SIMULATION STUDIES OF IONIZATION COOLING

D. Neuffer and A. Van Ginneken
Fermilab, P. O. Box 500, Batavia IL 60510

Abstract

A μ^+-μ^- collider must compress the beam phase-space volume by a factor of ~10^6 to obtain high luminosity, and this beam cooling must occur before μ-decay. In this paper we present simulations of ionization cooling which explore the various conditions needed for cooling to collider conditions. Cooling by large factors is demonstrated and directions toward complete cooling scenarios are discussed.

I. INTRODUCTION

The μ^+-μ^- collider [1, 2, 3] concept relies on ionization cooling to compress the beam phase-space volume to obtain high luminosity. This method has been described by Skrinsky et al.[2] and by Neuffer.[3] In ionization cooling, the beam loses transverse and longitudinal momentum while passing through a material medium, but regains only longitudinal momentum in acceleration cavities. Cooling by large factors requires successive stages of energy loss and reacceleration (20 to 50 stages). In this process the beam will evolve from a large phase-space volume to more compressed forms, and the cooling sections must change to match these. Also since the ionization cooling process does not directly cool the beam longitudinally, the beam must also pass through wedge absorbers at regions of non-zero dispersion to exchange longitudinal and (cooled) transverse phase-space.

The differential equation for rms transverse cooling is:

$$\frac{d\varepsilon_N}{ds} = -\frac{1}{\beta^2 E}\frac{dE}{ds}\varepsilon_N + \frac{\beta_\perp E_s^2}{2\beta^3 m_\mu c^2 L_R E} \qquad (1)$$

where the first term is the frictional cooling effect and the second is the multiple scattering heating term. ε_N is the normalized transverse emittance, E is the μ energy, and $\beta_\perp$ is the betatron amplitude. Similarly an equation for longitudinal cooling (reduction of energy spread σ_E) with energy loss can be written as:

$$\frac{d\sigma_E^2}{ds} = -2\frac{\partial\frac{dE}{ds}}{\partial E}\sigma_E^2 + \frac{d\left\langle \Delta E_{rms}{}^2 \right\rangle}{ds} \qquad (2)$$

in which the first term is the cooling term and the second is the heating term caused by random fluctuations in energy losses. The longitudinal cooling term is nearly zero, but it can be enhanced by placing the absorbers where transverse position depends upon energy (at nonzero dispersion) and where the absorber density or thickness also depends upon energy, such as in a wedge. In that case the cooling derivative can be written as:

$$\frac{\partial\frac{dE}{ds}}{\partial E} \Rightarrow \left.\frac{\partial\frac{dE}{ds}}{\partial E}\right|_0 + \frac{dE}{ds}\frac{\eta\rho'}{\beta cp\rho_0} \qquad (3)$$

where ρ'/ρ_0 indicates the change in density with respect to transverse position, ρ_0 is the reference density associated with dE/ds, and η is the dispersion (η = dx/d(δp/p)). Increasing the longitudinal cooling in this manner decreases the transverse cooling by the same amount.

In the long pathlength Gaussian-distribution limit, the energy heating term is given approximately by:

$$\frac{d\left\langle \Delta E_{rms}{}^2 \right\rangle}{ds} = 4\pi\left(r_e m_e c^2\right)^2 n_e \gamma^2 \left(1 - \frac{\beta^2}{2}\right) \quad , \qquad (4)$$

where n_e is the electron density in the material. This expression increases rapidly with higher energy (larger γ).

II. μ-COOLING REQUIREMENTS AND SCENARIOS

The μ-beam at the end of the rf rotation and decay section is expected to be ~300 MeV in energy with an rms energy spread of ~10%, an rms bunch length of ~3m and with a transverse emittance in x and y of ε_T = 0.015 m-rad (normalized). With the presently proposed high-energy μ^+-μ^- collider parameters the rms transverse emittance must be reduced to ~0.00005 at the end of the cooling system (300× smaller). The longitudinal emittance required in the 2×2TeV collider is 3mm bunch length by 3 GeV energy width, (only ~10× smaller). The overall 6-D cooling required is ~10^6. Cooling by these large factors requires a sequence of absorbers interspersed with reaccelerations. This sequence must include dispersion/wedge absorbers for energy cooling, and rf bunching for bunch length control. To minimize transverse heating (see Eq. 1), L_R (the material radiation length of the absorber) must be large, which means a light element such as Li or Be, and $\beta_\perp$ must be small and become progressively smaller as the beam is cooled, which means strong focusing at the absorbers. Scenarios for complete cooling have been developed; detailed simulations are needed to verify and optimize possible scenarios.

