THE BEAM LOADING EFFECT IN THE MULTICAVITY LINEAR ACCELERATOR AND THE REQUIREMENTS FOR THE RF CONTROL SYSTEM

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Abstract

This work has been done in the frame of ESS collaboration. The high intensity proton beam with 60 mA average current is supposed to be accelerated in the ESS linac up to 1.3 GeV. The ratio between the beam power and the losses power is changed along the accelerator from 0.6 to 0.7. With such a strong beam loading effect, the amplitude and phase RF field instabilities cause the growth of the effective longitudinal emittance at the exit of the multicavity linear accelerator. It gives rise to the difficulties with the injection of the beam in the ring. We define the requirements to the RF control system, taking this effect into account.

1 INTRODUCTION

At present the neutron source projects are developed in Europe, USA and Japan. Its are based on $1.0 \div 1.5$ GeV linac and the compressor ring or the high cycling synchrotron with the full maximum beam power of 5 MW[1]. The dominating design principle for the accelerators is the minimisation of beam losses. The maintenance and repair require that high energy beam losses are kept below 1 nA/m. In addition, the linac has to be optimised for low loss injection into the ring. Due to these problems the longitudinal and transverse emittance growth has to be comprehensively explored. The author of this work studies such a phenomena as the effective longitudinal emittance blow up. Owing to the random-regular character of the time-space deviation of the accelerating field, each single bunch undergoes the different exposure. With increasing of an energy the length and the relative momentum spread of the single bunch is adiabatically damped, but in the same time the effective sizes grow up with a number of the cavity. The distortion of the electromagnetic field has a lot of sources. We are interested the RF power or the beam switch off and on and the instability of the power supply or the beam current. Taking into account the feedback system, the reaction of the cavity on any random perturbation is described by the regular function, which has the time scale of change determined by the cavity inertia and the feedback delay. In other words two neighbour bunches have to have very similar parameters and the energy-phase deviation along the bunch train depends on the feedback and the cavity parameters. The fundamental mode change is recognised by the beam as the time distortion of the average field and it can be stabilised by the feedback system. The excitation of the modes with the variation along the cavity results in the space distortion. It is not under the control of feedback system and can be minimised by the relevant choice of the cavity parameters. The author has paid attention to this effect in the high intensity linear accelerator, working under ESS project[3].

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2 ERRORS COMPENSATED BY FEEDBACK

To stabilise the average field in the cavity the feedback system is used. The standard feedback system includes the object with the transition function $W_c = \frac{K_c e^{-p\tau_c}}{pT_c+1}$ and the feedback system itself with the transition function $W_f = \frac{K_f e^{-p\tau_f}}{pT_c+1}$, where K_c, K_f are the gain coefficients, τ_c, τ_f are the time delays and T_c, T_f are the inertia in the direct and feedback circuit respectively. The transition function of the cavity applied by the feedback is $W(p) = \frac{W_c(p)}{1+W_f(p)W_c(p)}$ and the original function of the closed loop is:

$$X(t) = \frac{X_0}{1+K_t}\left[1 - \frac{\omega_1}{\omega_0} e^{-\delta t}\sin(\omega_0 t + \theta)\right], \qquad (1)$$

where

$$\delta = \frac{T_f + T_c - K_t\tau}{2T_cT_f + K_t\tau^2}, \omega_1^2 = \frac{K_t + 1}{T_fT_c + 0.5K_t\tau^2},$$
$$\omega_0^2 = \omega_1^2 - \delta^2, K_t = K_fK_c, \tau = \tau_f + \tau_c, \tan\theta = \frac{\omega_0}{\delta},$$

where X_0 is the initial stepwise perturbation. The parameter δ determines the stability of the system. The maximum coefficient for the stable regime is $K_t \leq \frac{T_f+T_c}{\tau}$. Actually it has to be less, since the control time in response to the perturbation grows with K_t. In the same time the residual error equals to $\frac{X_0}{1+K_t}$. To realise the required gain coefficient the generator has to have the power reserve . The extra power depends on the ratio between the perturbation and the required residual error. For ESS this value equals to 30% of the nominal power to stabilise the amplitude and phase of $\pm1\%$ and $\pm1^0$ respectively. Thus the efficiency of the control system is determined in the first approximation by the ratio of the cavity feeling time and the time delay in the closed loop of the feedback system. There are lot of methods how to compensate the delay τ_f in the feedback system itself. One of them is the time leader unit applying the generator, which has to have the transition function similar to the direct circuit transition function. However no method compensates the natural delay of the signal in the cavity and the ultimate coefficient is restricted by the value $K_t \approx \frac{T_c}{\tau_c}$. The natural delay is equal to the one run time of the power along the cavity.

