RESONANT EFFECTS IN THE LONGITUDINAL COUPLING IMPEDANCE OF A RECTANGULAR SLOT IN A COAXIAL LINER

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Abstract

We earlier developed a general analysis, based on a variational formulation, which includes both the realistic coaxial structure of the beam pipe and the effect of finite wavelength in the calculation of the coupling impedance of a rectangular slot in a liner wall of zero thickness. In the present paper we use this analysis to study the frequency dependence of the coupling impedance of a longitudinal rectangular slot. Resonant effects in the coupling impedance are discussed.

1 IMPEDANCE OF A SMALL SQUARE HOLE EFFECT OF THE OUTER WALL

We use expressions obtained earlier[1] to obtain numerical values for a small square hole with the edge length equal to $w = l = 0.25\ a$, where a is the radius of the liner, and $b/a = 1.3125$. For a=16 mm this corresponds to the parameters used in the calculation of Scholz.[2] The frequency dependence of both the real and imaginary parts of the coupling impedance are presented in Fig. 1. Plots similar to Fig. 1 were obtained by Scholz,[2] but the peaks he obtains for the cutoffs of the modes in the coaxial region are in error.[3]

1.1 Imaginary Part

Using the expressions for the magnetic susceptibility ψ and electric polarizability χ for the square hole in a plane metallic wall in the well known Bethe small hole approximation, for the frequency 1 GHz and dimensions of the square hole given above, one obtains

$$Z = j0.0073[\Omega]. \qquad (1)$$

In the problem we consider, the wall of the outer pipe is at radius $b = 21$ [mm], with $b - a = 5$ [mm]. The result is expected to be influenced by the outer metallic wall, being less than the one given by Eq. (1), and to approach this result for large $(b-a)$. This behavior is consistant with the numerical results in Table 1. Results obtained by Scholz[2] for 1 GHz are a few percent higher, and at $b/a = 2$ the numerical value obtained by Scholz is already 10 percent above the static result in Eq. (1) for the hole in the pipe without an outer wall.

1.2 Real part

At the frequency 1 GHz ($ka = 0.3351$) with $a = 16$ [mm], our numerical results (Ref.[1]) and the results obtained by Scholtz[2] in Table 2 are significantly larger than those obtained by using the formula of Palumbo *et al.*[4]

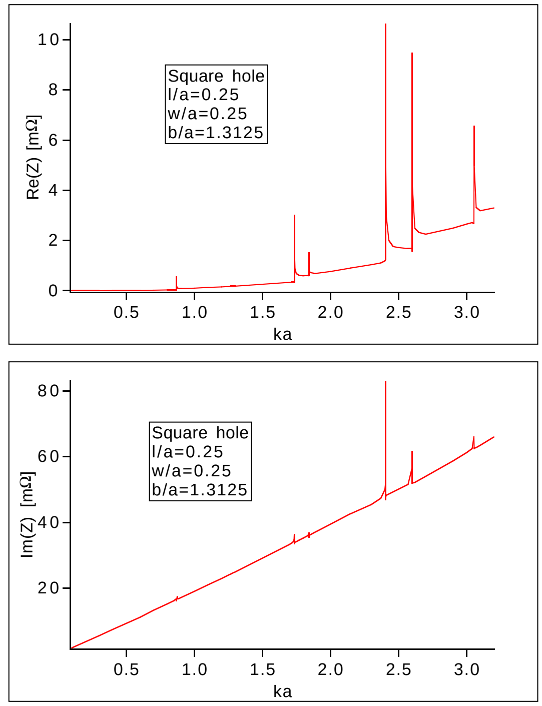

Figure 1: Real and imaginary parts of the impedance of a square hole (for a=16 mm, w=l=4 mm).

ka	$b/a = 1.2$	$b/a = 1.3125$	b/a=1.5
0.3351	6.46	6.57	6.60

Table 1: $Im(Z)$ $[m\Omega]$ for a square hole with 4 [mm] edge length at frequency 1 GHz.

Based on our earlier analysis[1] we can examine the low frequency behavior analytically. For frequencies below all possible cutoffs, we obtain

$$Re\left(\frac{Z}{Z_0}\right) = \frac{k^2}{64\pi^3 a^4 ln(b/a)}(\psi^2 + \chi^2) \qquad (2)$$

due to energy radiated in the TEM mode. The available static approximations for ψ and χ can be now used to estimate the real part of the impedance for holes of different shape. The value of the impedance obtained in this way will be a few percent higher than the real one due to the fact that expressions for ψ and χ are given in the literature

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Ref.	$b/a = 1.2$	$b/a = 1.3125$	$b/a = 1.5$
[1]	7.54	5.21	3.53
[2]	≈ 7.7	≈ 5.3	≈ 3.8
[4]	1.64	1.1	0.74

Table 2: $Re(Z)$ $[\mu\Omega]$ for frequency 1 GHz.

b/a	1.2	1.3125	1.5	$\to \infty$
$\chi/(2w^3)$	0.0998	0.1010	0.1015	0.1082
$\psi/(2w^3)$	0.2305	0.2339	0.2351	0.2532

Table 3: The electric polarizabilty χ and magnetic susceptibility ψ of a square hole ($w = l$).

for a hole in a plane metallic wall (without the outer wall which is present in the coaxial structure). As an example, in Table 3 we present the b/a dependence of ψ and χ, which approach the results available for a square hole in a metallic plate when $b/a \to \infty$. For the circular hole, using Eq. (2) we obtain

$$Re\Big(\frac{Z}{Z_0}\Big) = \frac{5k^2r^6}{36\pi^3a^4ln(b/a)}, \qquad (3)$$

where r is the radius of the hole. The expression given in Eq. (3) is a factor of 5 larger than the one obtained by Palumbo *et al*.[4] The results obtained by using Eq. (3) are now in good agreement with the numerical calculations.

