DESIGN AND VERIFICATION OF CONTROLLERS FOR LONGITUDINAL OSCILLATIONS USING OPTIMAL CONTROL THEORY AND NUMERICAL SIMULATION: PREDICTIONS FOR PEP-II

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Abstract

We present a technique for the design and verification of efficient bunch-by-bunch controllers for damping longitudinal multibunch instabilities. The controllers attempt to optimize the use of available feedback amplifier power - one of the most expensive components of a feedback system - and define the limits of closed loop system performance. Our design technique alternates between analytic computation of single bunch optimal controllers and verification on a multibunch numerical simulator. The simulator identifies unstable coupled bunch modes and predicts their growth and damping rates. The results from the simulator are shown to be in reasonable agreement with analytical calculations based on the single bunch model. The technique is then used to evaluate the performance of a variety of controllers proposed for PEP-II.

1 INTRODUCTION

The programmable digital signal processor based longitudinal feedback system described in [1] is currently being installed at PEP-II. In this paper, we present results from a study of the rms noise performance of the feedback system and closed loop damping rates, for PEP-II. The results were obtained using optimal control theory [2],[3] together with multibunch numerical simulation [4].

2 DESIGN APPROACH

For feedback design, the overall beam-feedback system can be abstracted into the block diagram in fig.1. The objective of the feedback is to damp the unstable modes of the beam, minimize steady state phase oscillations z in the presence of process noise w, and reject dc to allow each bunch to ride on its own synchronous phase. This must be done using the minimum amplifier power u and without amplifying the sensor noise v. The closed loop system, which we call G, has 2-inputs (w & v) and 2-outputs (z & u). When designing a controller H for Go, all four components of G (Gzw, Gzv, Guw, and Guv) must be taken into account.

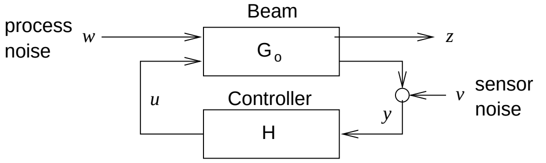

Figure 1: System block diagram.

2.1 Single Bunch Beam Model

Due to the very large number of bunches in the beam, it is computationally infeasible to directly design a controller by treating the whole beam as a single system. Therefore a decentralized or ”bunch-by-bunch” approach is used instead, where the feedback for each bunch is computed using measurements of that bunch only. So, the controllers are designed for the following open loop model Go: a single bunch beam, which is essentially a discrete-time simple harmonic oscillator with only radiation damping and a resonance at the synchrotron frequency. Fig.2 is the frequency response of Go for the PEP-II low energy ring (LER), on which we focus our study. Estimates for the noise statistics were obtained by scaling the values measured at the Advanced Light Source at LBNL.

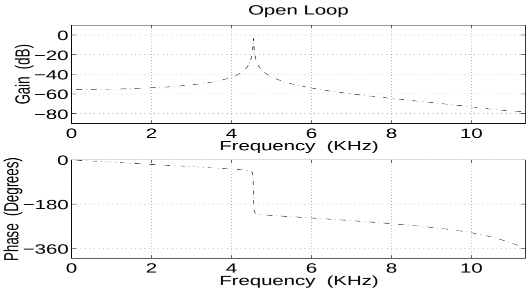

Figure 2: Single bunch beam model.

2.2 Trade Off Analysis

LQG controllers [2],[3] minimize rms beam motion z for given rms feedback amplifier power u. They are optimal with respect to this rms criterion. As the available rms u is varied, one obtains a curve which defines the limits of performance of the system, see fig.3. A similar curve can be obtained for the FIR controllers [2] proposed for PEP-II by simply varying the loop gain. One can then inspect these curves, pick a desirable operating point, and design the corresponding controllers.

