to find  $\alpha$ , at least for the longitudinal case, we have performed a series of MAFIA [10] time domain calculations. We have obtained the per cell loss factor  $k_{tot}$  of short Gaussian bunches of various lengths in structures consisting of a finite number of cells with infinitely long beam tubes. Although the SBLC structure is a constant gradient structure, for simplicity, we use only identical cells, with the dimensions of cell 144 in the real structure, for which  $a = 1.20 \text{ cm}$ . The bunch

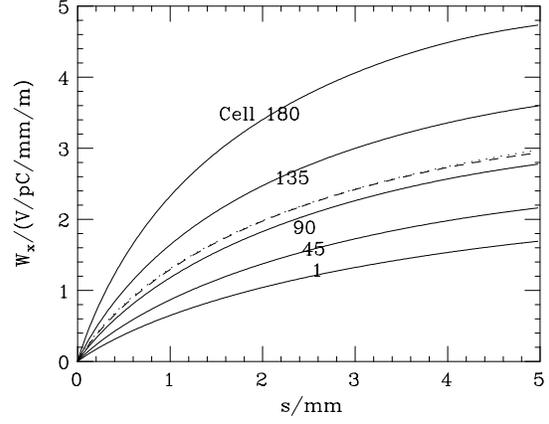


Figure 2: The transverse (dipole) wakefield of representative cells in the SBLC structure (solid curves). The dashes give the average, the dots the model fit, Eq. 5.

lengths vary from  $\sigma_z = 0.1 \text{ mm}$ ,  $0.3 \text{ mm}$ ,  $0.5 \text{ mm}$ , and then in  $0.25 \text{ mm}$  steps up to  $4 \text{ mm}$ ; the number of cells vary from  $N_{cell} = 1$  to  $10$ . A very fine mesh is used (e.g.,  $\sigma_z = 0.1 \text{ mm}$ ,  $N_{cell} = 10 \Rightarrow 65 \times 10^6$  meshpoints) in order to obtain good accuracy.

The per cell loss factor  $k_{tot}$  is plotted as function of  $\sigma_z$  in Fig. 3 (the results are connected by lines). The results are bounded by two curves representing the single cell solution, given by the Lawson diffraction model [2]:

$$k_{tot} = \frac{\Gamma(1/4) Z_0 c}{4\pi^{5/2} a} \sqrt{\frac{g}{\sigma_z}}, \quad (6)$$

with  $\Gamma(1/4) \cong 3.63$  and  $g$  the cavity gap (i.e.,  $L - t$ ), and the periodic solution:

$$k_{tot} = \frac{L}{2\sqrt{\pi}\sigma_z} \int_0^\infty ds W_z(s) e^{-\frac{s^2}{4\sigma_z^2}}, \quad (7)$$

with  $W_z$  given by Eq. 4. For bunch lengths larger than about  $2 \text{ mm}$  the loss per cell is independent of the number of cells. For shorter bunches we note that the single cell results approach the single cell asymptotes, as the bunch length decreases. Also, for a given bunch length, as the number of cells increases, the loss per cell asymptotically approaches the periodic structure curve, however, with a slight systematic offset (which is not understood). Note that for a finite number of cells, as the bunch length becomes ever shorter the loss again varies as  $\sigma_z^{-1/2}$ .

To determine  $N_{crit}$  we first calculate the differential loss factor for each cell from the MAFIA data,  $\kappa_n$ , defined as the total loss factor for an  $N_{cell}$  cell structure minus that of an  $N_{cell} - 1$  cell structure, which we then convert into a function of cell number through cubic spline interpolation (see Fig. 4). In Fig. 4 we note that for  $\sigma_z = 0.1 \text{ mm}$  the curve decreases throughout the range; for the longer bunch lengths it is rather constant throughout the range. For the intermediate bunch lengths, however,  $\kappa_n$  begins by decreasing gradually, and then levels off for high cell number.

