

ELECTRON BEAM PROPAGATION WITH PREMODULATED ENERGY AND CURRENT

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*Abstract**

This paper considers an electron beam whose energy and current are simultaneously premodulated at the injection point of a drift tube. A theoretical model is developed for the subsequent current and energy modulations which propagate downstream. A closed integrodifferential equation for the normalized beam current is obtained in terms of time and propagation distance. To make the nonlinear current modulation analytically tractable, a small signal theory is introduced into the modulation calculation. The current modulation in the linear regime is a linear combination of the forward and backward waves of the initial current and energy modulations. The downstream energy modulation is also expressed by a linear combination of the forward and backward waves of the initial current and energy modulations. It is shown that the initial energy modulation is a very effective means for downstream current modulation. Numerical data even for a large initial current and energy modulations agree reasonably well with analytical results predicted by the linear theory.

I INTRODUCTION

Prebunched charged particle beams have been investigated for various applications, including accelerator physics¹⁻⁷ and high-power traveling wave tubes (TWT).^{8,9} Prebunched beams may make possible compact traveling wave tubes, eliminating the amplification regions. The recent experiment^{8,9} on micro-field-emission gates indicates a strong possibility of the prebunched beams in near future. Unlike klystron amplifiers,¹⁰⁻¹⁴ initiated by energy modulation at the input cavity, the prebunched TWT is operated by a premodulated electron beam, where the initial current modulation is a driving factor. A large initial current-modulation may considerably reduce size of the amplification region in TWT. In this context, a nonlinear theory of a prebunched electron beam propagating through a drift tube has been reported in a previous paper.¹⁵ When the prebunched electron beam enters the drift tube, the initial electron current of the beam is modulated according to a prescribed current profile $f(\omega t_0)$, where ω is the bunching frequency of the beam and t_0 is the entering time of each beam slice. In addition to initial current modulation, we may also modulate the beam energy at the beginning. In reality, energy of the beam electrons¹³ in the klystron amplifiers is modulated at the injection point by the input cavity without the current modulation. It is useful to investigate the influence of the initial energy modulation

on the prebunched beam current during propagation. This article develops a theoretical model describing the current and energy modulations of an electron beam propagating downstream, when the beam's energy and current are simultaneously premodulated at the injection point.

II PROPAGATION THEORY OF AN ELECTRON BEAM

A premodulated electron beam enters a drift tube at $z = 0$. The electron beam is radially confined by a strong magnetic field. For simplicity in the subsequent analysis, we assume that the electron beam current at $z = 0$ is premodulated according to a periodic function $f(\theta)$. Here, the normalized time $\theta = \omega t_0$ represents the time t_0 at which the beam segment labeled by t_0 enters the drift tube and the parameter ω is the modulation frequency. The function $f(\theta)$ can be an arbitrary function that represents the initial current modulation. For example, the function $f(\theta) = 1 - h\cos(\theta)$ shows a sinusoidal current modulation with the strength of h . Electron beam energy is also premodulated according to

$$\gamma_0(\theta) = \gamma_b + g(\theta), \quad (1)$$

where γ_0 and γ_b are the relativistic mass factor at $z = 0$ and its average value over one period of the energy modulation function $g(\theta)$. The propagation distance z is related to the present time t and the injection time t_0 by making use of the velocity definition $dz/dt = \beta c$. Defining $\theta = \omega t_0$ and $\varphi = \omega t$, we obtain

$$\varphi - \theta = \int_0^\zeta \frac{d\zeta'}{\beta} = \int_0^\zeta d\zeta' \frac{\gamma}{\sqrt{\gamma^2 - 1}}, \quad (2)$$

where ζ is the normalized propagation distance defined by $\zeta = \omega z/c$ and $\gamma(\zeta, \theta) = (1 - \beta^2)^{-1/2}$ is the instantaneous relativistic mass factor of the beam segment labeled by θ .

The present time φ is uniquely described by the injection time θ and propagation distance ζ .

The self-electric field E , which exerts in the beam segment θ is calculated to be¹³

$$E(\zeta, \theta) = 2G(R_b) \frac{\omega}{\beta^2 \gamma^2 c^2} \left(\frac{\partial I}{\partial \varphi} \right)_z, \quad (3)$$

where the geometrical factor G is determined in terms of system configuration. For convenience in the subsequent analysis, we define the normalized current $F(\zeta, \theta)$ by

