EMITTANCE GROWTH OF AN INTENSE ELECTRON BEAM IN A FOCUSING CHANNEL

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Abstract

We use the single-particle radial equation of motion to identify nonlinear forces which lead to an emittance growth in a focusing channel consisting of solenoids. For a uniform density beam, the two dominant effects are the axial velocity variations within a solenoid due to the particles' azimuthal velocity and changes in the particle's energy due to radial motion and the radial electric space-charge field. We derive estimates for the emittance growth for a space-charge dominated beam due to these effects, both for the case of a hard focus to a small beam waist and for the case where there is gentle beam scalloping. We also briefly catalog less important emittance growth mechanisms.

1 INTRODUCTION

In this paper, we will analyze the emittance growth of a continuous, intense electron beam in a transport channel made up of short discrete solenoids. Our goal is to derive formulas for the emittance growth for the case that the beam is focused to a small waist and for the case the beam is radially oscillating (either due to a mismatch or the discreteness of the focusing solenoids). By far, the most important effect is due to beam-density nonuniformities, and there is a mistaken tendency to assume that there is no emittance growth if the density is uniform. However, we will show that particles in a uniform-density beam still have a nonlinear radial equation of motion. We will estimate the emittance growth for these cases for a uniform-density beam, which would be then useful for evaluating beamline designs. These estimates are also valid if the beam is in the emittance-dominated regime. We will assume that the electron beam does not reach an equilibrium or periodic phase-space distribution. For simplicity, we will assume that all elements are perfectly aligned and that the focusing elements are perfect with no fringe fields. The emittance growth will be dominated by effects arising from axial velocity variations within the soleniods (due to a radius-dependent azimuthal velocity, and which leads to particles at larger radii spending more time within the solenoid and thus being overfocused) and from changes in the particles' energy (due to the coupling between the radial space-charge force and a particle's radial velocity). For a nominal 4-kA, 6-MeV electron beam, the normalized emittance growths from these mechanism can easily exceed 200 mm mrad.

2 RADIAL EQUATION OF MOTION

The radial equation of motion for a particle within the beam within the central part of a solenoid (where the applied magnetic field from the solenoid is purely axial) is given by

$$m\frac{d(\gamma\dot{r})}{dt} = eE_r + e(v_\theta B_{dia} - v_z B_\theta) + ev_\theta B_{ext} + \frac{\gamma m v_\theta^2}{r}, \quad (1)$$

where γ is the relativistic mass factor, B_{ext} is the total external axial magnetic field (from both the solenoid and the diamagnetic effect from the image currents in the beampipe), B_{dia} is the diamagnetic axial magnetic field induced from the beam current opposing the solenoidal field, B_θ is the azimuthal magnetic field from the space charge, and E_r is the radial electric field from the space charge, all at the position of the particle, and e and m are the electronic charge and mass, respectively. For balanced flow, the solenoid strength is adjusted such that the linear part of the combination of $ev_\theta B_{ext}$ and the centrifugal force will cancel the linear part of the resulting space-charge force. There is also a potential depression within the beam (a variation of γ that is a function of radius). Our approach will be to expand the radial equation of motion in terms of the variation of γ, to lowest order, which we will then use to estimate the emittance growth for the two cases.

We will assume that the particles have no intrinsic angular momentum (there is no axial magnetic field at the location of the cathode) and that the external magnetic field is radially constant. Thus, the azimuthal velocity can found by application of Busch's Theorem [1] (the conservation of angular momentum):

$$v_\theta = -\frac{e}{2\gamma m}\int_0^r (B_{ext} + B_{dia})\nu\, d\nu \quad , \quad (2)$$

where ν is a dummy variable for the radial integration. We will use Gauss' Law to find the radial electric field,

$$E_r(r) = \int_0^r \frac{\rho(\nu)\nu}{\varepsilon r} d\nu \quad , \quad (3)$$

where ρ is the charge density. The diamagnetic field is given by

$$B_{dia} = \int_r^{r_b} \mu v_\theta(\nu)\rho(\nu)\, d\nu \quad (4)$$

where r_b is the radial edge of the beam. The diamagnetic field is small, and, to first order, only the azimuthal velocity depending on the externally applied solenoidal field needs to be considered in Eqn. (4). We can write the relativistic mass factor as $\gamma(r) = \gamma_a + \gamma_1(r)$, where γ_a is the mass factor along the axis ($r=0$). Let us assume that the space-charge density is of the form $\rho = \rho_o r^n$. Explicit evaluation of the above integrals for this charge density profile gives [2]

$$B = B_a\left(1 + \frac{\gamma_1}{\gamma_a}\frac{n+2}{2}\right) \tag{5}$$

where B is the total axial magnetic field and B_a is the field on axis. The azimuthal velocity in terms of the field and relativistic mass factor on axis is given by

$$v_\theta = -\frac{eB_a r}{2\gamma_a m}\left(\frac{1 + \dfrac{\gamma_1}{\gamma_a}\dfrac{n+2}{n+4}}{1 + \dfrac{\gamma_1}{\gamma_a}}\right) \quad . \tag{6}$$

