COHERENT NONLINEAR LONGITUDINAL PHENOMENA IN UNBUNCHED SYNCHROTRON BEAMS

L. K. Spentzouris and P. L. Colestock *,
Fermilab, P.O. Box 500, Batavia, IL 60510, USA

Abstract

Coherent nonlinear longitudinal dynamics have been investigated in unbunched hadron beams[1]. These phenomena may be described using a theoretical framework originally developed to describe interactions in plasmas[2]. Processes exhibiting the weakest degree of nonlinearity are known as wave-wave interactions. Manifestations of these interactions may be used to help characterize a beam. Echoes have been used to measure the collisional damping rate of weak diffusive processes which degrade a stored beam. Further information is contained in the echo shape, which depends on the form of the particle energy distribution. The echo shape may also be modified by the presence of wakefields, or nonlinearities in the machine lattice which affect the longitudinal motion of the particles.

1 ECHO MEASUREMENT AND DYNAMICS

Beam echoes may be generated transversely [3, 4], or else longitudinally in either a bunched beam [5] or an unbunched beam [1, 6, 7]. Although the underlying phenomena are the same in all cases, the discussion here pertains specifically to longitudinal echoes in an unbunched beam.

An echo is a coherent current oscillation which grows out of a quiet beam, with some delay after a sequence of two independent pulse excitations. In the absence of wakefields, the beam response to a longitudinal kick naturally decoheres, with a decoherence time that is inversely proportional to the energy spread of the beam. Even though the coherent motion of the beam damps away, the phases of the particles will remain correlated. Due to this correlation, the phase evolution of the decoherence is reversible. An echo is a partial reconstruction of the particle phase relation present during the coherent motion from the initial kicks. This reconstruction normally occurs some time after the beam response to the kicks has damped away.

Longitudinal echoes in an unbunched beam have been clearly observed in the Fermilab Accumulator. Figure 1 shows the time development of the peak beam current during echo production, as seen on a broadband longitudinal resistive-wall beam detector. The two large amplitude spikes within the first 100 ms correspond to the longitudinal kicks that were applied to the beam at harmonics $h = 9$ and $h = 10$, respectively. The frequency of the coherent response of the echo occurs at the difference frequency of the kicks, $f_{echo} = f_{kick2} - f_{kick1}$, which in this case is $h = 1$.

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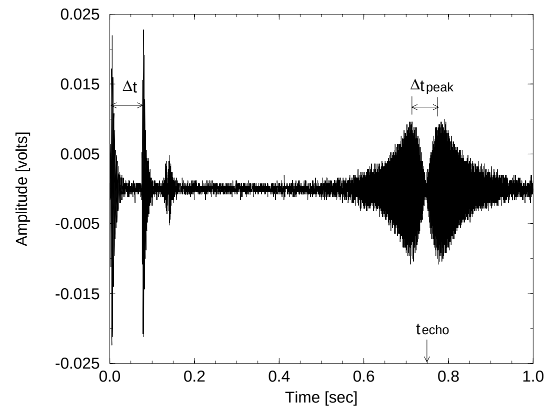

Figure 1: Beam response to a pair of impulse excitations separated in time by $\Delta t = .075$ seconds. The echo is centered at 0.75 seconds after the first kick. The beam parameters were: beam current $I_0 = 147$ mA, $\eta = .023$, total beam energy $E_0 = 8696$ MeV, beam energy spread $\sigma_\varepsilon = 3.2$ MeV, transverse normalized emittances $\epsilon_H = 1.75\pi$ mm-mrad, $\epsilon_V = .56\pi$ mm-mrad, and peak separation of the echo $\Delta t_{peak} = .07$ sec. Note the presence of a higher-order echo immediately following the second excitation pulse.

The timing of an echo is theoretically predicted to be $t_{echo} = [f_{kick2}/(f_{kick2} - f_{kick1})]\Delta t$, where Δt is the time separation between the two kicks. Verification of the scaling of t_{echo} with the kick frequencies is shown in figure 2.

