HALO FORMATION FROM AXISYMMETRIC BREATHING BEAMS

Masanori Ikegami and Hiromi Okamoto
Nuclear Science Research Facility, Institute for Chemical Research, Kyoto University
Gokanosho, Uji, Kyoto 611, JAPAN

Abstract

We study halo formation from mismatched axisymmetric beams propagating in a periodic solenoidal channel as well as in a uniform channel. Some fundamental properties of halos are self-consistently explored with a one-dimensional space-charge code designed particularly for breathing-mode study. We apply the code to three different types of phase-space distributions, i.e., Gaussian, waterbag and parabolic distribution. A possibility of removing halo particles is also discussed.

1 INTRODUCTION

In designing a linac system for intense beam acceleration, it is extremely important to have a clear understanding of space-charge-induced phenomena since the beam quality can easily be deteriorated by them. The halo formation is one such phenomenon which must be investigated in more detail. In fact, recent interest in using high-current ion linacs for the production of tritium, the transmutation of nuclear waste, etc. has greatly enhanced the activity of halo study, because these machines must operate with a extremely low beam loss to avoid serious radio-activation.

According to recent work on halos[1-4], beam mismatch is understood to be the primary factor of halo formation. In particular, it is speculated that the breathing mode-oscillations excited by a beam-size mismatch might have the most dominant effect in causing halos. Following this viewpoint, we develop, in the present paper, an essentially one-dimensional space-charge routine dedicated to breathing-mode study. Self-consistent simulation results are given to deepen our current understanding of halo formation in a periodic focusing channel as well as in a uniform channel. Finally, we try to figure out whether a halo may be scraped off, as pointed out in the previous work[5], by means of a multi-collimator system.

2 UNIFORM FOCUSING CHANNEL

First of all, we examine breathing-beam properties in a uniform focusing channel. In this case, the beam motion is governed by the Hamiltonian

$$H = \frac{1}{2}\left({p_r}^2 + \frac{{p_\theta}^2}{r^2} \right) + \frac{1}{2}\kappa^2 r^2 + \frac{q}{mu^2} V(r;s), \qquad (1)$$

where m, u and q are, respectively, the mass, speed and charge state of an ion, κ is a constant corresponding to the external focusing-field strength, and the independent variable s is the distance measured along the transport line. Note here that, because of the symmetry of breathing modes, the space-charge potential $V(r;s)$ is independent of the azimuthal coordinate θ, which enables us to put $p_\theta = L$=const. Scaling the variables, we reach the equation of motion

$$\frac{d^2\tilde{r}}{d\tilde{s}^2} + \tilde{r} = \frac{\tilde{L}^2}{\tilde{r}^3} + \tilde{K}\frac{\xi(\tilde{r};\tilde{s})}{\tilde{r}}, \qquad (2)$$

where $\tilde{r} = \sqrt{\kappa/\varepsilon}\, r$ with ε being the initial rms emittance, $\tilde{K} = K/\kappa\varepsilon$ with K being the generalized perviance, $\tilde{s} = \kappa s$, and $\xi(\tilde{r};\tilde{s})$ is the number of ions contained in the circular region of the radius $\tilde{r}$ relative to the total ion number. The parameter $\tilde{K}$ can be related to the tune depression η as $\tilde{K} = (1-\eta^2)/\eta$, where η has been defined as the ratio of the space-charge depressed betatron frequency to the zero-current frequency. The angular momentum $\tilde{L} = (xp_y - yp_x)/\varepsilon$ is a particle-dependent constant, and can be determined from an initial beam distribution generated in four-dimensional phase space (x, y, p_x, p_y). In this work, three types of realistic beam distribution, i.e., Gaussian-, waterbag-, and parabolic-type distribution, are adopted as the initial distribution. Once L and the initial shape of the function $\xi(\tilde{r};\tilde{s})$ are determined, we then integrate Eq. (2) fixing $\xi(\tilde{r};\tilde{s})$ within every time step. Since most beams come to roughly saturated state before arriving at $\tilde{s} = 20\lambda_p$, where λ_p is the scaled plasma wave length, we consider a uniform focusing channel of $20\lambda_p$ long.