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$$F(\zeta, \theta) = \frac{I(\zeta, \theta)}{I_b}, \quad (4)$$

where I_b is the average beam current in one period of the current modulation at the injection point and the normalized distance ζ is defined by $\zeta = \omega z/c$. Velocity modulation of the beam segment labeled by θ is obtained from

$$mc^2 \frac{d}{dz} \gamma = eE(z, \theta), \quad (5)$$

with the initial condition $\gamma = \gamma_0(\theta)$ at the entrance point $z = 0$ of the beam segment θ . Substituting Eq. (3) into Eq. (5), we obtain

$$\frac{d}{d\zeta} \left(\frac{\gamma^3}{3} - \gamma \right) = \kappa \left[\frac{\partial}{\partial \varphi} F(\zeta, \theta) \right]_{\zeta}, \quad (6)$$

where the self-potential depression κ is defined by¹⁶ $\kappa = 2I_b G/I_A$ and $I_A = 17$ kA is the Alfvén current. Carrying out the integration of Eq. (6) and making use of the initial condition, we obtain

$$\frac{\gamma^3}{3} - \gamma = \frac{\gamma_0^3(\theta)}{3} - \gamma_0(\theta) + \kappa \int_0^\zeta d\zeta' \left(\frac{\partial F}{\partial \varphi} \right)_{\zeta'}, \quad (7)$$

where the initial beam energy $\gamma_0(\theta)$ is determined from Eq. (1). Once the current modulation F is known, the energy modulation $\gamma(\zeta, \theta)$ is also known from Eq. (7). Differentiating Eq. (7) with respect to θ , we obtain

$$\frac{d\gamma}{d\theta} = \frac{\gamma_0^2}{\gamma^2 - 1} \frac{d g}{d\theta} + \frac{\kappa}{\gamma^2 - 1} \int_0^\zeta d\zeta' \left(\frac{\partial^2 F}{\partial \theta \partial \varphi} \right)_{\zeta'}, \quad (8)$$

where $g(\theta)$ is the energy modulation function defined in Eq. (1).

It is useful in subsequent analysis to obtain

$$\frac{\partial \varphi}{\partial \theta} = 1 - \int_0^\zeta \frac{d\zeta'}{(\gamma^2 - 1)^{3/2}} \left(\frac{\partial \gamma}{\partial \theta} \right)_{\zeta'}, \quad (9)$$

from Eq. (2). Substituting Eq. (8) into Eq. (9) gives

$$\left(\frac{\partial \varphi}{\partial \theta} \right)_{\zeta} = 1 - (\gamma_0^2 - 1) \frac{d g}{d\theta} \int_0^\zeta \frac{d\zeta'}{(\gamma^2 - 1)^{5/2}} - \kappa \int_0^\zeta \frac{d\zeta'}{(\gamma^2 - 1)^{5/2}} \int_0^{\zeta'} d\zeta'' \left(\frac{\partial^2 F}{\partial \theta \partial \varphi} \right)_{\zeta''}. \quad (10)$$

Note that the derivative $\partial \varphi / \partial \theta$ in Eq. (10) is uniquely described by the history of the current modulation F until the present time in which the beam segment labeled by θ arrives at the distance z .

The beam segment t_0 passes the injection point at time $t = t_0$. When this segment arrives at z in time t , it is stretched by a factor of dt/dt_0 . Thus, the beam current of the segment t_0 at z is proportional to $(d\theta/d\varphi)f(\theta)$. In this regard,

the normalized current ratio $F(\zeta, \theta)$ in Eq. (4) is expressed as

$$F(\zeta, \theta) = \frac{N(\zeta) f(\theta)}{|d\varphi/d\theta|}, \quad (11)$$

where the normalization constant $N(\zeta)$. After carrying out a straightforward calculation, we obtain the nonlinear integrodifferential equation

$$\frac{N(\zeta)}{F(\zeta, \theta)} f(\theta) = 1 - (\gamma_0^2 - 1) \frac{d g}{d\theta} \int_0^\zeta \frac{d\zeta'}{(\gamma^2 - 1)^{5/2}} - \frac{\kappa}{2} \int_0^\zeta \frac{d\zeta'}{(\gamma^2 - 1)^{5/2}} \int_0^{\zeta'} \frac{d\zeta''}{N(\zeta'')} \frac{\partial}{\partial \theta} \left[\frac{1}{f(\theta)} \left(\frac{\partial F^2}{\partial \theta} \right) \right]_{\zeta''}, \quad (12)$$

where use has been made of the relation $\partial \theta / \partial \varphi = F/Nf(\theta)$.

The term that is proportional to $dg/d\theta$ in Eq. (12) originates from the initial energy modulation at the injection. The initial condition of the integrodifferential equation (12) is $F(0, \theta) = f(\theta)$. Equations (7) and (12) are two principal results in this article, and can be used to find the nonlinear current evolution as the premodulated beam propagates downstream. The numerical calculation of Eq. (12) is not so simple. The main difficulty in numerical calculation of Eq. (12) is the double integration of ζ in the right-hand side, which requires a long computer time.

III SMALL SIGNAL THEORY

In order to find a scaling law, we linearize Eq. (12), assuming a small level of modulation. Therefore, the initial and later current modulations are expressed as

$$f(\theta) = 1 + \delta f(\theta), \quad (13)$$

$$F(\zeta, \theta) = 1 + \delta F(\zeta, \theta),$$

where the amplitudes of δf and δF are much less than unity.

