

GENERATOR-INVERTOR-DAMPER SYSTEM OF ELECTRON (POSITRON) BUNCHES MOVING IN COLD PLASMA.

A.Ts. Amatuni, S.G. Arutunian, M.R. Mailian

Yerevan Physics Institute, Alikhanian Brothers' St. 2, Yerevan 375036, Republic of Armenia

Abstract

It is shown that high accelerating gradient can be obtained in a specially constructed system of electron (positron) bunches, moving in cold plasma with definite density. These combined bunch systems do not generate the wake fields behind and can pass through the plasma column in a periodic sequence. The consideration is carried out numerically and analytically in one dimensional approach, (which can be applied to finite system when its transverse dimensions are larger than plasma wave length). The possibilities of the experimental tests by measuring the predicted energy gain are discussed on the examples of Argonne Wakefield Accelerator and induction linac with typical parameters.

1 INTRODUCTION

In the work [1], which is based on the previous results [2] used as a zero order approximation, nonlinear dynamics of the one-dimensional non rigid ultrarelativistic bunch of electrons, moving in the cold plasma, is considered by multiple scales perturbative approach. Square root of the inverse Lorentz factor of the bunch electrons is taken as a small parameter.

In the first approximation of multiple scales method, the change of the momentum of the bunch electrons is given by $\Delta p_b = -e E t$, where t is a acceleration (deceleration) time interval which, due to applicability of the multiple scales method, is $t \leq \omega_p^{-1} \gamma^{1/2}$.

The essential restriction on the acceleration rate in the considered cases comes from the steady state assumption, adopted in [1]. If plasma electrons momenta in the wake field behind the first dense relativistic bunch, excited same critical value, allowed by steady state assumption, the plasma electron density behind the bunch turn out to be negative, which probably means that wake wave breaking limit is achieved. Restriction on the plasma electrons momenta inside the bunch automatically restricts the maximum value of the generated electric field.

In the present work the proper combination of the (e^\pm) bunches is proposed, which is constructed in such a way, that it does not generate the wake fields behind the combined bunch at all. The plasma electrons momenta, inside the combined bunch being negative, remains in the allowed domain and their maximum value is restricted only by the total momentum conservation law. The maximum of the subsequent electric field then also can be large enough.

The combined bunch in the considered case consists (at least) of three parts: first one is a relativistic negative particles bunch, with density $n_b^{(1)} \gg \frac{1}{2} n_0$, which generates inside the bunch strong electric field, which brakes the bunch

particles (generator). The second bunch is a low density bunch $n_b^{(2)} < \frac{1}{2} n_0$ (it can be negative or positive particles bunch), which inverts (invertor) the sign of the electric field generated by the first bunch. The third bunch (dumper) with the parameters, which can coincide with that of the first one, provided on its rear part the zero values of electric field and plasma electrons momenta, coincided with the same values on the front of the first bunch. It means that plasma behind the system of bunches remains unperturbed and no wake field is generated.

Acceleration of the particles of the rear part of the second and of the third bunches, where electric field is negative, will take place. The goal of the following calculations is to estimate the maximum possible values of the acceleration parameter.

2 PHASE PLANE IMAGES OF THE COMBINED BUNCHES.

Consider a plane rigid electron bunch with the infinite transverse dimensions and length $[0, d]$ along the z -axis. Velocity of the bunch in the lab system is $v_z = v_0$ ($\beta = \frac{v_0}{c}$), plasma is cold with the immobile ions. The charge density of the plasma electrons is n_b ; the case, when $n_b > \frac{1}{2} n_0$ called underdense regime, $n_b < \frac{1}{2} n_0$ -overdense regime.