2 DIFFUSION RATE MEASUREMENTS

Echoes are a useful tool for measuring the diffusion rate in a beam. Current methods of measuring thermal effects in a beam take on the order of hours. Echoes effectively amplify the effects of scattering, allowing the measurement of small collision frequencies in a much shorter time.

The presence of a diffusion mechanism destroys the reversibility of the decoherence of particle bunching. In the absence of diffusion, the phase evolution of the spreading particles depends strictly on their relative energies. The phase development of the particles due to their energy change from the second kick combines with their phase development from the first kick in such a way as to rebunch the particles. Partial decorrelation will reduce the echo amplitude, and full decorrelation will completely inhibit the echo. A collisional process, for example intrabeam scat-

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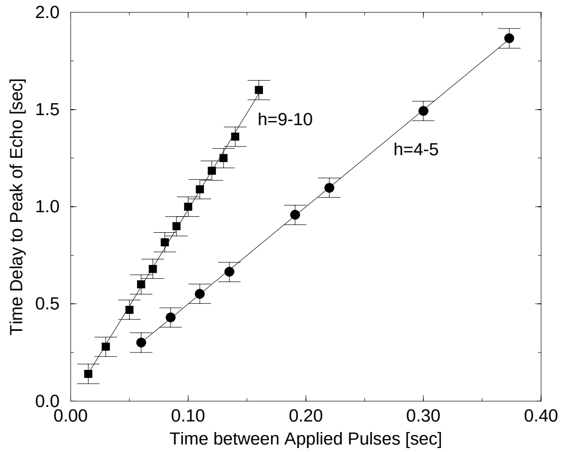

Figure 2: Measured echo delay time as a function of separation of the drive pulses. Upper curve: 1^{st} pulse at $h = 9$, 2^{nd} pulse at $h = 10$, giving expected time dependence $t_{echo} = [10/(10-9)]\Delta t = 10\Delta t$. Lower curve: 1^{st} pulse at $h = 4$, 2^{nd} pulse at $h = 5$, giving $t_{echo} = [5/(5-4)]\Delta t = 5\Delta t$.

tering, will break down the correlation between energy and phase by knocking individual particles off their original trajectories. A diffusion process is thus expected to change the dependence of echo amplitude on the time at which the echo occurs, decreasing the amplitude until there is no echo at all.

In the presence of diffusion, the beam current at the echo harmonic goes as

$$I_{echo} = AJ_1(k_1\delta\Delta t)\exp\left(-k_2\nu t_{echo}^3\right) \qquad (1)$$

where A is a constant, δ is proportional to the kick strength of the excitation, k_1 is a constant equal to one of the kick harmonics, Δt is the time between kicks, k_2 is a constant which depends on the kick harmonics and the echo harmonic, ν is the collision rate, and t_{echo} is the time from the first kick to the center of the echo. Decorrelation due to collisions results in a decay as t^3, which modifies the otherwise Bessel function form of the echo amplitude response. Thus, by fitting a measurement of echo amplitude versus the time of the echo, the diffusion rate in the beam may be determined. Such a fit is shown in figure 3.

The diffusion rate is large enough in the Fermilab Accumulator that only a portion of the first lobe of the Bessel function is visible in the beam response. This was further verified by performing an echo amplitude scan at two different sets of kick harmonics, as shown in figure 4. A comparison of the time when the echo amplitude vanishes in the two scans indicates whether or not this zero is a node in the Bessel function dependence, because $k_1 \neq k_2$ in equation 1. The ratio of the vanishing time in the two scans of figure 4 is consistent with the ratio of the decay time constant in equation 1, and is not consistent with the ratio of

