EFFECT OF NOISE IN BEAM-BEAM INTERACTION

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Abstract

Beam-beam instability induced by random fluctuations in size of a strong beam has been studied both numerically and analytically. In contrast with stochastic particle motion due to overlapping of nonlinear beam-beam resonance islands, noise beam-beam instability exists at any value of beam-beam parameter. Two conditions are essential to initiate instability: beam-beam kick has to be a nonlinear function of coordinate and parameter of the kick has to be a subject of noise. Meanwhile, the value of beam emittance is conserved even in noise regime if beam-beam kick is a linear function of coordinate. It gives an idea to prevent beam-beam instability using linearization of beam-beam kick. Compensation scheme utilizing higher order component in field distribution of the focusing elements is suggested.

1 INTRODUCTION

One of the main problems in ion-ion circular colliders is a small value of achievable beam-beam tune shift ξ=0.005. Physical reason for beam-beam limitation is usually attributed to the excitation of a set of nonlinear resonances due to a periodic nonlinear kick in linear system. Overlapping of nonlinear resonances is an universal mechanism of stochastic particle instability in nonlinear systems [1]. Another mechanism of unstable particle motion is a diffusion, created by a noise [2-5]. This noise can exist, for example, due to mismatch of the beam with the channel. In this paper we study the noise which appears in an incoherent beam-beam interaction. As it is shown below, such a noise can induce beam-beam instability in much more simple conditions than the overlapping of nonlinear resonances. Due to the diffusion character, noisy beam-beam instability does not have a threshold character and can exists under any value of beam-beam tune shift.

2 NUMERICAL SIMULATION OF NOISE BEAM-BEAM INSTABILITY

Let us consider for simplicity a one-dimensional model, which is suitable to demonstrate main features of noise beam-beam instability. Results of this study are valid for a multi-dimensional problem as well. Particle motion is described in coordinates (x, p =β dx/dz), where β is a value of beta-function of collider. Between subsequent collisions particle experience linear matrix transformation with betatron angle $\theta = 2\pi Q$. Beam-beam interaction is treated as a thin lens with nonlinear beam-beam kick Δp_n:

$$\begin{pmatrix} x_{n+1} \\ p_{n+1} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x_n \\ p_n + \Delta p_n \end{pmatrix}. \qquad (1)$$

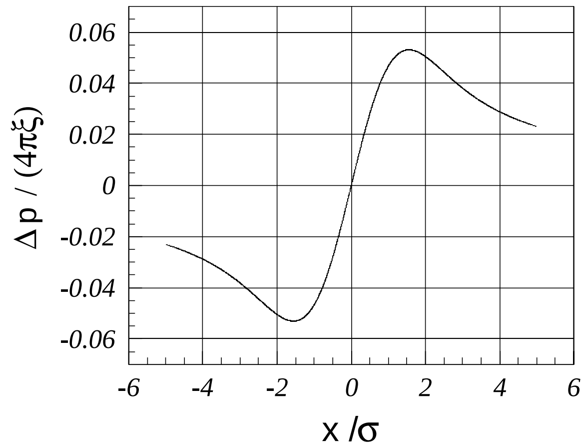

Fig. 1a

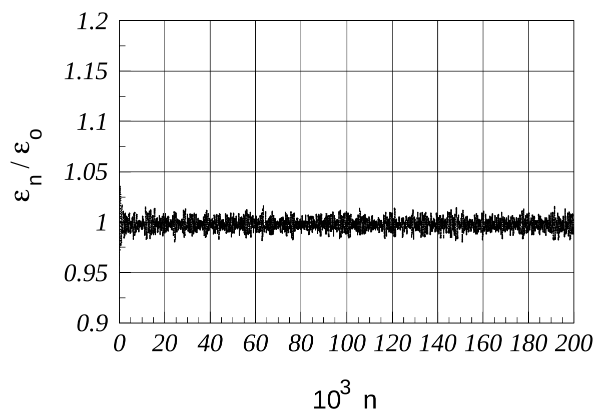

Fig. 1b

Fig. 1. Stable particle motion with the values of beam-beam tune shift ξ =0.005 and betatron tune Q=3.168: a) beam-beam kick, b) rms beam emittance.

