A STANDARD FODO LATTICE WITH ADJUSTABLE MOMENTUM COMPACTION *

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Abstract

An existing lattice made of identical FODO cells can be modified to have adjustable momentum compaction. The modified lattice consists of repeating superperiods of three FODO cells where the cells have different horizontal phase advances. This allows tuning of the momentum compaction or γ_t (transition) to any desired value. A value of the γ_t could be an imaginary number. A drawback of this modification is relatively large values of the dispersion function (two or three times larger than in the regular FODO cell design). This scheme also requires an additional quad bus for the modified cells.

1 INTRODUCTION

Particles travel along the reference orbit in an accelerator ring with momentum p_0 and period of revolution T_0. If they have a momentum deviation Δp, the time of the arrival a the point of observation will be different. An offset in the revolution period ΔT is given by:

$$\frac{\Delta T}{T_0} = \left(\alpha - \frac{1}{\gamma^2}\right)\frac{\Delta p}{p_0}\ , \qquad (1)$$

where α, the *momentum compaction* is a property of the lattice, and $\eta = \alpha - \gamma^{-2}$ is called the *phase-slip factor*; γ is the Lorentz relativistic factor for the on-momentum particle. The momentum compaction factor is a measure of the path length difference between the off-momentum particle and the on-momentum particle. The transition energy γ_t is the energy at which η vanishes, i.e. it equals $1/\sqrt{\alpha}$. In many accelerators γ_t lies in the acceleration range. We shall show that an existing FODO lattice can be modified so as to make γ_t either very large or even imaginary (negative α). This could be used to for example to avoid having to cross transition, or to make zero momentum compaction *isochronous* storage rings.

The momentum compaction of a lattice, to the first order, is an integral of the dispersion function D through the dipoles:

$$\alpha = \frac{1}{C_0}\oint \frac{D(s)}{\rho(s)}ds\ , \qquad (2)$$

where ρ is the radius of curvature and s is the longitudinal path length measured along the reference orbit with a circumference C_0. There are many ways to devise an accelerator lattice with either fixed or adjustable value of the momentum compaction [1],[3],[4],[5]. Vladimirski and Tarasov [1] propose use of reverse bend dipoles to make the momentum compaction negative. Teng [6] shows that a straight section with a phase advance of π can make the dispersion closed orbit negative at dipoles. Iliev [3] and Guignard [4] use a harmonic approach, where the betatron function is modulated to produce negative values of the momentum compaction by way of resonance conditions. We have reported earlier [5] and [8] the use of flexible-momentum compaction lattices to minimize dispersion values.

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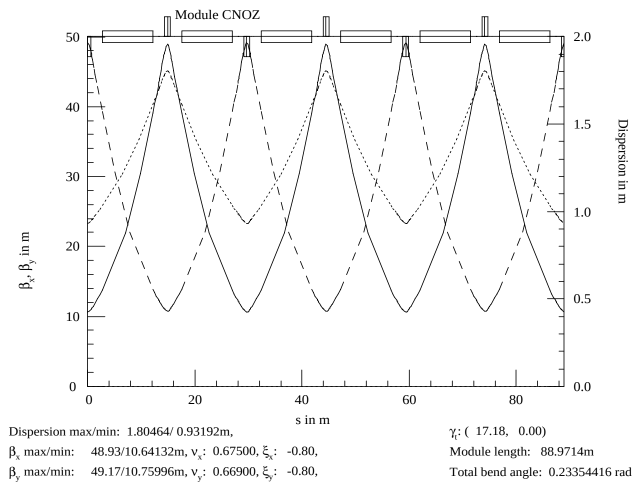

Figure 1: Courant-Snyder functions in the three RHIC cells under normal operating conditions.

2 NORMALIZED DISPERSION FUNCTION

The dispersion function D needs to be adjusted through the FODO cell to obtain a different integral of its values through dipoles. Because the dispersion function satisfies a second order inhomogeneous differential equation of motion [7] it is useful to use the normalized dispersion function with components ξ and χ as previously defined [5]:

$$\xi = \sqrt{\beta_x}D' - \frac{\beta_x'}{2\sqrt{\beta_x}}D,\ \ \chi = \frac{1}{\sqrt{\beta_x}}D, \qquad (3)$$

where β_x and β_x' are respectively the horizontal betatron amplitude function and its derivative [7], ξ and χ are projections of the normalized dispersion vector.

