EQUIPARTITIONING AND HALO DUE TO ANISOTROPY

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Abstract

Equipartitioning and certain aspects of halo formation in high-current linac beams are explained in terms of collective multipole oscillations in x-y geometry. For strong space charge tune depression and anisotropy (emittance and/or focusing strength) some eigenmodes can - in principle - become unstable leading to emittance exchange. It is shown that for parameters of practical interest in linac design beams can be un-equipartitioned without risk of instability. The effect of (stable) mismatch core oscillations on the halo is briefly discussed.

1 INTRODUCTION

Anisotropy in conjunction with space charge effects has its most important potential application in high-current linear accelerators for protons or ions (spallation neutron sources, radioactive waste transmutation linacs, heavy ion fusion linacs etc.). In such linac bunches one of the crucial beam dynamics issues is to what extent deviations from "equipartitioning" can be tolerated without risk of emittance growth (for more recent discussions see Refs. [1, 2].

Coupling resonances leading to amplitude exchange are a familiar subject in circular accelerators, where they are driven by deviations from ideal focusing. It will be shown here that beam self-fields in the space-charge-dominated regime can play a similar role in an ideal linear lattice: in the presence of internal energy anisotropy between different degrees of freedom initially small space charge coupling terms can grow exponentially due to collective instability. Although our theory is derived for cylindrical x-y geometry we assume that the basic arguments also hold for all three degrees of freedom in a bunched beam.

Our analysis contains as a special case the KV-"breathing" ("fourth-order") mode of round isotropic beams in constant focusing, which has recently been suggested as a driving mechanism for halo [3]. This isotropic "breathing" mode is, however, known to vanish if the KV δ-function distribution is slightly broadened [4]. We assume that anisotropy as a driving mechanism is much more robust with respect to the detailed form of the distribution function. While results for the isotropic case can be expressed in terms of one dimensionless parameter, ν/ν_0, anisotropy requires two further dimensionless parameters.

2 ANALYTICAL MODEL

Basic assumptions of the model are summarized in the following, whereas details of the analytical theory are presented elsewhere [5] (see also Ref. [6] for an earlier reference to certain aspects of this work). The unperturbed equilibrium beam is assumed to have uniform density within an elliptic cross section defined by $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 \leq 1$, with a, b the semi-axis of the boundary ellipse. Assuming linear and time-independent external focusing forces for the equilibrium beam ("smooth approximation") we can write separate Hamiltonians for the x- and y- motion:

$$\begin{aligned} H_{0x} &= (p_x^2 + m^2\gamma^2\nu_x^2 x^2)/(2m\gamma) \\ H_{0y} &= (p_y^2 + m^2\gamma^2\nu_y^2 y^2)/(2m\gamma) \end{aligned} \tag{1}$$

and define a generalized anisotropic Kapchinskij-Vladimirskij distribution as δ-function of a linear combination of the two separate Hamiltonians:

$$f_0(x, y, p_x, p_y) = \frac{NT\nu_y/\nu_x}{2\pi^2 m\gamma a^2}\,\delta\left(H_{0x} + TH_{0y} - m\gamma\nu_x^2\frac{a^2}{2}\right) \tag{2}$$

Here T is the ratio of oscillation energies in the x and y directions which can be readily written for harmonic oscillators as $T = (a^2\nu_x^2)/(b^2\nu_y^2)$. The ratio of emittances is given by $\epsilon_x/\epsilon_y = (a^2\nu_x)/(b^2\nu_y)$. The time-independent f_0 in Eq. 2 is a solution of Vlasov's equation since H_{0x}, H_{0y} are constants of the motion. For the perturbed distribution function $f \equiv f_0(H_{0x}, H_{0y}) + f_1(x, y, p_x, p_y)e^{-i\omega t}$ we linearize Vlasov's equation keeping only first order terms in f_1 and in the perturbed electrostatic potential Φ, which is expanded as polynomial in x, y in the interior of the beam. The order l of this polynomial is related to the spatial profile of the density perturbation as is shown in Fig. 1. It is