The beam-induced azimuthal magnetic field in Eqn. (1) is given in terms of the vector potential by

$$B_\theta = \frac{\partial}{\partial z}A_r + \frac{1}{r}\frac{\partial}{\partial r}rA_z \quad . \tag{7}$$

If the beam is converging or diverging, there is a nonzero radial vector potential, and if the beam is being focused in a solenoid, the axial derivative of the radial vector potential is nonzero [3]. In that case, the azimuthal magnetic field is approximated by

$$B_\theta = -\frac{\beta}{c}\frac{1}{r}\frac{\partial}{\partial r}(r\phi) + \frac{\beta}{c}\left(\frac{13 - 8r/r_b}{10}\right)\frac{d^2 r}{dz^2}\phi_b \quad , \tag{8}$$

where the scalar potential at the beam radius is $\phi_b = \gamma_b mc^2/e$, and where γ_b is the difference in the relativistic mass factor between the center and radial edge of the beam.

Using dots to refer to time derivatives and primes to refer to axial derivatives, we have

$$\frac{d}{dt}\gamma\dot{r} = \dot{r}\frac{d\gamma}{dt} + \gamma\ddot{r} = \frac{eE_r}{mc^2}\dot{r}^2 + \gamma\ddot{r} \tag{9}$$

and

$$\begin{aligned} \dot{r} &= r' v_a\left(1 + \frac{v(r)}{v_a}\right) \\ \ddot{r} &= r'' v_z^2(r) = r'' v_a^2\left(1 + 2\frac{v(r)}{v_a}\right) \end{aligned} \tag{10}$$

where we are defining v_a to be the axial velocity at the axis and v to be the relative axial velocity, $v_z(r) = v_a + v(r)$. After combining the focusing term and the centrifugal acceleration term, and combining the r'' terms and dividing through by a factor of γ, Eqn. (1) becomes

$$\begin{aligned} r'' m v_a^2\left(1 + 2\frac{v}{v_a} - \frac{\gamma_b}{\gamma_a}\chi\right) &= \frac{eE_r}{\gamma\gamma^*(r)^2} + \\ \frac{1}{\gamma}\left(ev_\theta\left(B_{ext} + B_{dia}\right) + \frac{\gamma m v_\theta^2}{r}\right) &- \frac{e}{c^2\gamma}v_o^2 E_r r'^2 \end{aligned} \tag{11}$$

where $\chi = 8r/10r_b - 13/10$ and γ^* is an effective relativistic mass factor. To lowest order in the small quantities, the radial force equation becomes in terms of the parameters evaluated on axis

$$\begin{aligned} r'' &= \left(1 - 2\frac{v}{v_a} + \frac{\gamma_b}{\gamma_a}\chi\right)\times \\ &\left[\frac{eE_r}{mv_a^2\gamma_a\gamma^*(r)^2}\left(1 - \frac{\gamma_1}{\gamma_a}\right) - \frac{e^2B_a^2 r}{4\gamma_a^2 m^2 v_a^2}\left(1 + n\frac{\gamma_1}{\gamma_a}\right) - \frac{eE_r r'^2}{\gamma_a mc^2}\right]. \end{aligned} \tag{12}$$

For a uniform density beam after a drift of length l, we find that the integrated radial divergence is given by

$$\begin{aligned} r' &= \left(1 - 2\frac{v}{v_a} + \frac{\gamma_b}{\gamma_a}\chi\right)\times \\ &\left[\frac{leE_r}{mv_a^2\gamma_a\gamma^*(r)^2}\left(1 - \frac{\gamma_1}{\gamma_a}\right) - \frac{r}{f} - \frac{le}{\gamma_a mc^2}E_r r_o'^2\right] \end{aligned} \tag{13}$$

where r_o' is the initial radial divergence and where we are using the definition of the focal length of a solenoid, $f = 4\gamma_a^2 m^2 v_a^2 / le^2 B_a^2$. Note that the effect of the potential depression of the beam exactly cancels the effect of the diamagnetic field, leading to a purely linear focusing force and no emittance growth. However, there are four terms present that lead to an emittance growth, even for a uniform density beam. These are: (1) the axial velocity shear (which will arise from the conservation of energy and the particles' azimuthal velocity), (2) the chi term (which is from the axial derivative of the radial vector potential), (3) the radial space-charge term (for balanced flow, γ^* is independent of radius, and the nonlinear term scales as I^2/I_A^2), and (4) the radial velocity term (which arises from the divergence of the beam leading to energy variations). For most cases of interest, terms 2 and 3 are small, and we will only consider terms 1 and 4 in the next section.