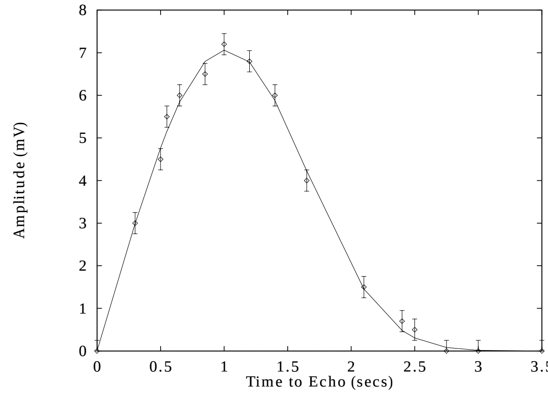

Figure 3: Peak echo response as a function of the time to echo following the initial pulse. The solid line represents a theoretical fit corresponding to a collision rate $\nu = (3.0 \pm 0.8) \times 10^{-4}$ Hz.

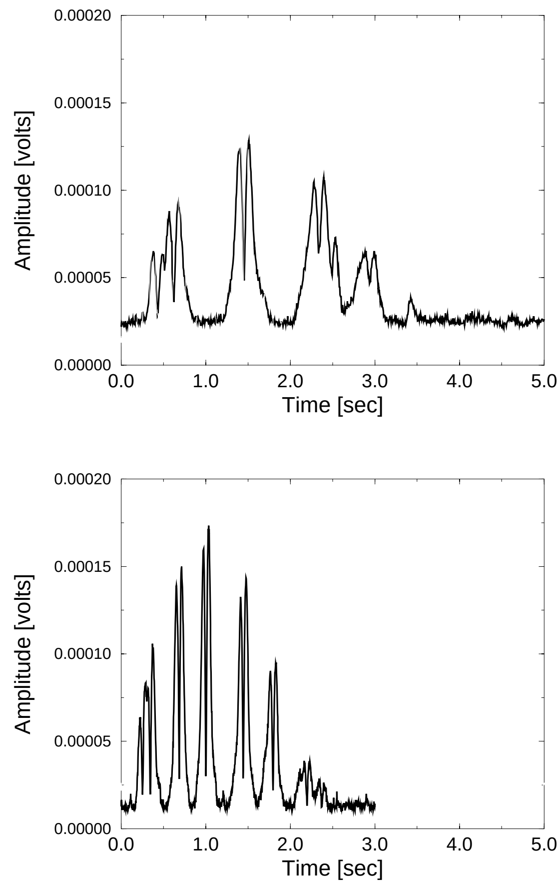

Figure 4: Echo amplitude vs. echo time t_{echo}. The top scan was done with kick harmonics $h_{kick1} = 9$ and $h_{kick2} = 10$, giving $h_{echo} = 1$. The bottom scan was done with $h_{kick1} = 8$ and $h_{kick2} = 10$, giving $h_{echo} = 2$. All other parameters were the same.

the argument of the Bessel function.

3 ECHO SHAPE

There is other information about the beam besides the diffusion rate residing in an echo. Knowledge of the beam energy sigma and the shape of the energy distribution of the particles may be extracted. It is perhaps even possible to learn about nonlinearities in the machine η function which governs the transit times of particles around the machine.

One of the major features of the observed echoes has been a deep notch in the center of the response. In the absence of wakefields, the echo current is dependent on the derivative of the unperturbed particle energy distribution. For a Gaussian beam, the slope of the distribution is zero at the center, hence the echo current goes to zero in the middle of the echo. The separation of the peaks on either side of this zero has a predicted dependence on the energy width of the beam. This goes as,

$$\Delta t_{peak} = \frac{\beta^2}{h_{echo}\pi f_{rev}|\eta|\frac{\sigma_E}{E_0}} \quad (2)$$

where β is the relativistic β of the beam, h_{echo} is the harmonic of the echo frequency, f_{rev} is the revolution frequency of the machine, η is the slip factor governing the particle transit time, and $\frac{\sigma_E}{E_0}$ is the energy sigma over the central energy of the Gaussian beam. In figure 5, experimental results are shown to agree with equation 1 to within 20%.