Beam-beam kick Δp is expressed by a Gaussian function of coordinate x, beam-beam tune shift ξ, and size of the opposite beam σ:

$$\Delta p = - 4\pi\xi x \frac{1 - \exp(- x^2/ 2 \sigma^2)}{(x^2/ 2 \sigma^2)} . \qquad (2)$$

Nonlinear kick (2) induces a set of nonlinear resonances, which are stable, until islands do not overlap each other. In Fig. 1 an example of stable beam-beam interaction in the vicinity of the 6th order resonance is presented. In simulations, one beam is presented as a collection of 3000 particles. Simulations are done for the value of betatron tune Q = 3.168, close to the 6th order resonance value 3.16666 (or 6Q=19). From the simulations it is clear that beam emittance is stable.

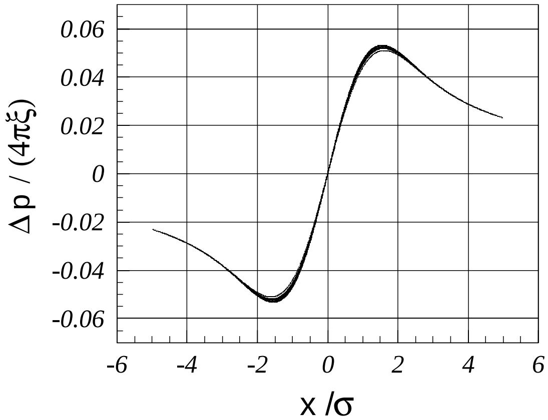

Fig. 2a

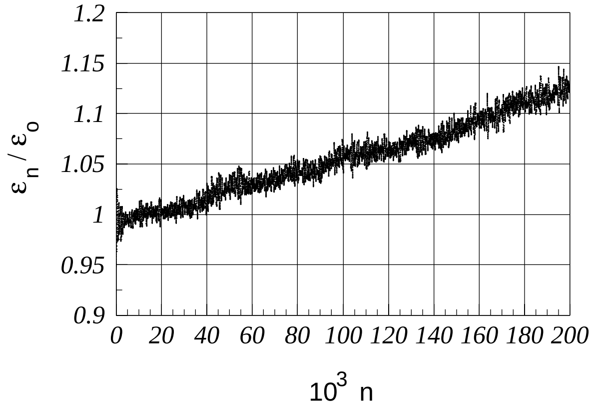

Fig. 2b

Fig.2. Beam-beam instability under 5% noise in the parameter σ during particle interaction with $\xi = 0.005$ and Q=3.168: a) beam-beam kick, b) rms beam emittance growth.

Another picture is observed if the parameter σ of beam-beam kick (2) is a subject of noise. In the calculations, presented in Fig. 2, parameters of the process are chosen to be the same as those in Fig.1, but standard deviation σ is changed from turn to turn according to the expression:

$$\sigma^{(n)} = \sigma^{(o)} \left(1 \pm \frac{u}{2} u_n\right) , \qquad (3)$$

where u is a noise amplitude and u_n is a random noise function within the interval (0,1). It corresponds to the noise in the size of the opposite beam, which can exist due to small beam mismatch with the channel. The value of noise amplitude u=0.05 is chosen arbitrary to demonstrate appearance of diffusion-type instability in the presence of small random perturbation of beam-beam kick. As shown in Fig. 2, this noise destroys the stability. In contrast with Fig. 1, beam emittance expand with time. Important feature of the noise regime is that this kind of instability can exist apart from the excitation of nonlinear resonances. Noise beam-beam instability appears if two conditions are met:

- beam-beam kick is a nonlinear function of coordinate
- parameter of beam-beam kick (beam standard deviation σ) is a subject of noise.