3 ERRORS UNCOMPENSATED BY FEEDBACK SYSTEM

The nature of these uncompensated errors lies in the difference between the irradiation of the generator and the beam. This is based on the different nature of the sources. In the first case the source is at rest:

$$j^g(z,t) = j_m\delta(z-z_0)e^{-i\omega t} \quad (2)$$

in the second one the source moves with the velocity v:

$$j^b(z,t) = j_m\delta(z-vt)e^{-i\omega t} \quad (3)$$

The electromagnetic field (electrical component) is represented as the sum of incident E_s and reflected E_{-s} waves:

$$E = \sum(C_sE_s + C_{-s}E{-s}) + \frac{4\pi}{i\omega\epsilon}j^b_z, \quad (4)$$

where

$$C_s(z,k) = \frac{1}{N_s(k)}\int_{z_1,z} j_kE_{-s}dv,$$
$$C_{-s}(z,k) = \frac{1}{N_{-s}(k)}\int_{z_2,z} j_kE_sdv$$

Substituting the expression for current in these integrals, we can find out:

- The generator current excites **all modes** of the cavity.
- The beam current excites that modes only, which have the phase velocity less than the beam velocity, what accords to **Cherenkov irradiation**.
- The generator irradiates in **both directions** from the power extraction point and the front of wave goes with the group velocity.
- The beam irradiates **back only**, but it fills the cavity with the phase velocity, that is practically instantaneously. So we should consider the beam as the distributed source along cavity.

In the waveguide approximation after "n" reflections we can write the expression for the total field irradiated by the generator located at one of the end of cavity. It is the stepwise function:

$$E = E_me^{i\phi}e^{-\alpha z}$$
$$\left[1 + ... + (1+e^{-2\alpha z}\dot{G}_1\dot{G}_2)e^{-2\alpha z}\dot{G}_1\dot{G}_2\right], \quad (5)$$

or for the steady value:

$$E = E_me^{i\phi}e^{-\alpha z}\frac{1-e^{-2\alpha Ln}}{1-e^{-2\alpha L}} \quad (6)$$

where $E_me^{i\phi}$ is the travelling wave field, α is the attenuation constant determined by $\alpha L = \frac{\tau_{gr}}{\tau}$. The one run time of the power along the cavity is $\tau_{gr} = \frac{L}{v_{gr}}$, where v_{gr} is the group velocity. The group velocity depends on the phase velocity v_{ph} and the coupling coefficient K_{coupl} of the accelerating structure and equals $v_{gr} = \frac{\pi}{4}K_{coupl}v_{ph}$. The filling time is determined by the quality factor Q and the resonant frequency ω. From (6) the ratio between the steady value of E and the travelling wave amplitude E_m is $\tau/2\tau_{gr}$, or using $E = (P_{generator}R_{sh})^{1/2}$,we have

$$E_m = \frac{2\tau_{gr}}{\tau}(P_{generator}R_{sh})^{1/2}. \quad (7)$$

Without beam all power is spent for the losses compensation $P_{generator} = P_{losses}$. To compensate the beam loading (fundamental mode) we have to extract the additional power from the generator $P_{generator} = P_{beam}/\cos\varphi_s$. Thus, the initial step amplitude for the stepwise function (5) is:

$$\frac{E_m}{E} = \frac{2\tau_{gr}}{\tau}\left[(\frac{P_{beam}}{P_{losses}\cos\varphi_s}+1)^{1/2}-1\right]. \quad (8)$$

In particular for ESS, where $P_{beam}/P_{losses} \approx 70\%$ the value $E_m/E \approx 7\%$. To minimise this perturbation the beam has to be injected with the finite front of the pulse current. However even in the steady regime any perturbation of few percents causes the extra power injection with the sharp front due to the high gain coefficient of the feedback system. In that case to estimate the perturbation we should use the extra power P_{extra} instead P_{beam} in the formula (8) and we get $E_m/E \approx 3\%$. Thus during the pulse current acceleration such a perturbation will "walk" along the cavity. The beam sees its as the accelerating field modulation. The characteristic time of this perturbation is determined by the frequency beat between the fundamental and the nearest modes. For ESS it is about $1 \div 2$MHz. To decrease these distortions we have to do one of the following:

- to use shorter cavity
- to use the accelerating structure with the higher coupling coefficient
- to use the structure with the higher quality factor

From this point of view it would be interesting to compare the normal cavity with the high coupling coefficient and the super conductive cavity. However, we should understand how these distortions influence the beam parameters.