3 FODO CELLS WITH ADJUSTABLE MOMENTUM COMPACTION

As an example we modify the lattice of the Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory, where the dipole length is $L_d = 9.45m$ and the cell length is $L_c \simeq 29.6m$. In our example we use superperiods of three cells as previously proposed by Guignard [4]. Figure 1 shows the Courant-Snyder functions for three cells under the standard operating conditions.

We chose a point of reflection symmetry of the orbit functions at the middle of the three cells. We modify the quadrupole strengths so as to obtain a negative momentum compaction.The quadrupole strengths which make the momentum compaction negative, with $\gamma_t = i358$, are presented in Table 1.

Table 1

Q. Strengths	κ(1/m)
κ_{QDA}	0.05648
κ_{QF}	0.07330
κ_{QD}	0.09634
κ_{QFC}	0.13057

The normalized dispersion plot for this case is shown in figure 2.

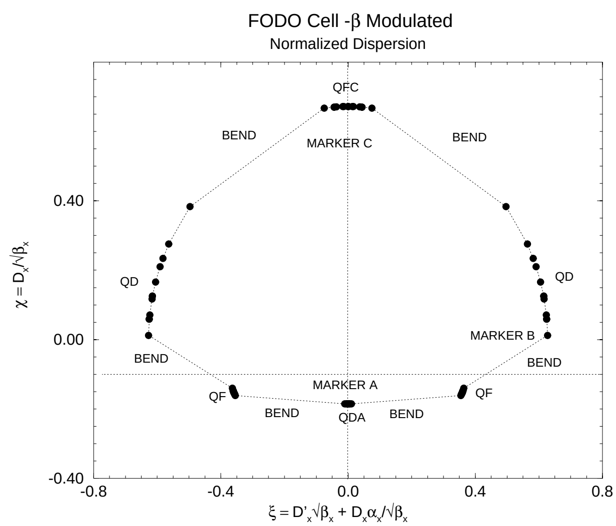

Figure 2: Normalized Dispersion function within the β modulated three FODO cells.

Figure 3 shows the orbit functions β_x, β_y and D in the modified cells (the centers of symmetry are at QDA and QFC).

We see that the penalty paid for making the dispersion negative is almost a doubling of the maximum β_x and β_y function and of the dispersion funcion. In addition the tunes ν_x and ν_y are changed substantially.

4 CONCLUSION

The momentum compaction of the standard FODO cell lattice could be adjusted by the modulating the *betatron functions* to any desired values with the drawback of larger values of the dispersion and betatron functions. A range of dispersion function offsets, obtained by the quad adjustments, falls within twice of the optimum FODO cell dispersion

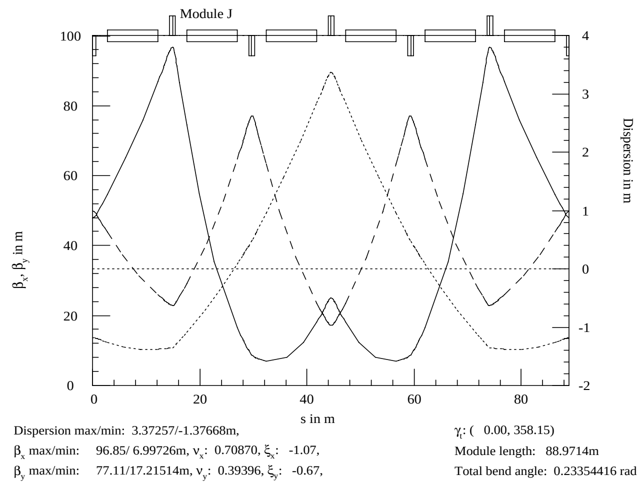

Figure 3: Courant-Snyder functions in three β perturbed cells.

values. The maximum values of β_x and β_y are less than two times the values at optimum betatron tunes ($\nu = \pi/2$). The beam size was less than $\sqrt{2}$ larger. This report shows that a resonance condition ([4], [3]) was not necessary to achieve different values of the momentum compaction within standard FODO cells. We used an existing FODO lattice to accommodate the momentum compaction value, but we do not recommend it the lattice of a new accelerator.

5 REFERENCES

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