3 ANALYTICAL TREATMENT OF EFFECTIVE BEAM EMITTANCE GROWTH

Let us provide analytical estimations of emittance growth under noise beam-beam interaction. Transfer matrix after n turns with arbitrary momentum kick at every turn, Δp_i , is given by the expression [6]:

$$x_n = a \cos (n\theta + \Psi) + \sum_{i=0}^{n-1} \Delta p_i \sin (n-i)\,\theta ,$$
$$p_n = - a \sin (n\theta + \Psi) + \sum_{i=0}^{n-1} \Delta p_i \cos (n-i)\,\theta , \qquad (4)$$

where Ψ is the initial phase of oscillations. Random beam-beam kick (Δp_i) can be expressed as a function of unperturbed trajectory. It gives an approximate treatment of the problem, valid for small values of perturbation. Suppose that perturbation is a linear function of coordinate:

$$\Delta p_i = \delta_i x_i. \qquad (5)$$

We study evolution of the root-mean-square (rms) beam emittance $\varepsilon_n = 4\sqrt{<x_n^2> <p_n^2> - <x_n p_n>^2}$, where brackets mean averaging on initial phases of particles. Calculation of beam emittance growth gives:

$$\frac{\varepsilon_n^2}{16} = \left[\frac{a^2}{2} + \frac{a^2}{4}\sum_{i=0}^{n-1}\delta_i^2\right]^2 -$$
$$- \left[\frac{a^2}{2}\sum_{i=0}^{n-1}\delta_i \sin(2n\theta-2i\theta) - \frac{a^2}{4}\sum_{i=0}^{n-1}\delta_i^2 \cos(2n\theta-2i\theta)\right]^2 -$$
$$- \left[\frac{a^2}{2}\sum_{i=0}^{n-1}\delta_i \cos(2n\theta-2i\theta) + \frac{a^2}{4}\sum_{i=0}^{n-1}\delta_i^2 \sin(2n\theta-2i\theta)\right]^2 . \qquad (6)$$

Analysis of the expression (6) shows that terms proportional to δ, δ^2, δ^3 are vanished. Rms beam emittance is conserved until high order of perturbation:

$$\frac{\varepsilon_n^2}{\varepsilon_o^2} = 1+\varsigma(\delta^4), \quad \varsigma(\delta^4) = \frac{1}{4}\sum_{i=0}^{n-1}\sum_{k=0}^{n-1}\delta_i^2\delta_k^2\,[1- \cos(2i\theta -2k\theta)]. \qquad (7)$$

Above derivations are approximate due to suggestion that linear beam-beam kick is proportional to the unperturbed trajectory. Numerical simulations exhibit exact conservation of rms beam emittance in the linear beam-beam kick regime. Preservation of beam emittance is explained by the fact that in case of linear kick a beam of particles experiences sequence of linear transformation, each of them conserves beam emittance.

Let us consider now the case when kick is a nonlinear function of unperturbed trajectory:

$$\Delta p_i = 4\ \delta_i x_i^3 . \qquad (8)$$

Calculation of rms beam emittance gives:

$$\frac{\varepsilon_n^2}{16} = \left[\frac{a^2}{2} + \frac{5}{2} a^6 \sum_{i=0}^{n-1} \delta_i^2\right]^2 -$$

$$- \left[\frac{3a^4}{2} \sum_{i=0}^{n-1} \delta_i \sin(2n\theta-2i\theta) - \frac{5a^6}{2} \sum_{i=0}^{n-1} \delta_i^2 \cos(2n\theta-2i\theta)\right]^2 -$$

$$- \left[\frac{3a^4}{2} \sum_{i=0}^{n-1} \delta_i \cos(2n\theta-2i\theta) + \frac{5a^6}{2} \sum_{i=0}^{n-1} \delta_i^2 \sin(2n\theta-2i\theta)\right]^2. \quad (9)$$