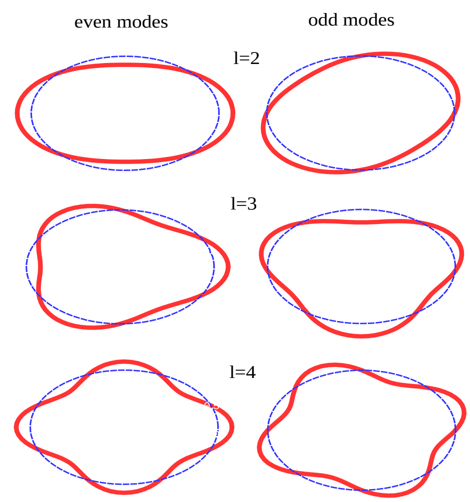

Figure 1: Beam cross sections for second , third and fourth order even and odd modes (schematic).

noted that the even modes are symmetric with respect to the horizontal (here $x-$) axis. The odd modes lack this symmetry; in 3-d these modes correspond to a lack of rotational symmetry around the longitudinal axis, hence they are suppressed in $r-z$ simulation codes.

The assumption of vanishing perturbed potential at infinity leads to a dispersion relation for the coherent frequency ω in the form of an algebraic expression depending on the three variables to describe the equilibrium beam. For this purpose we use ν_y/ν_{y0}, $\alpha \equiv \nu_y/\nu_x$ and $\eta \equiv a/b$ (≥ 1) and characterize the eigenfrequency by the dimensionless coherent frequency ω/ν_{y0}. The energy anisotropy is then given by η^2/α^2 and the ratio of emittances by η^2/α.

3 EIGENFREQUENCIES

Starting with second order modes the simplest modes are the well-known envelope oscillations. In addition, our analysis yields odd ("tilting") modes (see also Ref. [7] where a matrix formalism is used for the second order modes) which lead to a linear coupling between x and y and can become unstable for sufficiently large anisotropy. The coupling is caused by the space charge force corresponding to that of skew quadrupoles. The number of eigenfrequencies increases considerably with order l due to the anisotropy. In Fig. 2 this is shown for the $l=3$ odd mode and $\nu_x/\nu_y = 0.8$, $a/b = 1.94$ ($\epsilon_x/\epsilon_y = 3$ and $T = 2.4$). It indicates transition to an unstable solution ($Im\omega > 0$ with $Re\omega = 0$) for $\nu_y/\nu_{y0} < 0.39$ with a maximum growth rate of about 10% of the betatron frequency; note that there exists simultaneously a damped solution with $Im\omega < 0$ not shown here. The isotropic case is completely stable. We note that for the same parameters $l = 4$ yields already 16 different frequencies. For different values of α, η the thresholds for onset of instability may vary considerably.

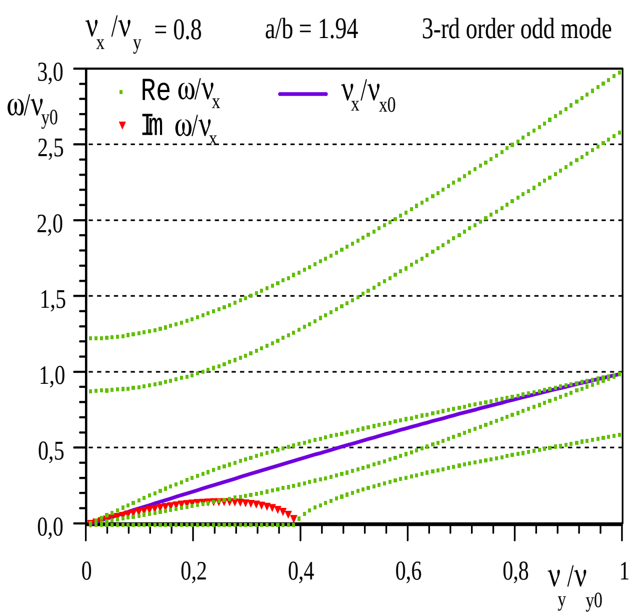

Figure 2: Example of frequencies for third order odd mode with $T = 2.4$ anisotropy.