We expect the rms performance of the multibunch beam to be close to the rms performance of a single bunch with the average damping rate of all the modes. To a good approximation, this may be taken to be the radiation damping rate. The controllers were therefore designed for the radiation damped beam. To check that these controllers also stabilize the unstable modes, tradeoff curves were plotted for the most unstable beam mode as well, producing the

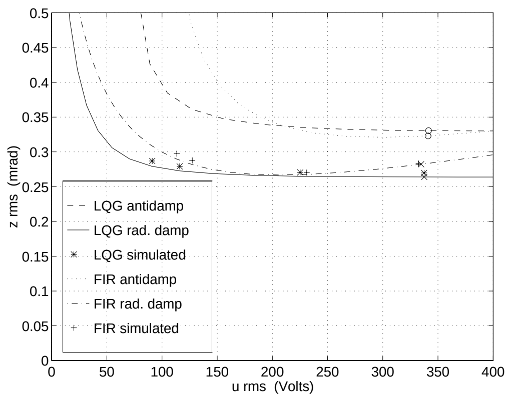

Figure 3: LQG & FIR Tradeoff curves.

remaining curves in fig.3.

3 RESULTS

The tradeoff curves show that the performance of the FIR controllers is quite close to optimal for a large range of operating conditions.

In this section we compare two controllers, one FIR and one LQG, which were designed to provide: a quiet beam in steady state, fast damping of unstable modes, some robustness to parameter variations and effective feedback without saturating the power amplifier on sensor noise. The rms performance of these two controllers is marked on the tradeoff curve with ‘x’s and ‘o’s.

3.1 Controllers

Fig.4 shows the frequency responses of two controllers tha we used in H for the single bunch LER model Go. FIR controllers (dashed) act as approximate differentiators; LQG controllers (solid) minimize rms beam motion using minimum rms amplifier power and are designed using optimal control theory. Both controllers provide dc rejection, which is a strict requirement.

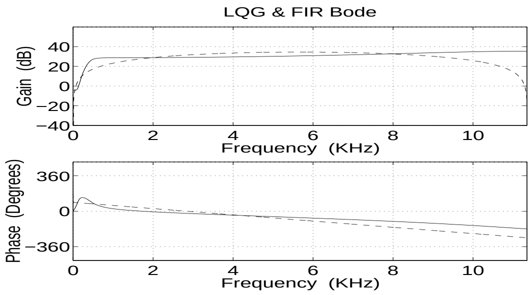

Figure 4: Controller frequency responses.

3.2 Closed Loop Performance

Fig. 5 shows the four closed loop frequency responses, Gzw, Gzv, Guw, and Guv obtained with the FIR controllers (dashed) and LQG controllers (solid). The (scaled) open loop response Go is plotted in dash-dot with Gzw. We note that the peak of Gzw is higher for the LQG than the FIR, showing that it provides lower damping than the FIR. The peaks of the other three responses are quite comparable. The LQG responses are narrower for Gzv and Guw, but broader for Guv. The following observations can be made for both controllers: There is a sharp notch in Guv, showing that the closed loop system rejects sensor noise at the synchrotron frequency. The fact that Guv is considerably larger than Guw over most of the frequency range, shows that the contribution of sensor noise v to amplifier voltage u is larger than that of process noise w. Thus at PEP-II the feeback system is sensor noise dominated.

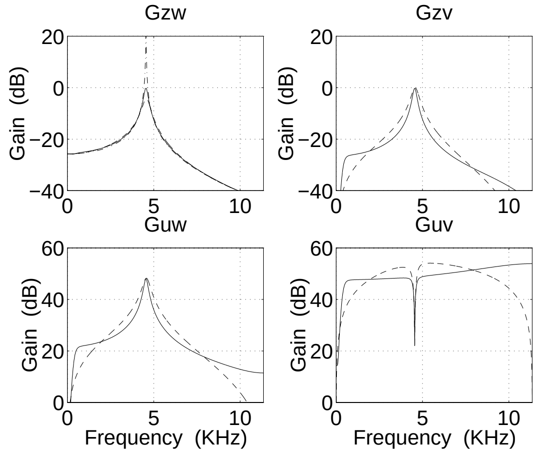

Figure 5: Closed loop frequency responses.

4 LONGITUDINAL SIMULATION RESULTS

A phase-space tracking simulation program is used to identify HOM-driven unstable coupled bunch modes in PEP-II and predict their growth rates. The action of the longitudinal feedback system is also simulated, so as to predict its damping effect. However, the dynamics of the fundamental cavity resonance are not simulated, and the fundamental is assumed to be rigid.

The rms noise performance of four pairs of controllers (FIR & LQG) were verified on the numerical simulator. These controllers ranged from the “knee” to the right end of the tradeoff curves. The results of the simulation, shown with ‘*’s and ‘+’s in fig.3, are in good agreement with the curves for the radiation damped bunch, as expected.