The real part of the impedance of a small hole in a coaxial structure varies as k^2 in contrast to the k^4 behavior for the radiation of a hole into free space and is now relatively more important.

2 IMPEDANCE OF A LONGITUDINAL SLOT

2.1 Validity of the static approximation for low frequencies

The frequency correction for the elliptical slot[5] suggests that the imaginary part of the impedance of a long elliptical slot should be strongly reduced at finite frequencies. Therefore, it is important to understand whether a similar effect is present for a rectangular slot at finite frequencies. Numerical study of this question for a long rectangular slot shows that the reduction in the imaginary part of the impedance at low frequencies is insignificant. The available static approximations for ψ and χ, give a reasonably good description for low frequencies even when the slot length is larger than the radius of the pipe, starting to lose its accuracy at large l/a (Fig. 2). For higher frequencies ($ka \geq 1$) the static approximation loses its accuracy at much smaller values of l/a (Fig. 3). However, the dependence of the imaginary part of the impedance on slot length is insignificant (Figs. 2 and 3).

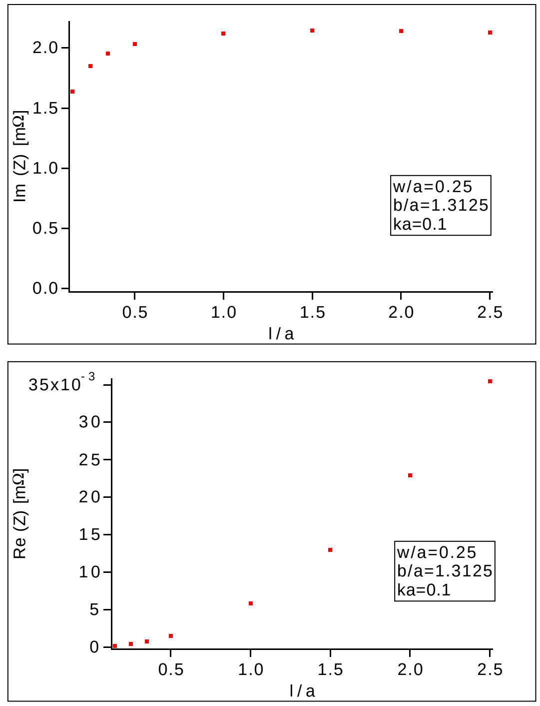

Figure 2: Real and imaginary parts of the impedance for low frequencies ($ka = 0.1$).

2.2 Resonances

One can see the appearance of resonant behavior for the real part of the impedance in Fig. 4. For the imaginary part we identify the resonances as the place where the impedance changes its sign. There appears to be resonant behavior for the odd and even parts of the impedance separately (Fig. 4), but due to the strong cancelation between the even and odd parts no resonant behavior is seen for the imaginary part of total impedance (Fig. 6). This makes it possible to design relatively long slots. The first two resonances can be seen in Fig. 5 for slot lengths $l/a = 3$ and $l/a = 5$. The frequency behavior of the real and imaginary parts of the total impedance for a very long rectangular slot ($l/a = 5$) is shown in Fig. 6.

3 REFERENCES

[1] A.V. Fedotov, R.L. Gluckstern, General Analysis of the Longitudinal Coupling Impedance of a Rectangular Slot in a Thin Coaxial Liner, preceding paper.

[2] T. Scholz, Impedance of rectangular slots in a round coaxial tube. Proceeding of the EPAC-96, Spain, 1996.

[3] T. Scholz, Private communication.

[4] S. De Santis, M. Migliorati, L. Palumbo and M. Zobov, Impedance of a hole in coaxial structures. Proceeding of the EPAC-96, Spain, 1996.

[5] A.V. Fedotov and R.L. Gluckstern, Phys. Rev. E, **54**,1930 (1996).