III. SIMULATION METHODS

Simulation of particle transport through a cooling section starts from a description of the phase space of the incident muons. To evaluate the cooling progress the kinematic variables of the particles are noted upon crossing some fixed set of planes perpendicular to the central trajectory—including the start and finish of the absorber. For a Hamiltonian formulation, this makes z, the distance along the nominal trajectory, the logical choice of independent variable. The cooling channel may include absorbers of arbitrary composition and dimensions as well as magnetic fields of arbitrary specification. Absorber

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material and magnetic field as a function of location are supplied to desired accuracy either by a field map or by an analytical prescription. Except when traversing a field-free void or a void in which the field is simple enough to permit an exact analytic solution, particles are traced through the absorber geometry in a series of small steps—each typically of the order of a few mm.

The physics content of SIMUCOOL is essentially unchanged from that presented in some detail in ref. [5]. Briefly, the main ingredients are: *ionization energy loss* as described by the Vavilov distribution modified for spin one-half particles and with inclusion of an energy threshold above which μ-e collisions are simulated individually, and *multiple Coulomb scattering*, in which an angular threshold is adopted below which it is treated in the Gaussian approximation and above which as coherent individual μ-nucleus scattering events. Lesser contributions (small at the low energies of interest here) such as incoherent Coulomb scattering between muons and nuclear protons, bremstrahlung, e^+-e^- production, and deep inelastic μ-nucleus collisions are also included in the simulations.

Energy loss and multiple scattering are treated as continuous processes and are thus applied during each step of the Monte Carlo which takes place in a material. All other processes, including large energy losses and large angle scattering, are treated event-by-event.

In each simulation a beam of 5000—25000 particles, generated within 6-D gaussian distributions is tracked through absorbers plus through transport and acceleration modules. Beam properties at the end of the transport are calculated and compared with rms equations.

IV. SIMULATION RESULTS

Simulations of ionization cooling have been obtained for a variety of situations corresponding to critical portions of a cooling scenario. The cases which have been explored include the following: absorbers at a β* focus, absorbers within solenoids, Be or Li lens absorbers, and wedge absorbers with cooling over a range of energies from multi-GeV levels to 20-MeV. We discuss some of these below; a more detailed discussion, with more cases, is found in ref. 6.

1. Absorbers in free space

The simplest cooling case is a field-free absorber at a beam focus. A number of these cases have been studied, using a variety of materials, and with beams at momenta from 200 MeV/c to 1.2 GeV/c. Overall very close agreement with rms equations is obtained, and very little particle-loss by scattering to large angles or large δp/p occurs. For example, a 400 MeV/c beam was tracked through 40cm of Be (matched to β*=20cm at the center). With p reduced to 277 MeV/c, the rms emittance is reduced from 0.01 to 0.0077 m-rad while δp_{rms} increases from 8 to 10.4 MeV/c.

Cooling was tracked over a broad range in initial energies, and figure 1 shows energy spread before and after a 20 cm Be rod, showing the increased width as well as the non-gaussian loss pattern with a "Landau tail". Energy straggling increases with increasing energy (see Eq. 4) and becomes too large for p > ~800MeV/c.

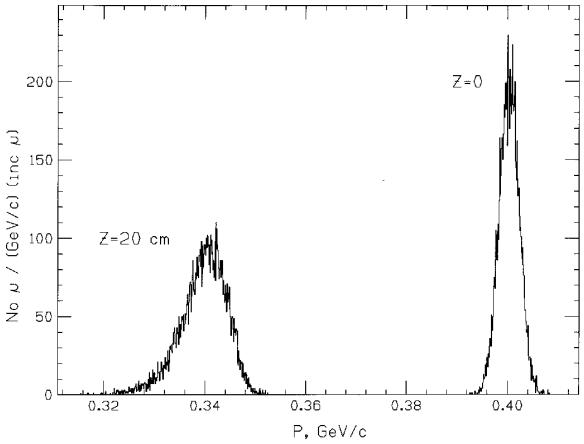

Figure 1: momentum distribution of a beam before and after a 20 cm Be absorber, showing increase in δp, as well as change in distribution shape.

2. wedge absorbers

μ-cooling naturally cools only transversely, and is accompanied by a gradual increase in δp_{beam}. 6-D cooling requires periodic exchanges in phase space, and this can be obtained if the beam is given a non-zero dispersion (position dependence on momentum) and then passes through a wedge absorber, which is placed so that the high-momentum side of the beam passes through more material and loses more energy. The resulting process reduces δp/p while increasing the intrinsic transverse size, thereby increasing ε_T by the same factor.