Equation of plasma electrons motion can be written in steady state approximation as:

$$E = -\frac{d\Phi}{dz} \equiv -\Phi', \Phi \equiv \sqrt{1 + \rho_e^2} - \beta \rho_e \quad (1)$$

and continuity equation for plasma electrons and Coulomb law give the equation for Φ :

$$\Phi'' - \beta \gamma^2 \Phi (\Phi^2 - \gamma^{-2})^{-1/2} = \alpha - \gamma^2 \quad (2)$$

Eq. (2) has an "energy" integral

$$\mathcal{E} = \frac{1}{2} \Phi'^2 + \gamma^2 [\Phi - \beta (\Phi^2 - \gamma^{-2})^{1/2}] - \alpha \Phi, \quad (3)$$

where $\alpha = n_b/n_0$, and dimensionless arguments and functions are introduced:

$$t' = \omega_p t, z' = k_p z, \omega_p^2 = \frac{4\pi e^2 n_0}{m}, k_p = \frac{\omega_p}{c},$$

$$E = \frac{mc\omega_p}{e} E', n'_e = \frac{n_e}{n_0}, n_b \equiv \alpha = \frac{n_b}{n_0}, \rho_e = \frac{\rho_{ez}}{mc},$$

$$\beta = \frac{v_0}{c}, \beta_e = \frac{v_{ez}}{c}, \gamma = (1 - \beta^2)^{-1/2}, \tilde{z} = z' - \beta t'$$

(in what follows the prime is omitted)

The integral (3) allowed to interpret equations (1-3) as equations for the point with unit mass with the "coordinate" Φ and "velocity" Φ' moving in the potential $U = \gamma^2 \left[\Phi - \beta (\Phi^2 - \gamma^{-2})^{1/2} \right] - \alpha \Phi$. The negative values of $\alpha = \frac{n_b}{n_0}$ correspond to the bunch of positive charged particles, positive α corresponds to the bunch of negative charged particles. For $\alpha < 0, \gamma^{-1} \leq \Phi \leq 1$ and $\Phi \geq 1$ for $\alpha > 0$. Boundary conditions, when $\tilde{z} = d$, are $\Phi = 1$ and $\Phi' = 0$. For $\alpha < 0$ the motion is always periodic, it is also periodic for $0 \leq \alpha \leq 1/2$, and non periodic for $\alpha > 1/2$ [2].

Phase plane trajectories can serve to construct the phase portraits of the combined bunches, moving in cold plasma. In the case, when wake field is absent, it must be represented by a closed loop, starts (and accomplished) from the boundary condition $\Phi = 1, \Phi' = 0$ on the phase plane. An example of such a loop is presented on Fig. 1. The loop on Fig. 1 starts from the point 0 : $\Phi = 1, \Phi' = 0$ ($\rho_e = 0, v_e = 0, E = 0$, when $\tilde{z} = d$) and the curve OA represents the motion of the bunch of negative charged particles (generator) $\alpha^{(1)} = 10$, when at the rear side of the bunch very strong positive electric field $E_0 = -\Phi'$ can be generated. Taking this point as a new boundary conditions for the second bunch of positive charged particles (invertor), the trajectory is turned to the point B , where electric field is zero and then to the point A' where the field $-E_0$ is obtained. From point A' to the point 0 the trajectory described the motion of the third bunch of the negatively charged particles (damper) on the rear side of which the initial boundary conditions exist. It means that no wake field excites and the plasma behind the third bunch remains unperturbed. As a consequence, the above mentioned restriction on the value of the field E_0 , which arouse from the condition of the stability of the wake field, is removed. This is not only the