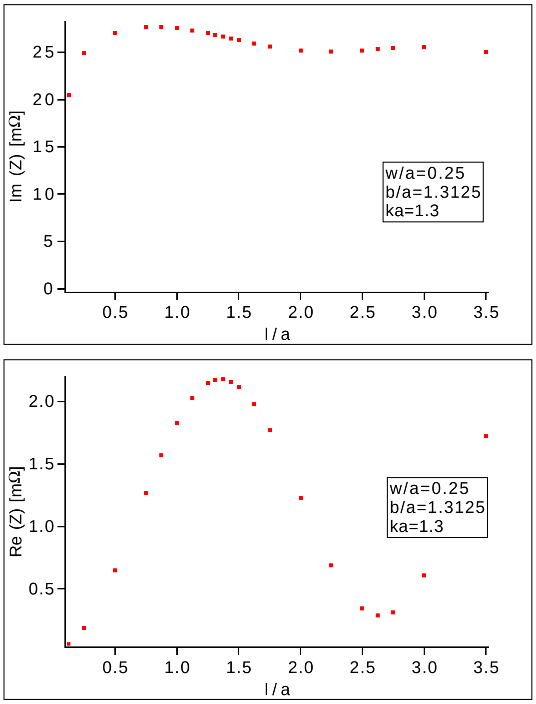

Figure 3: Real and imaginary parts of the impedance for high frequencies ($ka = 1.3$).

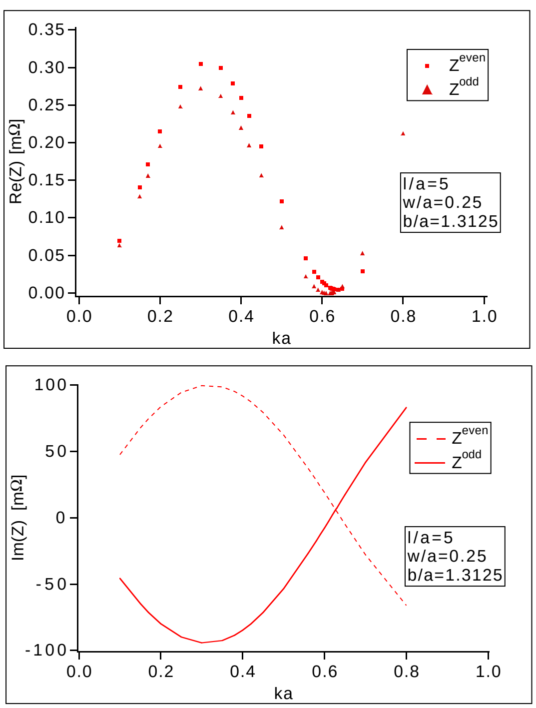

Figure 4: Resonant behavior in the even and odd parts of the impedance.

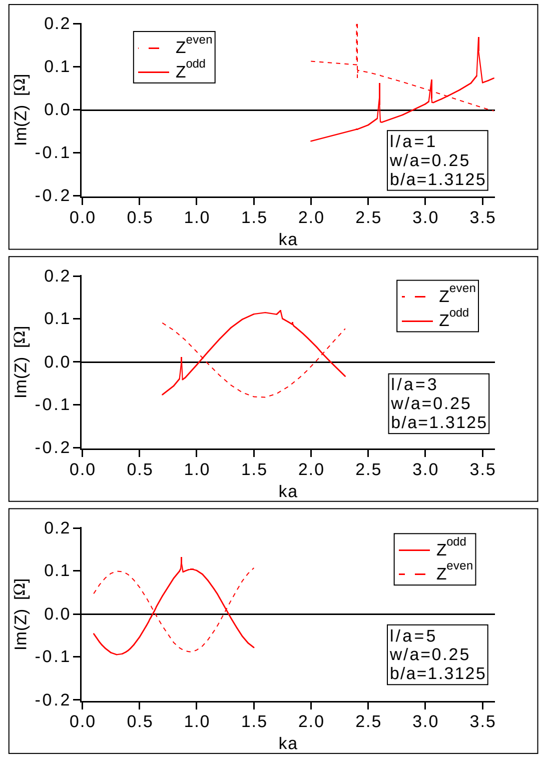

Figure 5: Resonant behavior of the imaginary part of the impedance for slots of different length.

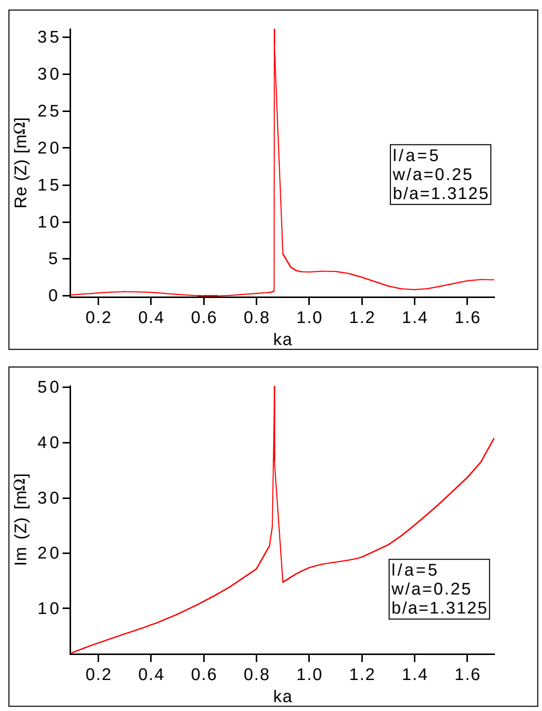

Figure 6: Real and imaginary parts of the impedance for a very long slot ($l/a = 5$).