

SPIN TRACKING STUDY OF THE AGS*

Haixin Huang, Thomas Roser, Alfredo Luccio
Brookhaven National Laboratory, Upton, NY 11973, USA

Abstract

In the recent polarized proton runs in the AGS, a 5% partial snake was used successfully to overcome the imperfection depolarizing resonances. Although some depolarization at intrinsic resonances are expected, the level of the depolarization does not agree with a simple model calculation. A spin tracking program is then used to simulate the real polarized proton beam in the AGS. The results show that, due to the linear coupling introduced by a solenoidal 5% partial snake, the polarized beam will be partially depolarized also at the so-called coupling resonance, which is related to the horizontal betatron tune. The synchrotron oscillation also affect the beam polarization to a smaller extent.

1 INTRODUCTION

The acceleration of polarized beam will encounter depolarizing resonances whenever the spin precession frequency exactly matches the frequency with which the protons encounter depolarizing horizontal magnetic fields. In the recent polarized proton run in the AGS, a 5% partial snake [1] was used to overcome the imperfection depolarizing resonances [2]. Fig. 1 shows the measured absolute value of the vertical polarization at $G\gamma = n + \frac{1}{2}$ up to $G\gamma = 22.5$ (solid points). For these measurements, the betatron tunes were set at $\nu_x = 8.80$, $\nu_y = 8.70$, and the acceleration rate α was about 1.1×10^{-5} . The pulsed tune-jump quadrupoles were not used in this experiment. As shown in Fig. 1, the depolarization resulted only from the three intrinsic resonances, located at $G\gamma = 0 + \nu_y$, $24 - \nu_y$, and $12 + \nu_y$, which the 5% partial snake could not overcome. The observed level of depolarization at the intrinsic resonances were -65%, 63% and -49%, respectively.

Although some depolarization at intrinsic resonances are expected, the level of the depolarization does not agree with a simple model calculation. When calculating the intrinsic resonance depolarization, one has to take into account the beam distribution. Suppose the beam distribution is Gaussian, the effective polarization after passing through the resonance becomes,

$$\frac{P_f}{P_i} = \frac{[1 - \frac{\pi|\epsilon(\epsilon_0)|^2}{\alpha}]}{[1 + \frac{\pi|\epsilon(\epsilon_0)|^2}{\alpha}]}, \quad (1)$$

where P_f and P_i are the polarization before and after the resonance crossing, respectively, $\epsilon(\epsilon_0)$ is the resonance strength with rms emittance ϵ_0 , and α is the resonance crossing rate. The measured normalized 95% emittance

* Work performed under the auspices of the U.S. Dept. of Energy

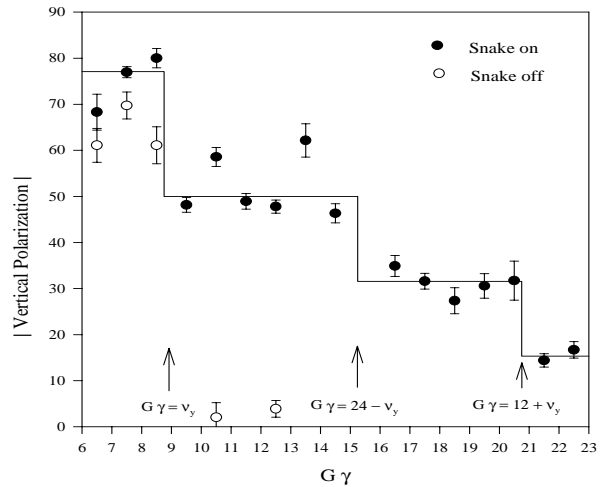


Figure 1: The measured absolute value of the vertical polarization at $G\gamma = n + \frac{1}{2}$ up to $G\gamma = 22.5$ (solid points). The acceleration rate α is about 1.1×10^{-5} .

$\epsilon_{N,95\%}$ in the AGS is about $25 \sim 30 \pi$ mm-mrad; the acceleration rate α is 1.1×10^{-5} ; and the resonance strengths of $0 + \nu_y$, $24 - \nu_y$ and $12 + \nu_y$ given by DEPOL [3] for a $\epsilon_N = 10\pi$ mm-mrad beam are 0.0154, 0.00062 and 0.0054, respectively. With these numbers, the polarization after crossing each of the three resonances are $\frac{P_f}{P_i} = -0.94$ at $0 + \nu_y$, $\frac{P_f}{P_i} = 0.90$ at $24 - \nu_y$, and $\frac{P_f}{P_i} = -0.61$ at $12 + \nu_y$, respectively. They are different from the measured values. Thus a spin tracking simulation is needed to understand the spin dynamics when a partial snake is inserted in the ring.