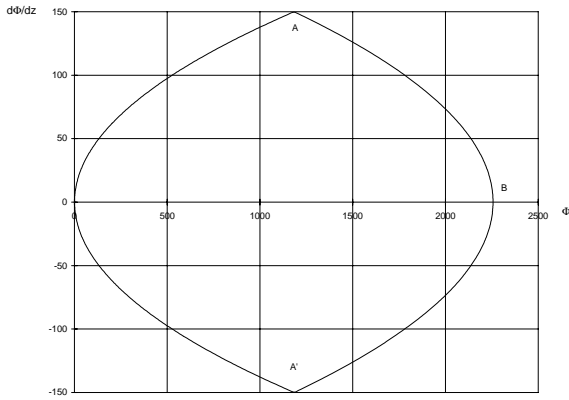


Figure 1:
condition for the applicability of steady state regime for the

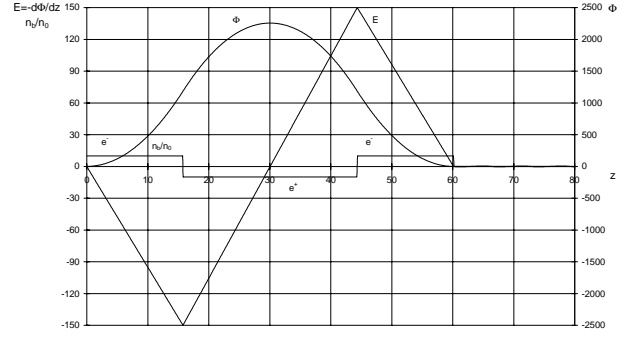


Figure 2:

description of the zero order approximation in the multiple scales approach, but also open the possibility to repeat more than once the procedure, sending the next combined bunches into the unperturbed by previous bunches plasma.

The Lorentz factor of all three bunches in the presented case is, of course, the same $\gamma = 100$, and

$$\alpha^{(2)} = -\frac{n_b^{(2)}}{n_0} = -10, \alpha^{(3)} = \alpha^{(1)} = \frac{n_b^{(3)}}{n_0} = 10.$$

The total length of the positron bunch is $28.57 \frac{\lambda_p}{2\pi}$ and lengths of the first and third electron bunches are $\sim 15.77 \frac{\lambda_p}{2\pi}$ each.

On Fig. 2 the potential Φ , electric field $E = -\frac{\partial \Phi}{\partial z}$ and ratio $\frac{n}{n_b}$ as functions of \tilde{z} are presented. It is evident that positrons of the first part of the second bunch, where $E > 0$, can be accelerated as well as all electrons of the third bunch ($E < 0$). The similar picture may be obtained by construction of the combination of three bunches with negatively charged particles (electrons). The role of the bunch-invertor may play also the plasma itself ($n_b^{(2)} = 0$). The charge distribution inside the bunches can be nonuniform.

3 ANALYTICAL APPROACH. PROPOSED EXPERIMENTAL TEST

In this section the results of the exact analytical solutions of eq. (2) for the combined bunch with the uniform charge distributions inside the constituting bunches are presented.

The possible largest plasma electron momenta inside the first bunch in the considered case differs from that found in by wake wave breaking limit [2] and can be estimated from momentum conservation (see also [1]),

$$n_b^{(1)} \beta \gamma d^{(1)} \geq \int_0^{d_1} \rho_e(\tilde{z}) n_e(\tilde{z}) dz \approx \frac{n_0 |\rho_0| d_1}{4} \quad (4)$$

For the estimate in (4) the expression for n_e , approximated by linear one in interval from $n_e = n_0$ at $p_e = 0$ up to the value $n_e = \frac{n_0\beta}{1+\beta}$ for large ρ_e [2].

From (4), $1 \ll |\rho_0| \leq 4n_b^{(1)}\gamma$, and corresponding electric field from is $|E_0| = 2 \left(n_b^{(1)} \right)^{1/2} |\rho_0|^{1/2} \leq 4n_b^{(1)}\gamma^{1/2}$

As it was shown in [1] the change of the bunch electron momenta Δp , in the first approximation in ordinary units is

$$\Delta p_b = eE_0 t \leq 4 \frac{n_b^{(1)}}{n_0} mc\gamma = \frac{4n_b^{(1)}}{n_0} p_b; \quad t \leq \omega_p^{-1} \gamma^{1/2} \quad (5)$$

