USING LUMINOSITY DISTRIBUTIONS TO DETERMINE LUMINOSITY IN A COLLIDER WITH A COUPLED LATTICE

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Abstract

The traditional method for calculating luminosity in the Fermilab Tevatron during collider operations assumes a knowledge of the machine lattice at the locations of the flying wires and the luminous regions. In this paper we investigate an alternative method of determining the luminosity which does not require a knowledge of the lattice functions and can be applied even in the case of a transversely coupled lattice. By measuring a set of longitudinal luminosity distributions each with a different intentionally added separation between the proton and antiproton closed orbits the transverse widths of the luminous region can be determined and used to calculate the luminosity. As an example we apply this method to a set of Monte Carlo generated luminosity distributions using a Tevatron lattice that is transversely coupled due to a roll in one of the low beta quadrupole magnets.

1 INTRODUCTION

During Collider Run Ib operations in the Fermilab Tevatron there was disagreement by about 20% between the luminosity calculated from measured beam emittances and the luminosity as measured by CDF and D0. Traditionally the luminosity is calculated by measuring the proton and antiproton beam parameters such as emittance and bunch length and using these parameters, along with a set of lattice functions, to perform the overlap integral for the luminosity [1]. With this method errors in the lattice functions translate into errors in the calculated luminosity. Measurements of the lattice functions have been attempted in the Tevatron by making one-bump orbit distortions but these measurements give errors of at least 10% and furthermore has not led to a quantitative understanding of the coupling present in the Tevatron.

As an attempt to better understand the lattice in the interaction regions we investigate a method for measuring the transverse widths of the luminous region and use these results to calculate the luminosity. The method takes advantage of the fact that the longitudinal distribution of luminosity in the interaction region changes if the proton and antiproton orbits are separated using the electrostatic separators. As an example the dashed line in Figure 1 shows the expected luminosity distribution as a function of longitudinal position when the proton and antiproton beam closed orbits are separated horizontally by 180μm in the interaction region by using the electrostatic separators. The luminosity distributions generated in this example used a Tevatron lattice which was coupled due to a rolled low beta quadrupole.

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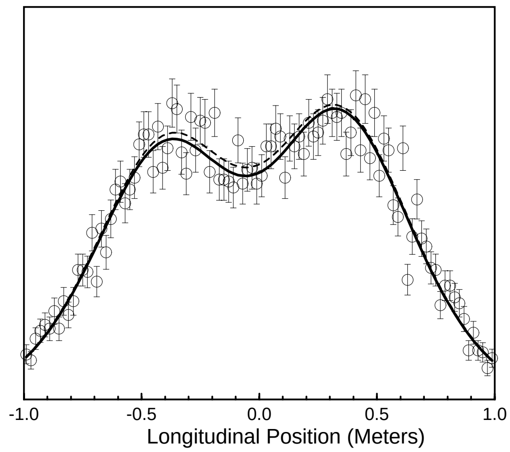

Figure 1: Luminosity distribution with added orbit separation of $\Delta X = +180\mathrm{um}$. Circles with error bars are a Monte Carlo generated distribution. The solid line is the result of a fit to the data. The dashed line is the distribution used to generate the Monte Carlo data.

Two features are immediately evident in Figure 1. First, the luminosity distributions dips near the center of the interaction region due to the hourglass shape of the beams. The magnitude of the dip gives an indication of the transverse proton and antiproton beam widths. The second notable feature is that the luminosity distribution is not symmetric as would be expected from the ideal design lattice. In this example the asymmetry is caused by the coupling introduced by the rolled low beta quadrupole.

The method we investigate in this paper uses these features of the luminosity distributions to determine the transverse beam widths of the luminous region and to calculate the luminosity. One of the important features of this method is the inclusion of transverse coupling.

The next section of the paper introduces the luminosity formula used to handle transverse coupling. The section following this discusses the method of calculating the luminosity from a set of luminosity distributions and applies the method to a simulated set of distributions. The final section makes some conclusions and lists some additional sources of error which have not been considered in this analysis.