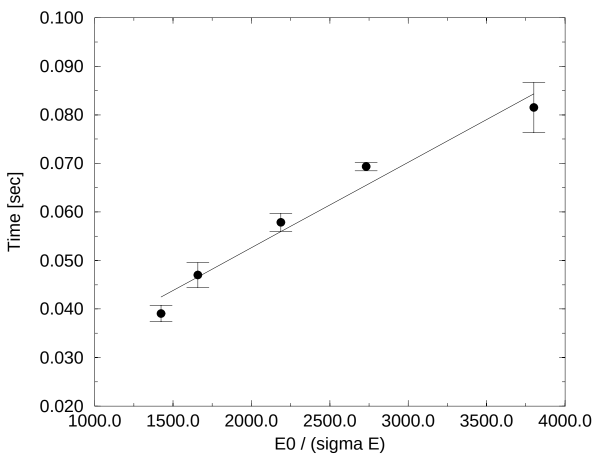

Figure 5: Peak separation of double-peaked echoes, Δt_{peak}, versus the inverse of $\frac{\sigma_\varepsilon}{E_0}$, the energy spread of the beam. The value of the slope from the linear fit is 1.76E-5. The beam parameters were $I_0 = 147$ mA, $\eta = .023$, $\beta^2 = .988$, $h_{echo} = 1$, and $f_{rev} = 629$ kHz.

Departures of the particle energy distribution from a Gaussian will be reflected in the shape of the echo. For example, during one study period an instability developed which knocked particles out of the center of the distribution. This left a beam energy profile with a depression in the center and shoulders on either side. The corresponding shape of the beam echoes is shown in figure 6. The

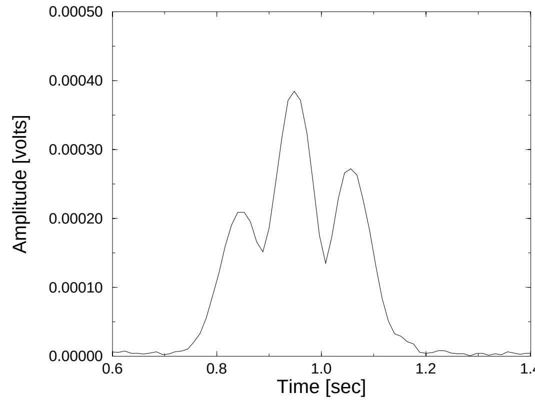

Figure 6: Beam echo from a non-Gaussian beam.

shape of the resulting beam profile can be constructed with a superposition of two offset Gaussians. By superposing the two corresponding normally notched echoes, a three-peaked echo such as that shown in figure 6 results. Since the echo shape is dependent on the slope of the beam distribution, perhaps a more careful and systematic algorithm for extraction of the beam profile can be developed.

While doing echo amplitude scans at various beam energy spreads, it was found that for large enough σ_ε the notch in the echo disappeared. Leaving all other parameters the same, scans were done for σ_ε in the range of 2.3 to 8 MeVc. As the beam energy spread increased, the echo amplitude decreased, and the central notch in the echo filled in, eventually vanishing completely. Figure 7 shows echoes from three of these scans. The echoes occur at approximately the same time relative to the driving pulses, but their shape is quite distinctive. Although only three echoes are shown in the figure, the trend is assiduously followed in all seven scans which were done. One possible explanation is that nonlinearities in the η function are mixing the particle distribution in such a way as to destroy the notch. The relation between the spread in particle revolution frequencies and the spread in particle energies is given by,

$$\begin{aligned} \frac{\Delta f}{f_0} &= -\frac{\eta}{\beta^2}\frac{\Delta E}{E_0} \\ &= -\frac{1}{\beta^2}\frac{\Delta E}{E_0}\left[\eta_0 + \eta_1\frac{1}{\beta^2}\frac{\Delta E}{E_0} + \cdots\right] \end{aligned}$$

As the energy spread of the beam gets larger, the nonlinear terms will make a greater contribution to the motion, and destroy the linear correlation between the energy and phase of the particles.