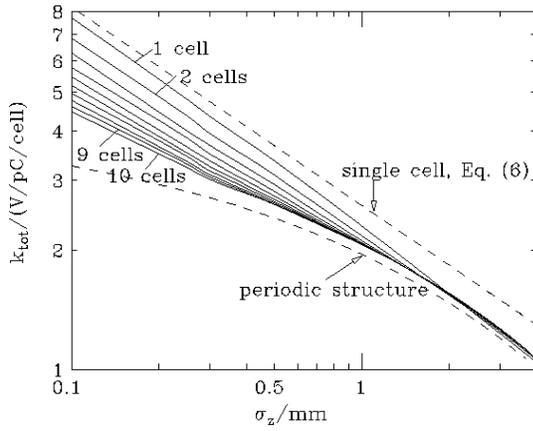


Figure 3: The loss factor  $k_{tot}(\sigma_z)$  obtained by MAFIA for a Gaussian bunch in a structure consisting of a finite number of cells (1-10) Also shown are the single cell (Eq. 6) and the periodic structure loss factor.

Upon careful inspection one notices that for these curves there is also a slight ( $\sim 3\%$ ) dip before the curve reaches the final, asymptotic value. In Fig. 5 numerical errors probably account for the slight anomalies that we see in the high end of the curves for short bunches; nevertheless, the general trend of these curves is still correct.

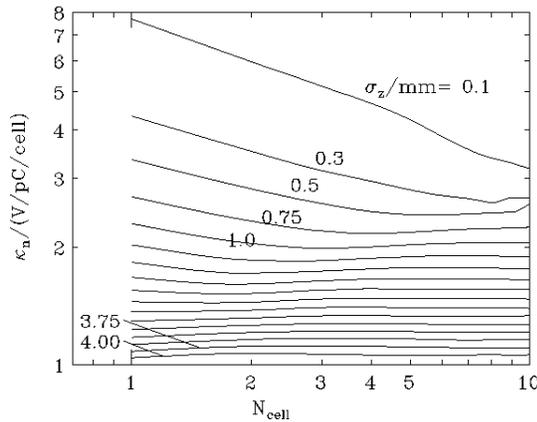


Figure 4: The differential loss factor of cell  $N_{cell}$ ,  $\kappa_n$ , for various bunch lengths, as obtained by MAFIA (the results are connected by smooth curves).

Let us consider 3 different criteria that result in a  $\kappa_n$  that is within a few percent of the asymptotic value: (1) the point in the curve, before the dip, where  $\kappa_n$  reaches the same value as the asymptote, (2) the dip position, and (3) the position (after the dip) where the curve reaches to 98% of its asymptotic value. The  $\sigma_z = 0.1$  and  $0.3$  curves, as well as those for which  $\sigma_z \geq 2.0$  mm, are not included; as asymptote we take the value of  $\kappa_n$  at  $N_{cell} = 8$ . The results are shown in Fig. 5, with case 1 given by the x's, case 2 by the diamonds, and case 3 by the +'s. Fitting the data to a power law ( $N_{crit}$  as a function of  $\sigma_z$ ) we obtain as exponent  $-1.06$  in the first case and  $-0.85$  in the oth-

ers. However, given the accuracy of the calculation, we can say that the results are consistent with the  $-1$  power dependence of Eq. 1. As for the coefficient  $\alpha$ , when fitting to Eq. 1 we obtain 0.5, 0.7, and 1.0 for cases 1, 2, and 3, respectively. In summary, to obtain an average loss at the end of a finite length, repeating structure that is within a few percent of the average loss in the truly periodic structure the critical number of cells needed is given by Eq. 1, with  $\alpha \sim 0.5 - 1.0$ .

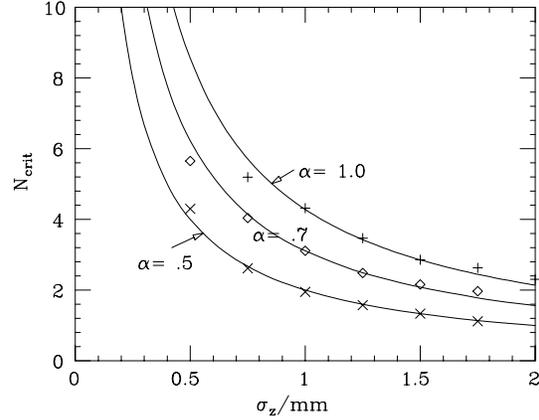


Figure 5:  $N_{crit}(\sigma_z)$  is obtained from Fig. 4 following 3 different criteria (discussed in the text). The curves are fits to Eq. 1, taking,  $\alpha = 0.5, 0.7, 1.0$  respectively.

## 4 REFERENCES

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