2 LUMINOSITY FORMULA

Without giving the details of the derivation (see [2]), the expression for the luminosity generated by a single proton

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and antiproton bunch colliding can be reduced to

$$\mathcal{L} = \frac{N_p N_a f_{\rm rev}}{\sqrt{2\pi}^3 \sigma_z} \int \frac{1}{|\det \underline{\underline{C}}|^{\frac{1}{2}}} \exp(-\frac{1}{2}\underline{\chi_0}^T \cdot \underline{\underline{C}}^{-1} \cdot \underline{\chi_0} - \frac{1}{2}(s-z_o)^2/\sigma_z^2)\, ds. \quad (1)$$

where $\mathcal{L}$ is the total luminosity, N_p (N_a) is the number of protons (antiprotons), $f_{\rm rev}$ is the revolution frequency, and s is the longitudinal position. The integral extends over the entire luminous region and $s = 0$ is located at the center of the detector. The longitudinal bunch widths of the protons and antiprotons are combined into the single parameter σ_z^2 which is equal to the proton and antiproton rms bunch lengths added in quadrature and divided by 2. It should be noted that this expression was derived by assuming the transverse and longitudinal phase space distributions of the proton and antiproton bunches are ideal Gaussians.

As written, Equation 1 has no mention of lattice functions, Edwards and Teng parameters [3], or beam emittances. Instead we use the 4×4 covariance matrix $\underline{\underline{C}}(s)$ which is a measure of the transverse width of the luminous region as a function of longitudinal position s. More explicitly the covariance matrix is

$$\underline{\underline{C}}(s) = \begin{pmatrix} \sigma_x^2 & \sigma_{xy}^2 \\ \sigma_{xy}^2 & \sigma_y^2 \end{pmatrix} \quad (2)$$

where the σ are the rms widths of the luminous region and each of the σ^2 is a quadratic function of the longitudinal position s. In the ideal case with no coupling σ_{xy} would be zero and σ_x (σ_y) is determined from the convolution of the proton and antiproton horizontal (vertical) beam widths. Because $\underline{\underline{C}}(s)$ is a symmetric matrix there are a total of 9 parameters needed to completely specify $\underline{\underline{C}}(s)$.

An important part of Equation 1 is the inclusion of the possibility of proton and antiproton closed orbits which are separated. If there is a difference in the closed orbits the bunches will not collide head-on but instead will pass by each other with some separation and crossing angle. The total separations between the proton and antiproton orbits is given by $\underline{\chi_0}^T = (\Delta x(s), \Delta y(s))$. The final parameter in Equation 1 is the cogging offset, z_0 which is the longitudinal distance between the center of the detector and the position at which the centers of the proton and antiproton bunches collide as they pass by each other.

As expressed in Equation 1 there are a total of 18 parameters ($\underline{\underline{C}}(s)$, $\underline{\chi_0}^T(s)$, σ_z, z_0, N_p, N_a, and $f_{\rm rev}$) needed to calculate the luminosity. In the next section we will show how 15 of these parameters (all except N_p, N_a, and $f_{\rm rev}$) can be determined by making measurements of the luminosity distributions with different orbit separations intentionally introduced.

3 LUMINOSITY DETERMINATION FROM LUMINOSITY DISTRIBUTIONS

The integrand of Equation 1 is the longitudinal luminosity distribution. This can be measured using the experimental detectors in the luminous regions to make event by event vertex reconstructions of the longitudinal position. Intentionally changing the separation of the proton and antiproton closed orbits will change the luminosity distributions as demonstrated in Figure 1. By collecting a set of distributions with a variety of closed orbit offsets it is possible to fit the distributions for most of the parameters in Equation 1.

To determine the sensitivity of the method just described a set of nine simulated luminosity distributions was generated from a given set of lattice parameters and beam emittances. For this particular example the lattice used was calculated by assuming that a low beta quadrupole near the interaction region was rolled by 10 mrad and the resulting global coupling was corrected using the skew quadrupole correction circuits in the Tevatron. (This is similar to a situation that existed in the Tevatron early in Collider Run Ib.) The emittances used for the Monte Carlo generation are typical of Tevatron beam in collider operations. The horizontal and vertical proton (antiproton) emittances were 25π (15π) mm-mrad (95%, normalized) and the rms bunch lengths were 65 cm.

Distribution Number	Δx μm	Δy μm	Δz cm
1	0	0	0
2	$+180$	0	0
3	-180	0	0
4	0	$+140$	0
5	0	-140	0
6	$+140$	$+140$	0
7	-140	-140	0
8	0	0	$+40$
9	0	0	-40

Table 1: Orbit separations and cogging offsets introduced for the simulated luminosity distribution measurements. The values are the added separation between the proton and pbar orbit. A positive value indicates that the proton orbit was moved vertically up (or radially out) with respect to the centered orbit.