4 THE EFFECTIVE PARAMETERS OF THE BEAM

We are interested in the motion in the perturbed electromagnetic field:

$$E = E_0 + \delta E(z,t) = E_0(1 + \frac{\delta E_m}{E_0}\sum_k e_k\cos k\nu t), \quad (9)$$

where $\nu = \frac{2\pi}{T_c}$ is the minimum frequency of perturbation with the period of the length cavity T_c. This kind of perturbation gives arise the parametric or external resonance in the longitudinal plane. To avoid these resonances the parameters of the cavity have to be in compliance with the condition[2]:

$$T_c \le \frac{4\pi k}{\Omega_0 \cos^{1/2}\varphi_s}, \delta E e_k \le \frac{2\Omega_0 \sin\varphi_s}{\cos^{1/2}\varphi_s}, \qquad (10)$$

where Ω_0 is the synchronous frequency when the synchronous phase $\varphi_s = -90^0$. Obviously this condition restricts us in the cavity length or the accelerating field amplitude. Let us suppose we have designed the accelerator, which avoids the resonance condition. In this case any perturbation can be considered as a high frequency kick. It causes the phase deviation of the bunch from the separatrix centre after passing through n cavities:

$$\langle\Delta\psi\rangle_{centre} = \left(\langle\delta\varphi\rangle^2 + \langle\cot\varphi_s \frac{\delta E}{E}\rangle^2\right)^{1/2} n^{1/2}\mathcal{K}(k) \quad (11)$$

and the energy deviation

$$\langle\Delta W\rangle_{centre} = \gamma^2 p_s v_s \frac{\Omega}{\omega}\langle\Delta\psi\rangle_{centre}, \qquad (12)$$

where $\mathcal{K}(k)$ is the harmonic number function. In the first approximation $\mathcal{K}(k) = \frac{\pi k/\mu}{(\pi k/\mu)^2 - 1}$, where μ is the advanced phase per the cavity. Thus, the energy spread of the bunch centres grows, as

$$\langle\Delta W\rangle_{centre} \propto n^{1/2}\gamma^{3/2}\beta^{3/2}\mathcal{K}(k), \qquad (13)$$

and the energy spread of single bunch:

$$\langle\Delta W\rangle_{bunch} \propto \gamma^{3/4}\beta^{3/4}, \qquad (14)$$

These expressions show the effective momentum spread is significantly determined by the spread of the centres. This effect has been calculated numerically for ESS side coupled linac in the work[4].

5 THE REGULAR COOLING OF BEAM

So, the effective longitudinal emittance is determined by the time-space distortions in the cavity. The lowest characteristic frequency of this process is most sensitive for the beam and inversely proportional to the inertia of units included in the closed loop. In particular for ESS this frequency is 150 kHz. Let us consider such a control system, when the phase perturbation of the accelerating field is compensated by the amplitude or contrary. Ideally it would be, when $\delta\varphi = \cot\varphi_s \delta E/E$. The energy deviation δW in that case equals zero. But the question is how to realise such a simple idea? The answer is the time of flight procedure. This procedure supposes the measurement of the flight phase and the correction of it either by the RF phase or by the RF amplitude until the coincidence with the designed value with accuracy Φ :

$$\left|\omega\int_0^{L_{cavity}} \frac{dz}{c\beta} - \omega\int_0^{L_{cavity}} \frac{dz}{c\beta_s}\right| \le \Phi \qquad (15)$$

In that case the bunch energy deviation after passing through the cavity will be:

$$\Delta W \le \left|\gamma_2 p_s v_s \frac{\Omega\Phi}{\omega\sin\mu} + \Delta\psi\tan\frac{\mu}{2}\right|. \qquad (16)$$

Schematically this procedure is shown for the relative velocity on figure 1. Thus, we do not have the factor $n^{1/2}$ and the velocity (energy) deviation is determined by the current cavity.

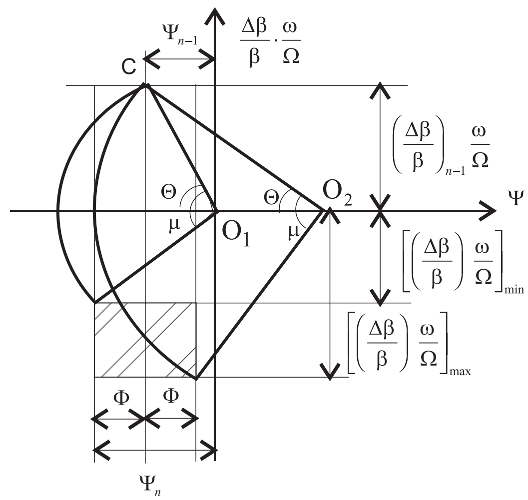

Figure 1: The phase diagram for the flight time procedure.

6 CONCLUSION

This material allows us to compare the normal and the super conductive accelerators and how each option influences the beam quality. On the one hand due to small ratio τ_{gr}/τ the superconductive cavity has no spatial distortion of the electromagnetic field and we do not have the errors uncompensated by feedback system. On other hand the higher accelerating field amplitude, the stronger influence of the residual errors to the spread of the bunch centres. From this point of view the most appropriate variant is the normal structure with the high coefficient of the coupling $\sim 50\%$.

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8 REFERENCES

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