DEPOL [3], a program written by E.D. Courant, calculates the depolarizing resonance strength by Fourier analysis. The inputs of DEPOL are the outputs of a machine code such as MAD or SYNCH. DEPOL is simple but it requires a smooth lattice condition which is not satisfied by a ring with a snake inserted. Moreover, there is no way to include the effect of synchrotron motion, and linear coupling effect of a solenoidal snake. So a spin tracking program is needed to analyze the data obtained from the AGS partial snake experiment.

2 SPIN TRACKING PROGRAM

A tracking program SPINK [4] is used to track the polarized proton beam in the AGS. The idea is to track a group of protons, randomly generated with certain distribution in the phase space, through the machine lattice. Each proton is characterized by four transverse coordinates, two longitudinal coordinates and three spin components. Orbit matrices are built from a TWISS file, output of MAD.

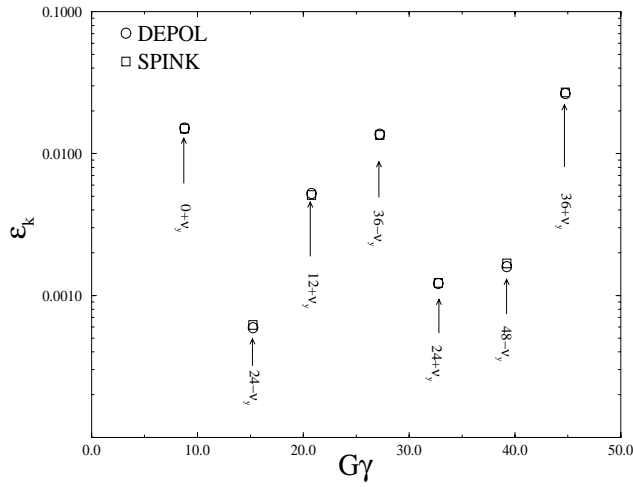


Figure 2: The comparison of results of DEPOL and SPINK for intrinsic resonances. The acceleration rate α used for SPINK is 4.5×10^{-5} .

Spin matrices are built for each piecewise constant magnet, which includes bending magnets (both separate function and combine function), quadrupoles (both regular and skew), snakes, rf cavities, spin flippers.

SPINK can be used to check the strength calculated by DEPOL if the resonances are well-separated and are not too strong. The first condition insures that the polarization is stable at the beginning of the tracking. The second condition should be met to avoid complete spin flip.

Suppose the initial polarization P_i is 1, then the resonance strength can be extracted from the final polarization $\langle P_f \rangle$ simulated by the tracking:

$$|\epsilon_k| = \sqrt{-\frac{2\alpha}{\pi} \ln \frac{(1 + \langle P_f \rangle)}{2}}, \quad (2)$$

where $\langle P_f \rangle$ is the average of the polarization over a number of turns after crossing the resonance.

An example is given for the AGS lattice without partial snake. The betatron tunes are set to be $\nu_x = 8.717$ and $\nu_y = 8.766$, respectively. The same TWISS file is used as inputs for DEPOL and SPINK. For the imperfection resonances, the random error is chosen as $\Delta y = 0.3$ mm. For the intrinsic resonances, the particle is chosen on the boundary of the 10π mm-mrad normalized emittance. The intrinsic resonance strengths are plotted in Fig. 2. The results of SPINK for the imperfection and intrinsic resonances agree well with those of DEPOL.

3 TRACKING FOR THE AGS BEAM

The SPINK program is then used to simulate the real polarized proton beam in the AGS. In the simulation, the betatron tunes, acceleration rate and the normalized longitudinal emittance are chosen as the experiment values; and for the simplicity, the two transverse emittances are set at the same value $\epsilon = 30\pi$ mm-mrad. A group of 200 particles randomly chosen with a Gaussian distribution in trans-