. The accelerating gradient is

$$G = \frac{c\Delta p_b}{el} \approx 4\sqrt{n_0}\gamma \left(\frac{n_b^{(1)}}{n_0} \right) \frac{V}{cm} \quad (6)$$

The lengths of the first and third electron bunches are:

$$d_1 = d_3 \leq \frac{\lambda_p}{\pi} \gamma^{1/2} \left(\frac{n_b^{(1)}}{n_0} \right)^{1/2}, \quad \lambda_p = \frac{2\pi c}{\omega_p};$$

The length of the second electron bunch is

$$d_2^e \leq \frac{8\lambda_p}{\pi} \gamma^{1/2} \left(\frac{n_b^{(1)}}{n_0} \right), \quad n_b^{(2)} \ll n_0 \ll n_b^{(1)}$$

The length of the second positron bunch is:

$$d_2^p \leq \frac{4\lambda_p}{\pi} \gamma^{1/2}, \quad \bar{n}_b^{(2)} = n_b^{(1)} \gg n_0;$$

The lengths of bunches, increases as $\sim \gamma^{1/2}$, so it seems that the bunches must be from high current accelerators with the energies up to tens of Mev. Even at these energies maximal acceleration gradient is high enough, it can exceed the acceleration gradient of the ordinary linacs by a several orders of magnitude.

Consider the experimental possibilities of proof-of-principle experiment on existing accelerators. It seems that Argonne Wakefield Accelerators (AWA) [3], which accelerates very dense bunches, is suitable for above mentioned propose.

For the existing experimental parameters of AWA: $d_1 = 0,5cm$, $d_2 = 23cm$, total charge of bunch $70nC$, $n_b^{(1)} = 1.4 \cdot 10^{14}cm^{-3}$ results of the numerical calculations, are shown on Fig. 3. The role of bunch-invertor plays the plasma column, with density n_0 as a free parameter, which was been chosen from the necessity to invert the field, produced by the first bunch, in such a way, that the successive bunch, with the same parameters as the first one, moving at fixed (by RF-frequency) distance from the first one, $d_2 = 23cm$, will be able to dump the field to zero. The sought density of plasma n_0 was found equal to $n_0 \approx 1.17 \cdot 10^{13}cm^{-3}$, $\lambda_p = 0.973cm$, $n_b/n_0 = 11.95$. Then the maximum value of the accelerated electric field is equal to $E_{max} \approx 100MV/cm$, and $E < 0$ on the rear part of the

second (third) bunch. So about the half of the electrons of the second (third) bunch, as it can be seen from Fig. 3, can be accelerated during the time $t \sim \omega_p^{-1} \gamma^{1/2}$ up to energy $\mathcal{E}_f \approx 70MeV$. Plasma remains practically unperturbed

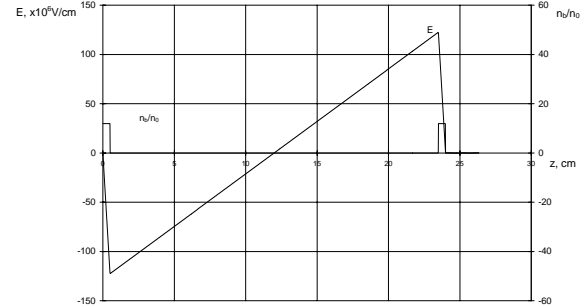


Figure 3:

after the passage of the first and the second bunches, so it is possible to inject the next couple of the bunches and also obtain acceleration, again leaving the plasma behind the bunches in practically unperturbed state and so on. In the considered case of AWA the diameter of the bunch is $D = 0.1cm \leq \frac{c}{\omega_p} = \frac{\lambda_p}{2\pi} = 0.15cm$, so for the existing parameters of AWA predictions of 1-D nonlinear theory can be valid [4]

Another possibility to check the predicted effect is based on the use of induction linacs with the typical parameters.

4 ACKNOWLEDGMENTS

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