The distributions were generated with 10000 events divided into bins 2 cm wide. The separations introduced for the 9 distributions are shown in Table 1. These values were chosen since they are close to the maximum separations currently achievable with the electrostatic separators in the Tevatron. The distributions were simulated to include Poisson statistical deviations from the nominal value in each bin. Typically there were about 100 counts per bin in the peaks of the distributions. One of the distributions is shown in Figure 1 as circles with the error bars being the Poisson statistical errors.

These nine Monte Carlo distributions were then fitted simultaneously for 24 parameters, $\underline{\underline{C}}(s)$ (where $\sigma_x^2(s) = \sigma_{x0}^2 + \sigma_{x1}^2 s + \sigma_{x2}^2 s^2$, etc), $\underline{\chi_0}$, z_0, σ_z^2, and 9 scale factors which scale the number of events in the luminosity distributions to the the actual luminosity. (In principle there needs to be only one overall scale factor but in practice it is simpler to have one scale factor for each distribution. The ratios of the fit scale factors values can then be compared to the ratios of the measured scale factors and used as a consistency check.) Using the fit values of the 24 parameters the luminosity distributions and luminosity can be calculated using Equation 1. In Figure 1 the dashed line is the calculated distribution using the fit parameters.

Fit Parameter	Mean fit value	Std dev. fit value	Actual value
z_0 (cm)	0.07018	0.290	0.0
$\sigma_z^2 (\mathrm{cm}^2)$	2125.0	35.2	2112.5
horz sep (μm)	0.07834	1.29	0.0
horz ang (μrad)	0.3959	1.59	0.0
vert sep (μm)	.004826	1.52	0.0
vert ang (μrad)	-0.3417	1.65	0.0
σ_{x0}^2	7231	253	7129
σ_{x1}^2	2445	377	2389
σ_{x2}^2	26860	2170	25920
σ_{xy0}^2	-2556	231	-2499
σ_{xy1}^2	-1156	365	-1173
σ_{xy2}^2	-7290	1340	-7386
σ_{y0}^2	7000	320	6930
σ_{y1}^2	2468	507	2491
σ_{y2}^2	21340	1960	21300
Scale factor 1	0.007511	0.000256	0.0075
Scale factor 2	0.04768	0.00256	0.0487
Scale factor 3	0.04758	0.00267	0.0487
Scale factor 4	0.02613	0.00109	0.0261
Scale factor 5	0.0261	0.00125	0.0261
Scale factor 6	0.1405	0.00829	0.1444
Scale factor 7	0.1402	0.00861	0.1444
Scale factor 8	0.00928	0.000317	0.0092
Scale factor 9	0.008631	0.000277	0.0086
luminosity	14.5576	0.453	14.7462

Table 2: Average and standard deviation of fit values for 100 simulated data sets compared to actual values for the distributions shown.

To determine statistically accuracy of this method 100 independently generated sets of Monte Carlo distributions were fitted and the mean and standard deviation of the fit values calculated. The results are listed in Table 2 and show that the luminosity can be determined with a 95% confidence level of $\pm 6\%$. The fit value of the luminosity was also systematically 1.3% lower than the actual value although it is not understood why. (Notice also that σ_{xy0}^2 is not zero due to the coupling of the lattice.)

4 CONCLUSIONS

In practice there are a number of difficulties which would add error to these measurements. Among these are the possibilities of detector errors, a smearing of the distributions from finite resolution of the z-vertex, and multiple interactions per crossing which may distort the luminosity distribution as well.

There were also several assumptions in this analysis which if not true will add error. First it was assumed that the intensities of the proton and antiproton bunches were known exactly. The uncertainties in these measurements will add at least several percent error to the calculated luminosity. Second, the formula for the luminosity in Equation 1 also assumes the proton and antiproton bunches are ideal Gaussians. Finally, it was assumed in this analysis that the actual separation introduced in the closed orbits was known exactly. In practice the amount of separation added between the closed orbits may not be known accurately.

Another complication not considered in this paper is the changing beam conditions while the luminosity distributions are being collected. Collecting 10000 events per luminosity distribution for several distributions can take on the order of an hour. During this time the proton and antiproton bunch lengths and transverse emittances are increasing leading to changing beam conditions and luminosity.

It may be possible to reduce the errors on this analysis but this has not yet been investigated. For instance measurements of the bunch lengths to within several percent could be used to further constrain the fits. Finally the choice of orbit separations used in this Monte Carlo (those listed on Table 1) were chosen as an example and were not chosen to optimize the error on the luminosity.

Despite limitations on the accuracy of the luminosity determined by this method, it still may be possible to gain information about the coupling of the lattice from the measurement of the covariance matrix $\underline{\underline{C}}(s)$.

5 ACKNOWLEDGMENTS

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