The simulated experiment illustrated in figure 6 starts at t = 0 with the 1658 bunch, 2.25A, LER beam at equilibrium. Noise excitation produces growth of unstable beam motion until feedback is turned on at t = 18ms. From this point on the bunch phases are passed through feedback filters and fed back as voltage kicks that damp the oscillations down to steady state.The longitudinal oscillations are projected onto symmetric beam eigenmodes by taking FFTs of the bunch phases on each turn [5]. The figure traces the evolution of the amplitudes of 1746 coupled bunch modes (folded at mode 873) over the whole grow/damp transient. We can see that there is a spectrum of unstable modes that grow from the noise floor in the absence of feedback. These modes are damped back down fairly rapidly once the 4-tap FIR feedback is turned on. Symmetric beam calculations indicate that modes 644-818 should be unstable at the design current, with growth rates ranging from 0 to 0.2 per ms. Exponential fits to the growing transients in the figure yield modes 677-689 and 758-789, growing at rates from 0.15 to 0.23 per ms. Exponential fits to the damping transients (after t = 18ms) yield damping rates clustered around 1 per ms. These rates were consistent with those obtained from the single bunch model analytical design. In fig.7, the simulation displayed in figure 6 is repeated with an LQG controller coming on 8ms later than before. As before, two bands of modes (677-689 and 758-789) show unstable growth in the absence of feedback. As a result of the delay in turning on the feedback, most modes grow to amplitudes 5-6 times larger than in the previous simulation before they are acted upon by the controller. The feedback power amplifier is thus heavily saturated, and hence fails to control the unstable modes. The threshold for saturation induced failure of the feedback system is approximately at 0.4 deg of rms beam motion for this controller.

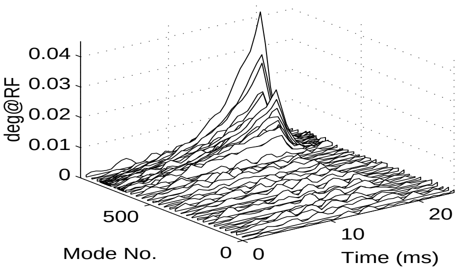

Figure 6: Growth and damping of unstable modes.

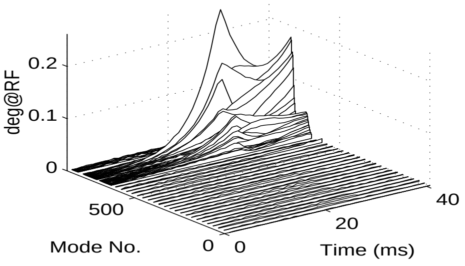

Figure 7: Instability due to saturated feedback.

5 SUMMARY AND CONCLUSION

We have presented a general technique for the design and verification of decentralized controllers for stabilizing longitudinal coupled bunch instabilities in a circular accelerator. The technique involves estimating growth rates of unstable modes using numerical simulation, analytical design of controllers for a radiation damped single bunch model, and verification of the controllers' performance on a multibunch beam simulation. This allowed the use of optimal control theory and trade off analysis to obtain the limits of the closed loop rms performance of the PEP-II feedback system in the presence of process and sensor noise.

It has been shown that the technique of designing controllers for a radiation damped single bunch model yields accurate calculations of the rms performance of the multibunch beam. The simulations show that it is possible to stabilize HER and LER beams under the estimated noise conditions with 1.5KW of amplifier power per ring. However, as demonstrated in fig.7, more power might be needed to control oscillations of the order of 8 mrad ($\approx$0.4 degree at rf) due to saturation of the back end. Reflections at the kickers, cable losses, and other nonidealities could also be larger than expected.

The damping rates calculated from the single bunch model were also consistent with those of coupled bunch modes in the numerical simulations. The tradeoff analysis showed that the feedback systems for the PEP-II beams are expected to be sensor noise dominated, for the gains required to produce good damping of unstable modes. The four-tap FIR controllers studied here were comparable to the LQG controllers in rms noise performance. They yielded better damping rates at the high end of the gain spectrum at the expense of only a 10-20% increase in rms beam motion. They also have better robustness to parameter variations, though this is not a significant issue for the conditions of PEP-II due to the fact that modal tune shifts are small in the two rings.

6 ACKNOWLEDGEMENTS

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