Fig. 1 shows the maximum extent of halos plotted as a function of tune depression η. The parameter μ is the mismatch factor defined as the ratio of the initial rms beam radius to the matched rms radius ρ_0[6]. The matched rms radius can be evaluated from

$${\rho_0}^2 = \left(\tilde{K} + \sqrt{\tilde{K}^2 + 4} \right) \Big/ 4. \qquad (3)$$

It is evident from Fig. 1 that the halo extent divided by $R_0 \equiv \sqrt{2}\rho_0$ is almost independent of η. The same tendency

0-7803-4376-X/98/$10.00©1998IEEE

as shown in Fig. 1 has been confirmed with different values of μ unless μ is too close to one.

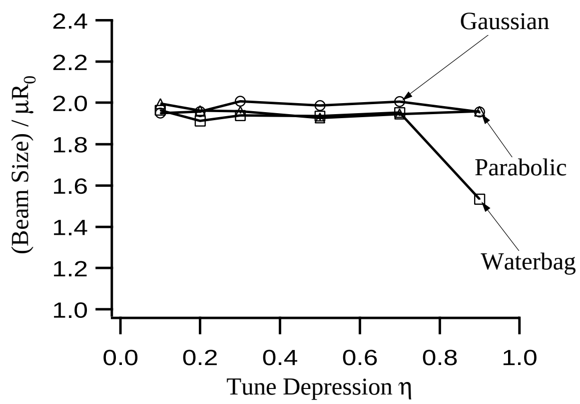

Figure 1: Maximum halo extent vs. tune depression η. The abscissa is the maximum halo size scaled with μR_0 where $R_0 = \sqrt{2}\rho_0$. We have here assumed the mismatch factor of 1.3. We find that the results are quite insensitive to both η and initial distribution type.

3 PERIODIC FOCUSING CHANNEL

Let us now proceed to a periodic focusing situation, generalizing the code. We here consider a periodic solenoidal channel consisting of 150 focusing cells, each having 50% filling factor. The equation of motion is

$$\frac{d^2\hat{r}}{d\hat{s}^2} + \vartheta(\hat{s})\hat{r} = \frac{\hat{L}^2}{\hat{r}^3} + \hat{K}\frac{\xi(\hat{r};\hat{s})}{\hat{r}}, \qquad (4)$$

where $\hat{r} = r/\sqrt{\varepsilon s_f}$ with s_f being the length of a focusing period, $\hat{K} = Ks_f/\varepsilon$, $\hat{s} = s/s_f$ and $\hat{L} = L/\varepsilon$. The periodic step function $\vartheta(\hat{s})$ has the periodicity of one, and its step size corresponds to the focusing-field strength. In the same scaling as employed in Eq. (4), the envelope equation is written as

$$\frac{d^2\hat{\rho}}{d\hat{s}^2} + \vartheta(\hat{s})\hat{\rho} = \frac{\hat{K}}{2\hat{\rho}} + \frac{1}{4\hat{\rho}^3}, \qquad (5)$$

from which the time evolution of the matched rms beam radius $\rho_0(\hat{s})$ is numerically evaluated.

The density-dependence of maximum halo extent is shown in Fig. 2 where we have considered mismatched Gaussian beams with μ=1.3. The definition of the mismatch factor μ is $\mu \equiv \rho_{in}/\rho_0^{\max}$ where ρ_{in} denotes the initial rms beam radius, and $\rho_0^{\max} \equiv \rho_0(0)$. Note here that $\rho_0(0)$ corresponds to the maximum rms radius of a matched beam since the origin of the coordinate s has been set at the middle of a focusing magnet. Similar to the result in Fig. 1, the maximum extents of halos are insensitive not only to the tune depression η but also to the zero-current phase advance σ_0.