In contrast with linear beam-beam kick the terms proportional to δ^2 in expression (9) do not vanish. Effective beam emittance growth is then as follows:

$$\frac{\varepsilon_n^2}{\varepsilon_0^2} = 1 + \delta^2 a^4 n + \zeta(\delta^4), \quad (10)$$

$$\zeta(\delta^4) = 25 a^8 \sum_{i=0}^{n-1} \sum_{k=0}^{n-1} \delta_i^2 \delta_k^2 [1 - \cos(2i\theta - 2k\theta)]. \quad (11)$$

In the case of nonlinear kick, the beam emittance growth does not vanish in the first positive term δ^2, which indicates that nonlinearity is an essential point for expansion of emittance growth under noise conditions.
Comparison of eq.(10) with eq. (2) gives an expression for a diffusion coefficient D in beam emittance growth under noise regime $\varepsilon_n/\varepsilon_o = \sqrt{1 + D\, n}$:

$$D = \pi^2 (\xi u)^2. \quad (12)$$

Diffusion coefficient exhibits quadratic dependence on noise amplitude and on value of beam-beam parameter, which is confirmed by computer simulations. It indicates, that noise instability in beam-beam interaction appears under arbitrary small values of u and ξ.

4 COMPENSATION OF BEAM-BEAM KICK

From results of previous section it follows, that the linear beam-beam kick preserves beam emittance even in the case of noise. Therefore linearization of the kick is expected to be a way to improve particle stability. A trivial approach assumes that the opposite beam is uniformly populated. Another method suggests utilization of high order components in focusing lenses.
In an axial -symmetric magnetic lens a particle with a larger radius experiences stronger focusing as compared with linear law:

$$\Delta r' = -\frac{r}{f}\left(1 + \frac{C_s}{f}\frac{r^2}{f^2}\right), \quad (13)$$

where f is the focal length of the lens, and C_S is the spherical aberration coefficient. The dimensionless ratio C_s/f indicates significance of lens aberrations and is a figure of merit of the lens quality. The nonlinear term $\sim r^3$ can be used for compensation of the corresponding nonlinear term in the beam-beam collision. Magnetic lens

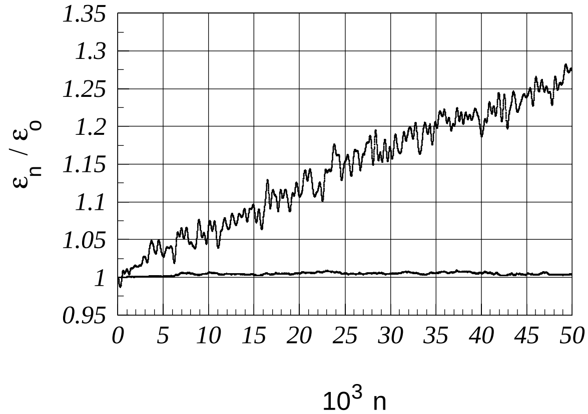

Fig.3. Beam emittance evolution in circular beam-beam interaction with kick (14), Qx = Qy =3.175 , ξ=0.015 and 5% noise in σ: (top) without compensation; (bottom) with compensation.

always focuses particles, therefore it can be used to compensate for beam-beam effect of colliding beams with opposite charge which also provides mutual focusing. Suppose beam-beam kick includes only the linear ~ r and first nonlinear ~ r^3 terms:

$$\Delta p = -4\pi\xi\, r \left(1 - \frac{r^2}{4\sigma^2}\right). \quad (14)$$

Nonlinear terms of beam-beam kick and of aberration have to be equal each other. Taking into account that $p=\beta\cdot dr/dz$, the condition for compensation is :

$$\left(\frac{C_s}{f}\right)\frac{\beta}{f^3} = \frac{\pi\xi}{\sigma^2}. \quad (15)$$

Fig.3 illustrates the effect of compensation of the cubic term in beam-beam kick. As can be seen, beam emittance is kept constant if a compensation lens is applied. Nevertheless, the effect of compensation vanishes if beam-beam kick includes all higher order nonlinear terms of the Gaussian kick and the compensation kick is still proportional to r^3. Therefore, the suggested compensation by lens aberration can be applied only to a weakly nonlinear beam-beam kick.

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