4 LINAC DESIGN STABILITY CHARTS

For the design of high-current linacs it is desirable to identify regions in parameter space where growth rates leading to emittance exchange might occur. For this purpose we have created charts (see Fig. 3) which show the tune

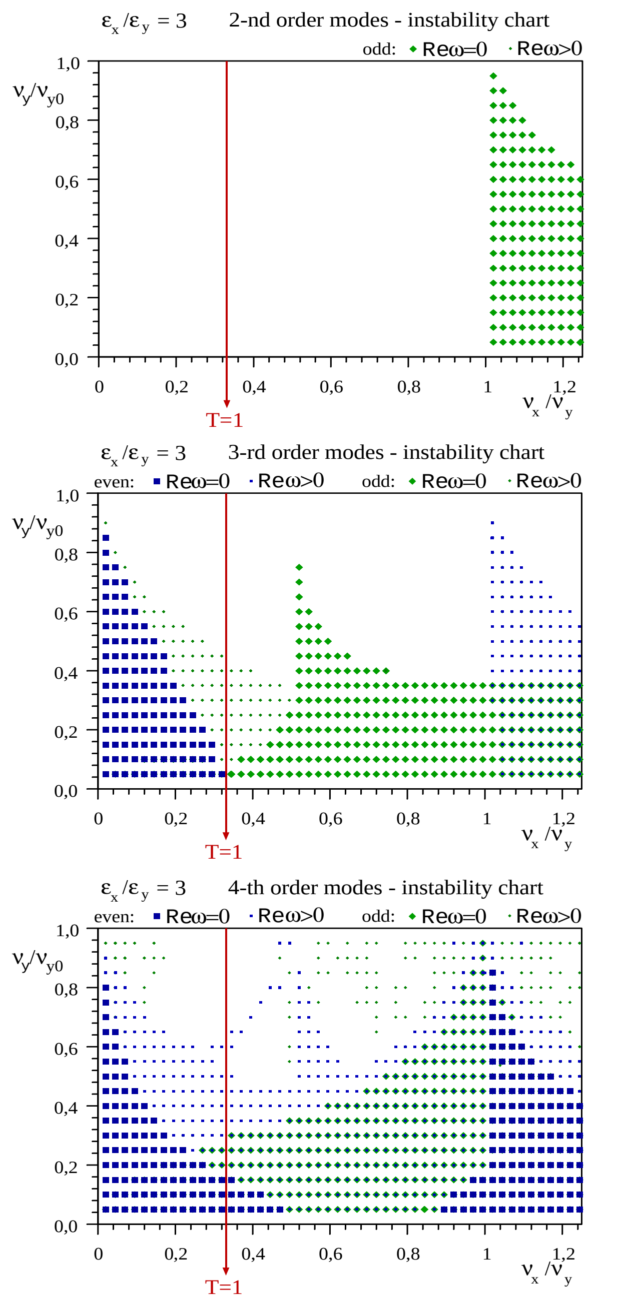

Figure 3: Stability charts for second, third and fourth order modes assuming $\epsilon_x/\epsilon_y = 3$.

depression ν_y/ν_{y0} versus tune ratio for a given ratio of emittances, and corresponding marks whenever an eigenfrequency indicates instability. Hence, at the boundaries of the marked regions growth rates vanish. The temperature anisotropy T is given by the product of tune ratio and emittance ratio and can be larger or smaller than unity. In

Fig. 3 we have assumed $\epsilon_x/\epsilon_y = 3$. We note that for different values of the emittance ratio ($>> 1$) the charts are qualitatively similar. ν_x/ν_{x0} is determined by these three parameters: one finds that for $T < 1$ it is the more strongly depressed one of the two tunes (assuming $\epsilon_x/\epsilon_y >> 1$), and the less depressed for $T >> 1$. Seriously large growth rates are found only for the non-oscillatory instabilities with $Re\omega = 0$; for completeness we also show in Fig. 3 the oscillatory instabilities with $Re\omega > 0$ (small marks)). We find that for $T = 1$ none of the modes are of concern; for $T > 1$ first the odd modes grow unstable, whereas for $T < 1$ only the even mode seems of concerns.