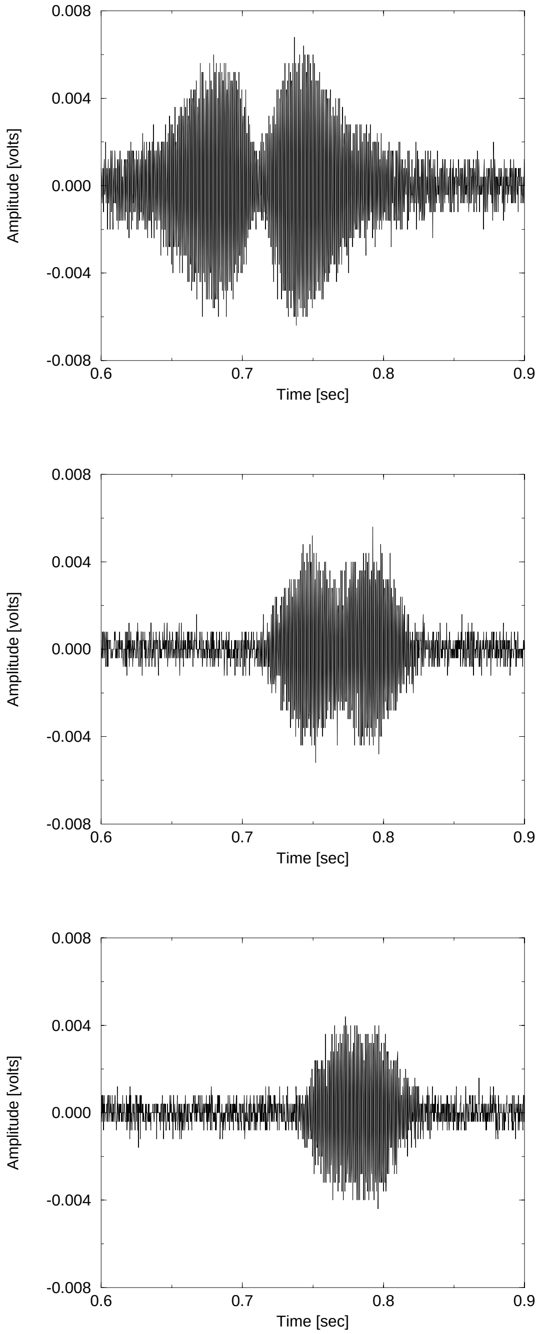

Figure 7: Three longitudinal beam echoes occurring in beams with different energy widths. The beam energy sigmas (σ_ε) were from top to bottom, 4.0, 6.1, and 8.0 MeVc. The energy spread was controlled with longitudinal stochastic cooling systems, all other beam conditions were the same. The beam intensity was 147 mA. Echoes which occurred at nearly the same time relative to the driving pulses were chosen for comparison.

4 CONCLUSIONS

High energy hadron beams are rich in nonlinear effects which are as of yet in the early stages of exploration. Echoes, which are a manifestation of weakly nonlinear wave-wave interactions, have been studied using an unbunched beam in a storage ring. Echoes can be used to explore the nature of the beam. The peak separation of the echoes generated in a Gaussian beam is proportional to the beam energy spread. In general, in the absence of wakefields, the shape of the echo depends on the slope of the energy distribution of the particles.

Echoes are sensitive to anything in a machine which disrupts the linear relation between particle energy and particle phase. As a result, it has been possible to make a fast measurement of the diffusion rate in a beam using longitudinal echoes. In the Fermilab Accumulator storage ring it has been observed that the notch normally present in the center of the echo fills in at large beam energy spreads. It may be that nonlinearities in the η function are responsible for this.

The investigation of beam echoes has demonstrated that studying nonlinear effects can be a useful tool in beam physics, allowing a deeper understanding of the behavior of particle storage rings.

5 ACKNOWLEDGEMENTS

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