When σ_0 exceeds 90°, we may encounter the strong instability caused by the periodic nature of the focusing force. Emittance growth rate is plotted in Fig.3 as a function of tune depression, where matched Gaussian beams with σ_0=105° have been considered. In waterbag and parabolic beams, some additional weak instabilities caused by higher-order resonances are observed in the region σ_0>60°, while no such instability has so far been identified in the region σ_0<90° for Gaussian beams.

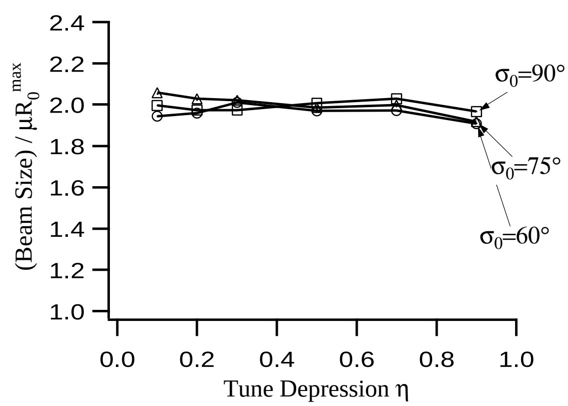

Figure 2: Maximum halo extent vs. tune depression η. The abscissa is the maximum halo size scaled by $\mu R_0^{\max}$ where $R_0^{\max} = \sqrt{2}\rho_0^{\max}$. Gaussian beams with the initial mismatch factor of 1.3 have been assumed. Three different values of zero-current phase advance, i.e., σ_0=60°, σ_0=75° and σ_0=90°, have been considered.

4 A POSSIBILITY OF HALO SCRAPING

In a previous work[5], we found that a halo was formed largely by the particles initially located around the tail portion of a phase-space distribution. This suggests that the beam core may roughly be stable and, accordingly, there is a possibility to efficiently reduce the halo intensity. In this section, we consider a simple halo-scraping system consisting of several collimators with a circular hole of the radius r_s. Each collimator is installed in the middle of drift space. For simplicity, we assume that both the geometrical and resistive-wall wake fields induced by the collimators are negligible.

As an example, let us take a Gaussian distribution which initially satisfies the conditions σ_0=60°, η=0.3 and μ=1.3. As shown in Fig. 4(a), a clear halo ring is formed around the central core region after the beam traverses 150 focusing periods. The beam is then delivered into a multi-collimator system of 14-cell long to scrape the halo. We have here set the radius of the collimator hole to be

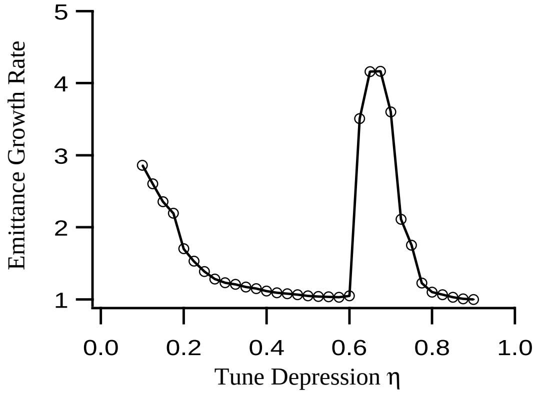

Figure 3: Emittance growth rate vs. tune depression η at σ_0=105°. Matched Gaussian beams have been considered.

$r_s/\mu R_0^{max}$=1. It is demonstrated, in Fig. 4(b), that the halo has been successfully removed by the collimators, although the beam intensity is reduced by 9.3% compared to the original intensity. The collimated beam in Fig. 4(b) further travels through a 150-cell transport channel to reach the final state in Fig. 4(c). We now recognize that the regeneration of halo has been completely suppressed while the core beam still has a mismatch executing a significant breathing oscillation.