We also assume that the initial energy modulation $g(\theta)$ is much less than the mean value γ_b of the relativistic mass factor. A previous study¹⁵ indicates that results from the small signal theory in the subsequent analysis agree remarkably well with results from numerical solution of Eq. (13), even when the modulation amplitude is close to unity.

The function $\delta f(\theta)$ in Eq. (13) is a periodic function with a periodicity of 2π . The initial current $f(\theta)$ and energy $g(\theta)$ profiles must satisfy the normalization condition that the average values of the functions $\delta f(\theta)$ and $g(\theta)$ must be zero over one period. Substituting Eq. (13) into Eq. (12) and linearizing, we find

$$\delta F(\zeta, \theta) = \delta f(\theta) + \xi \frac{d}{d\theta} g(\theta) + \eta^2 \int_0^\zeta d y \int_0^y d x \left[\frac{\partial^2}{\partial \theta^2} \delta F(x, \theta) \right]_x, \quad (14)$$

where the frequency η of the amplitude oscillation is defined by

$$\eta^2 = \frac{\kappa}{(\gamma_b^2 - 1)^{5/2}}, \quad \xi = \frac{\zeta}{(\gamma_b^2 - 1)^{3/2}}, \quad (15)$$

and the normalization constant $N(\zeta)$ in Eq. (12) has been approximated by $N(\zeta) = 1$ for small oscillations.

Differentiating both sides of Eq. (14) by ζ , we obtain

$$\frac{\partial^2}{\partial \zeta^2} \delta F(\zeta, \theta) = \eta^2 \frac{\partial^2}{\partial \theta^2} \delta F(\zeta, \theta), \quad (16)$$

which is a typical equation for wave propagation. The physically acceptable solution to Eq. (16) is

$$\delta F(\zeta, \theta) = A(\theta + \eta\zeta) + B(\theta - \eta\zeta), \quad (17)$$

where the functions A and B must satisfy the boundary conditions

$$A(\theta) + B(\theta) = \delta f(\theta) \quad (18)$$

and

$$\left(\frac{\partial A}{\partial \theta}\right)_{\zeta=0} - \left(\frac{\partial B}{\partial \theta}\right)_{\zeta=0} = \frac{1}{\eta} \left(\frac{dg}{d\theta}\right) \left(\frac{d\xi}{d\zeta}\right) \quad (19)$$

at the injection point $\zeta = 0$. Equation (19) is equivalently expressed as

$$A(\theta) - B(\theta) = \frac{g(\theta)}{\sqrt{\kappa} (\gamma_b^2 - 1)^{1/4}}, \quad (20)$$

where the definitions in Eq. (15) have been used. The functions $A(\theta)$ and $B(\theta)$ are determined to be

$$A(\theta) = \frac{1}{2} \left[\delta f(\theta) + \frac{g(\theta)}{\sqrt{\kappa} (\gamma_b^2 - 1)^{1/4}} \right], \quad (21)$$

$$B(\theta) = \frac{1}{2} \left[\delta f(\theta) - \frac{g(\theta)}{\sqrt{\kappa} (\gamma_b^2 - 1)^{1/4}} \right],$$

from Eqs. (18) and (20). The solution in Eq. (17) is therefore given by

$$\begin{aligned} \delta F(\zeta, \theta) &= \frac{1}{2} [\delta f(\theta + \eta\zeta) + \delta f(\theta - \eta\zeta)] \\ &+ \frac{1}{2\sqrt{\kappa} (\gamma_b^2 - 1)^{1/4}} [g(\theta + \eta\zeta) - g(\theta - \eta\zeta)], \end{aligned} \quad (22)$$

which is expressed by a combination of the initial energy and current modulations. We thus conclude that the downstream current modulation is described by a linear

combination of the initial energy and current modulations. Substituting Eq. (22) into Eq. (7) and carrying out the linearization, we obtain the downstream energy modulation

$$\begin{aligned} \gamma(\zeta, \theta) &= \gamma_b + \frac{1}{2} [g(\theta - \eta\zeta) + g(\theta + \eta\zeta)] \\ &+ \frac{\sqrt{\kappa} (\gamma_b^2 - 1)^{1/4}}{2} [\delta f(\theta + \eta\zeta) - \delta f(\theta - \eta\zeta)]. \end{aligned}$$

The current modulation $F(\zeta, \theta)$ in the linear regime is a linear combination [Eq. (22)] of the forward and backward waves of the initial current and energy modulations (f and g). Properties of the current and energy modulations were numerically investigated from the integrodifferential equation (12) for a broad range of system parameters. Magnitudes of the energy and current modulations were determined in terms of the modulation frequency, initial energy $g(\theta)$ and current $f(\theta)$ profiles, geometrical configuration, beam intensity and initial kinetic energy of the beam. Numerical data from Eq. (12), even for large initial current and energy modulations, agree reasonably well with analytical results predicted by the linear theory.

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