Linac Design: We suggest that the charts presented above give a useful orientation not only for the x-y coupling case but also for the longitudinal-transverse coupling (z-y or z-x), which is of real interest in linac bunches. If $\epsilon_l/\epsilon_t > 1$ we identify l with x and t with y in Fig. 3. We find that there is sufficient space free of instabilities right and left of the equipartitioning line $T = 1$. For $T = 1/3$ (3 times higher transverse oscillation energy), for instance, the transverse tune depression must be below 0.6 to enter into the unstable region of the third order even mode (and even lower for the fourth order even mode). We find that the odd mode instabilities come into play only for T sufficiently larger than unity. Hence we conclude that linac beams can be moderately "un-equipartitioned" without risk of emittance transfer, even for relatively strong tune depression.

In computer simulation of infinitely long coasting beams it was recently observed that a transverse to longitudinal temperature equilibration occurs, presumably driven by a similar mechanism [8].

5 COUPLING EFFECT ON HALO

While the above theory describes collective behaviour driven by the core of the beam we also expect that excitation of some of these eigenmodes causes a coupling in the halo. It is thus appropriate to extend the core/test-particle halo studies developped originally for round, isotropic beams [9] to anisotropic situations.

As a first step in this direction we have examined a particular case by exciting the second order odd mode at the level of 20% mismatch for different parameters ($a/b = 1.414$ fixed), where this mode is stable. We have traced 2 halo test particles with initial $x = 0.9a$ (crosses) and $x = 1.9a$ (triangles) assuming $a = 1.414$, and set initially p_x, y, p_y equal to zero. We have integrated their motion by a symplectic integrator (leap-frog) in the presence of the space charge field of the periodically oscillating core over 30 betatron periods. Fig. 4 shows the full time history of these 2 particles. While for $\nu_y/\nu_{y0} = 0.99$ we find practically no coupling into the $y-$plane, a significant effect occurs for stronger tune depression ($\nu_y/\nu_{y0} = 0.5$) due to the coupling space charge force, which compares with a skew quadrupole force. For the small amplitude particle the weakly anisotropic beam case (top, with $T = 2$) shows a stronger excursion in the x-direction, whereas the strongly anisotropic case (bottom, with $T = 8$) shows an enhanced y-amplitude. The coupling does, however, not lead to a full exchange of "temperatures". Hence, this example demonstrates that anisotropy in the halo is only partially removed by the effect of the space charge force. Obviously a more extensive exploration of the three-dimensional parameter space is required to establish decisively to what extent mismatch oscillations lead to equipartitioning in the halo.

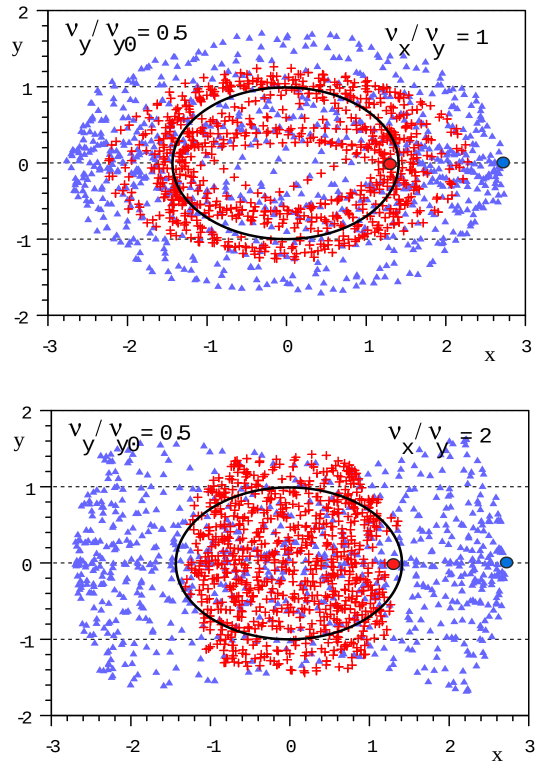

Figure 4: Halo development driven by second order odd (tilting) mode for different anisotropy ($\nu_x/\nu_y = 1, 2$) and tune depression.

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