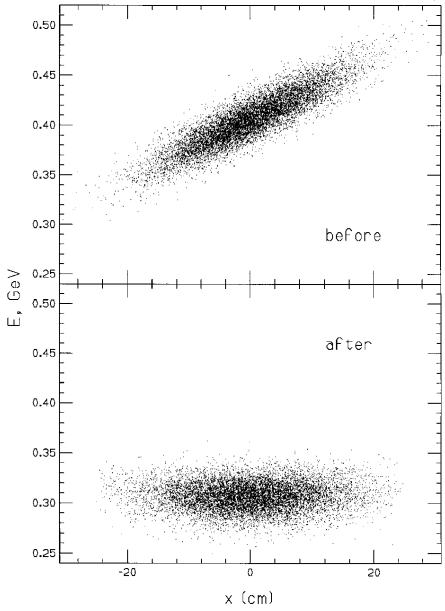

Figure 2. Beam (x-E distribution before and after a wedge + absorber for x-p emittance exchange. Note that δE is reduced by a factor of 2, and dispersion is reduced (1m → ~0).

Wedge absorbers of various materials have been simulated with vaious beam conditions (p, δp_{rms}, ε_T, etc.) and phase-space exchanges in agreement with cooling models have been obtained.

As discussed in ref. 7, the wedge and beam transport can be optically matched to obtain $\eta \cong 0$ at the exit of the wedge. Figure 2 shows simulation results of such a case, in which a hot 400 MeV/c beam with δp_{rms} =7.4%, ε_T = 0.015, at β^* = 0.34m and dispersion η = 1m, passes through a Be wedge + absorber. In the simulation, the energy spread is reduced by a factor of 2 while ε_x is nearly doubled, in agreement with expectations. (ε_y is slightly cooled.) As seen by the reduction in the x-p correlation, the dispersion η is cancelled to ~ zero.

3. Cooling in Be (Li) lens and in Solenoids

From Eq. 1, cooling requires small β^* or strong focusing at the absorbers. This can be most readily sustained if there is strong focusing at the absorber, and this can be obtained if the absorber is itself a focusing lens; i. e., a Li or Be lens, which is a conducting rod carrying a high current (up to ~1MA). Fields up to ~20000T/m can be developed in such rods. Simulations of μ-beam transport within conducting rods are able to obtain excellent cooling, in agreement with the rms equations. The cooling was insensitive to initial conditions and occurs with very large momentum spread (δp_{rms}).

Another suggested method for focusing within absorbers is to place the absorber within a high-field solenoid. Our simulations showed that this was not effective. The difficulty occurs because the solenoid introduces axial motion (angular momentum) into the beam which is damped within the absorber. However on exiting the absorber, the original axial motion is removed, and the beam has a net axial motion which dilutes the (projected) emittance.

4. Multistep cooling

To demonstrate the possibility of cooling by large factors, a beam was transported through a sequence of 8 Be lens. In each of these, momentum loss by a factor of 2 (from 400 to 200 MeV/c) was obtained and followed by reacceleration, while focussing gradients were increased from lens to lens from 30T/m to 2000T/m, matching the decreasing beam size. To control δp_{rms} a set of wedges was placed in the center of the cooling sequence, which reduced δp_{rms} by ~2×. Fig. 3 shows transverse phase-space (p_x, x) at the beginning and at the end of the sequence. Transverse cooling from 0.01 to 0.0004(~25×) and 6-D cooling by ~500× was obtained. Extrapolation to a complete cooling scenario seems possible.

V. DISCUSSION

We have obtained simulations of μ-cooling within absorbers in close agreement with rms equations. These simulations cover the range of conditions which occur within a complete collider cooling scenario, and support μ-cooling feasibility. However the simulations do not yet track a complete scenario, and they do not include a complete representation of the nonlinear beam transports and rf acceleration details. An integrated transport + energy loss simulation is needed, and this integrated code should develop into an optimizing design tool. μ-cooling simulations are also being developed by Fernow, Kirk, MacDonald, Palmer, et al.[8, 9] and their contributions will be essential in developing consistent design tools.

Also, there is as yet no experimental verification of ionization cooling, and cooling experiments in agreement with design calculations will be needed to establish the practicality of the μ-cooling concepts and requirements.

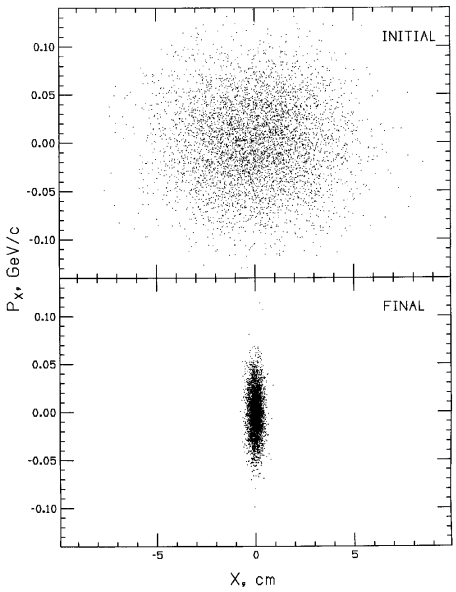

Figure 3: Transverse phase space before and after an 8-rod Be lens cooling system, with wedges for δp control. ε_T is reduced by a factor of 25 and 6-D ε is cooled by 500×

VI. REFERENCES

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