Nonlinear space-charge forces arising from a nonuniform charge density can easily dominate these effects; special care must be made to ensure the beam density is uniform.

3 EMITTANCE GROWTH ESTIMATES

3.1 Emittance growth in the hard focusing case

There are two effects worth considering when the beam is focused to a tight waist. First, inside the solenoid itself there is an axial velocity shear. Second, there is a significant convergence of the beam as it travels to the waist. We will assue as nominal parameters $\gamma_a = 12.74$, $l = 0.1$ m, $f = 0.6$ m, and the beam radius in the final focus solenoid is $r_b = 0.03$ m.

Inside the solenoid, the variation in the axial velocity $(v_z = v_a + v)$ is given by

$$v = -\frac{c^2}{2v_a}\beta_\theta^2 = -\frac{e^2 B_a^2 r^2}{8\gamma_a^2 m^2 v_a} \tag{14}$$

The divergence in the beam introduced by the solenoid is then (ignoring all terms except for the terms depending on the solenoid's focal length)

$$r' = -\frac{r}{f}\left(1 - 2\frac{v}{v_a}\right) = -\frac{r}{f}\left(1 - \frac{r^2}{lf}\right) \quad . \tag{15}$$

The normalized, 90% emittance growth from the nonlinear part of this divergence is

$$\varepsilon = \frac{1}{3\sqrt{2}}\gamma_a \frac{r_b^4}{lf^2} \quad . \tag{16}$$

For the nominal parameters given, the emittance growth is about 70 mm mrad.

After the solenoid, the divergence term in Eqn. (13) dominates. We need to be a little careful because the beam radius is changing over the axial range we are interested in. The growth of the radial divergence is given by

$$dr' = \frac{eE_r}{\gamma_a mc^2}\frac{r^2}{\zeta^2}d\zeta \tag{17}$$

where ζ is the distance from the beam waist. The differential emittance growth is

$$d\varepsilon = d\zeta\frac{\sqrt{2}}{3}\frac{r_b^2}{\zeta^2}\frac{I}{I_A} \quad , \tag{18}$$

where $I_A = 4\pi\varepsilon mc^3/e$. This expression is easy to integrate, using $r_b = \zeta r_{lens}/L$, where r_{lens} is the beam radius at the solenoid and L is the total separation between the solenoid and beam waist. We find the total accumulated emittance growth at the final focus to be

$$\varepsilon = \frac{\sqrt{2}}{3}\frac{r_{lens}^2}{L}\frac{I}{I_A} \quad . \tag{19}$$

For the previous parameters and a final focus length L of 60 cm, the emittance growth is about 180 mm mrad.

3.2 Emittance growth for a scalloping beam

In general, the beam is not in completely balanced, uniform flow or being focused hard to a waist. The solenoids are discrete, and the beam-edge radius gently undulates down the beamline. We can estimate the emittance growth for a length l of scalloping motion, by using the divergence term in Eqn. (13) while assuming that the beam radius is a constant.

In this case, the accumulated nonlinear divergence after a length l is given by

$$r' = l\frac{eE_r}{\gamma_a mc^2}\frac{r^2}{r_b^2}\overline{\alpha}^2 = 2\frac{I}{\gamma_a \beta I_A}\frac{lr^3}{r_b^4}\overline{\alpha}^2 \tag{20}$$

where now $\overline{\alpha}$ is the rms divergence of the scalloping of the radial beam edge (let us say it is on the order of 20 mrad). For this case, the normalized, 90% emittance growth is given by

$$\varepsilon = \frac{\sqrt{2}}{3}l\frac{I}{I_A}\overline{\alpha}^2 \quad . \tag{21}$$

Note that neither the beam radius nor the beam energy enters this equation. For the other parameters used in the previous example, the normalized, 90% emittance growth is about $6(10^{-5})l$. This emittance growth will tend to accumulate, and can become very large over long beamlines of several tens of meters.

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