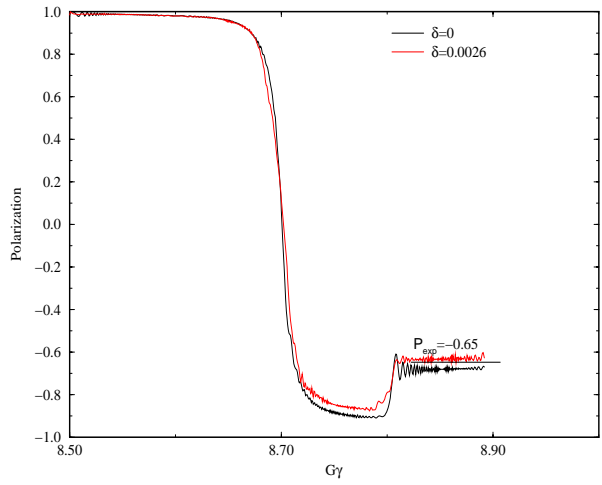


Figure 3: The simulation of crossing $G\gamma = 0 + \nu_y$, where $\nu_x = 8.80$ and $\nu_y = 8.70$. The momentum spread $\delta = 0.0026$ corresponds to emittance 0.8 eVs. The transverse emittances $\epsilon_x = \epsilon_y = 30\pi$ mm-mrad. The final polarization $P = -0.63$, which agrees with the experimental value $P_{expt} = -0.65$ within the error bar ± 0.05 . The acceleration rate α is 1.1×10^{-5} .

verse phase space and a parabolic distribution in longitudinal phase space are used in the tracking. The tracking results for $0 + \nu_y$, $12 + \nu_y$ and $24 - \nu_y$ are plotted in Figs. 3, 4 and 5, respectively.

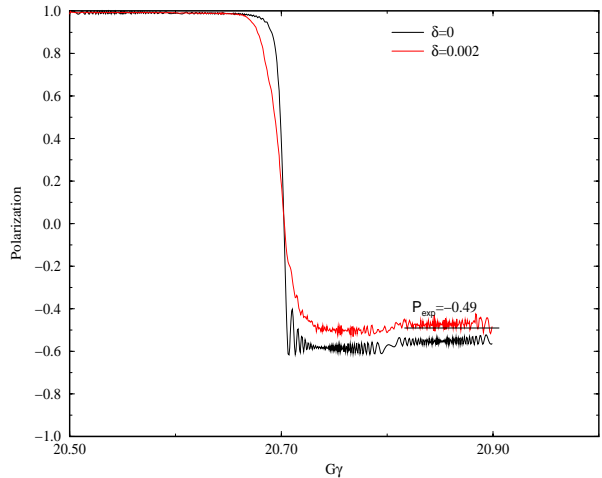


Figure 4: The simulation of crossing $G\gamma = 12 + \nu_y$, where the beam condition is the same as $G\gamma = 0 + \nu_y$. The momentum spread $\delta = 0.002$ corresponds to emittance 0.8 eVs. The final polarization $P = -0.47$, which agrees with the experimental value $P_{expt} = -0.49$ within the error bar ± 0.05 .

Figs. 3 and 4 clearly show that there is an extra resonance adjacent to the intrinsic resonance, which causes depolarization. This additional resonance can be easily understood as a linear coupling effect. The solenoidal partial snake introduces considerable linear coupling between the two transverse betatron motions. Due to the coupling,

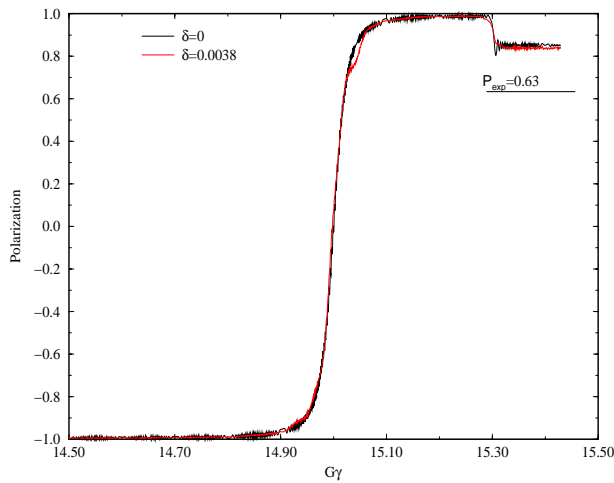


Figure 5: The simulation of crossing $G\gamma = 24 - \nu_y$, where the beam condition is the same as $G\gamma = 0 + \nu_y$. The momentum spread $\delta = 0.0038$ corresponds to emittance 0.8 eVs. The final polarization $P = 0.83$, which is higher than the experimental value $P_{expt} = 0.63$.

the vertical betatron motion also has a component with the horizontal betatron frequency. As a consequence, the beam will see an additional resonance, the so-called coupling resonance. Besides the linear coupling effect, the synchrotron motion also affect the beam polarization but to a smaller extent. The results show good agreement with the experimental data.