5 SUMMARY

We have explored some characteristics of halo formation from axisymmetric mismatched beams propagating in a periodic solenoidal channel as well as in a uniform focusing channel, developing a 1D space-charge code for breathing-mode study. It has been shown that the maximum halo extent divided by the matched rms radius ρ_0^{max} is quite insensitive to the tune depression η. In particular, it always takes the value around $2\sqrt{2}$ times the size of initial mismatch regardless of the shape of distribution function, when the beam is subjected to a sufficiently large mismatch. It may thus be said that the minimum aperture size of a high-power linac should be well above $2\sqrt{2}\,\mu_{max}\,\rho_0^{max}$ when we expect the possible maximum mismatch factor to be μ_{max}.

It has been demonstrated that halo intensity may significantly be reduced by the multiple halo-scraping scheme, provided that the wake fields generated by the collimators are negligible. It, however, seems that the practicability of such a collimator system depends on how much we could minimize the effect of wake fields neglected here. In addition, it is indeed necessary to include multi-dimensional effects to draw more definitive conclusions on this subject .

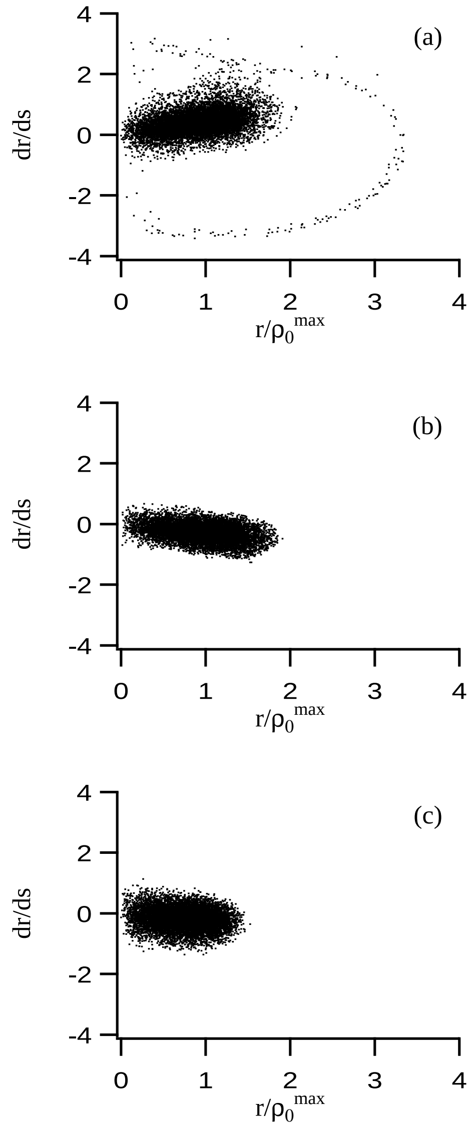

Figure 4: Phase-space configurations of a mismatched Gaussian beam initially with η=0.3 and μ=1.3. The zero-current phase advance has been set to be 60°. (a): the distribution after traveling through 150 focusing periods. (b): the beam collimated with a 14-cell halo scraper. (c): the collimated beam after traveling through another 150-cell focusing channel.

6 REFERENCES

[1]. R. A. Jameson, Los Alamos Report LA-UR-93-1029 (1993).

[2]. R. L. Gluckstern, Phys. Rev. Lett. **73**, 1247 (1994).

[3]. J. S. O'Connell, T. P. Wangler, R. S. Mills and K. R. Crandall, in Proceedings of the Particle Accelerator Conference, Washington DC, 3657 (1993).

[4]. J. M. Lagniel, Nucl. Inst. Meth. A345, 46 (1994).

[5] H. Okamoto and M. Ikegami, Phys. Rev. E. **55**, 4694 (1997).

[6]. In this paper, μ is always taken to be larger than one.