The simulation results for $24 - \nu_y$ are shown in Fig. 5. The coupling resonance is very weak as expected and there is no visible effect. The unexpected high depolarization at $G\gamma = 24 - \nu_y$ is believed to be caused by the coincidence with the AGS transition energy (For AGS, $\gamma_t = 8.5$, which corresponds to $G\gamma_t = 15.24$. When $\nu_y = 8.70$, the intrinsic resonance is located at $G\gamma = 24 - \nu_y = 15.3$). Further simulation study is needed to understand this depolarization.

The coupling resonance strength can be decreased by separating the betatron tunes. Thus it seems that the linear coupling can be diminished by well-separated betatron tunes. The betatron tunes of the AGS can be separated by more than one unit, so a simulation with $\nu_x = 7.70, \nu_y = 8.80$ is performed (see Fig. 6). Although there is no depolarization at $G\gamma = 0 + \nu_x = 7.70$, beam depolarization happens at $G\gamma = 1 + \nu_x = 8.70$ and the depolarization level is about the same as the one with $\nu_x = 8.80, \nu_y = 8.70$ (see Fig. 3). Because there is only one coupling element, the solenoidal partial snake, in the AGS, the coupling kick has all Fourier components $G\gamma = N \pm \nu_x$ and with the same amplitude. Whichever Fourier component is closest to the vertical betatron tune, it will pick up the strength of the intrinsic resonance and become the strongest coupling resonance.

The ratio of the coupling resonance strength to the intrinsic resonance strength with a 5% partial snake and 0.1 unit tune separation is about 0.06. With a larger tune separation

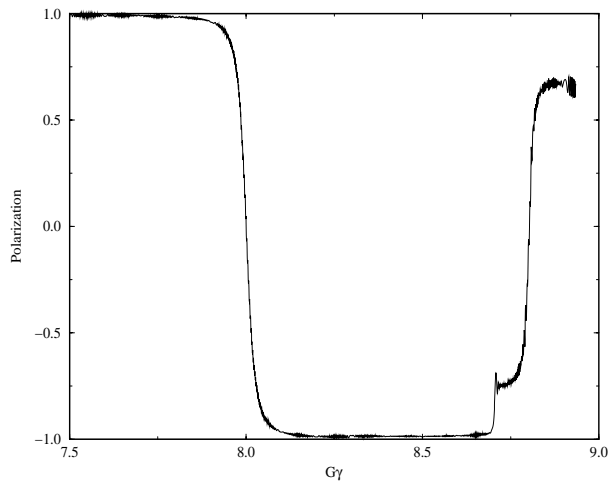


Figure 6: The simulation results for $\nu_y = 8.80$ and $\nu_x = 7.70$. $\varepsilon_x = \varepsilon_y = 30\pi$, $\delta = 0$, $\alpha = 1.1 \times 10^{-5}$.

of 0.3, the ratio goes down to 0.03. This ratio makes it impossible to cross those strong intrinsic resonances ($0 + \nu$, $12 + \nu$, and $36 \pm \nu$) at a single speed without losing polarization. A novel energy-jump method has been tested in the AGS to cross the coupling resonance and the results are summarized in [5].

4 CONCLUSION

As shown in the simulations, it is clear that depolarization level after crossing the strong intrinsic resonances is due to the combined effect of the coupling resonance and synchrotron oscillation. The coupling resonance strength can be decreased by separating the betatron tunes. However, separating the two tunes for more than 0.5 unit does not help. Spin tracking is a very useful tool to understand the experimental data and to prepare for the future experiments. Further simulation is going on for the AC dipole methods[6].

5 REFERENCES

- [1] T. Roser, in *High-energy spin physics: 8th international symposium*, ed. K.J. Heller, Minneapolis, 1988, AIP Conference Proceedings No. **187**(AIP, New York, 1989), p.1442.
- [2] H. Huang, et al., *Phys. Rev. Lett.* **73**, 2982 (1994).
- [3] E.D. Courant and R. Ruth, BNL report, BNL-51270(1980).
- [4] A. Luccio, BNL Report, BNL-52481(1995).
- [5] H. Huang, et al., *These Proceedings*.
- [6] M. Bai, et al., *These Proceedings*.