DAMPING OF INJECTION OSCILLATIONS

V. M. Zhabitsky
Joint Institute for Nuclear Research, 141980 Dubna, Moscow Region, Russia

Abstract

The increasing dimensions and beam intensities of the new circular accelerators and colliders impose special demands on the performance of injection and transverse feedback systems. Injection errors blow up the emittance of a beam. The emittance increment with active feedback depends on the combination of injection error amplitude, tune spread and extra damping (above the needs for stability) by the feedback. A description of the transverse feedback system (TFS) for damping of injection oscillations is given. The TFS includes a subsystem for damping of transverse instabilities and a pulse subsystem for damping of injection errors with high damping rate. The optimization of the subsystem parameters is discussed, and the results obtained for damping rate of the beam are presented.

1 INTRODUCTION

Transverse feedback system (TFS) is used now to damp not only coherent transverse instabilities but also injection oscillations. As rule, two separate subsystems are used: one subsystem for damping of instabilities and a pulse subsystem for damping of injection errors. Each subsystem consists of one pick-up (PU), delay, filter and power amplifier in feedback path, and a kicker (DK) (see Fig. 1).

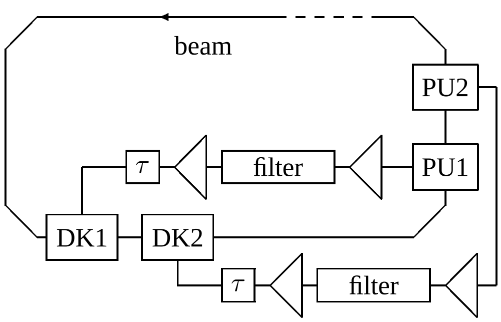

Figure 1: Damper scheme

The kicker corrects the beam angle, and this correction depends on the deviation of the beam from the closed orbit in the PU location. The gain of TFS amplifier is chosen to provide the needs for an amplitude decrease per revolution. All parameters of two subsystems are calculated independently. Clearly, these estimations can be used as a first approximation. Indeed, each kicker corrects the beam angle in accordance with its PU signal. But a change of beam angle per revolution is determined by two corrections produced by two kickers. This change depends also on PU signals due to "feedback via the beam". Hence, the damping rate depends on the feedback gains and betatron phase advances between pick-ups and kickers. Further the theoretical description of the feedback with two subsystems is given.

2 THEORY

2.1 Basic Equation

The arrangement for damping of transverse beam oscillations is sketched in Fig. 1. A pulse subsystem for damping of injection errors consists of a pick-up PU1 and a kicker DK1. Subsystem for transverse instabilities includes a feedback loop with a pick-up PU2 and a kicker DK2. Each pick-up measures bunch transverse deviation and the kicker corrects the beam angle. The kicker should change the angle of the same bunch that was measured by the PU. The delay τ in the feedback loop is adjusted to provide such a synchronization.

Taking into account the results obtained in [1, 2] the study of the transverse coherent motion bunch dynamic is started for independent bunches. In this case the bunch coupling, which occurs due to resistive wall instability, is neglected and the matrix method becomes suitable for the beam motion description.

Let the column matrix $\widehat{X}[n,s]$ determine the bunch state at the n-th turn at point s of the circumference C_0. The first element of this matrix equals the beam deviation $x[n,s]$ from the closed orbit and the second one is $x'[n,s]$. After a short DK the x' value of the beam is changed by $\Delta x'[n,s_K]$, while deviation remains the same as before the DK at point s_K^-. Hence, after DK at point s_K^+, the beam state is

$$\widehat{X}[n,s_K^+] = \widehat{X}[n,s_K^-] + \widehat{T}\Delta\widehat{X}[n,s_K], \tag{1}$$

where $\widehat{T}$ is the 2×2 matrix in which $T_{21}=1$ and the other elements are zero. The kick is determined with column matrix $\Delta\widehat{X}[n,s_K]$, where the first element equals $\Delta x'[n,s_K]$ and the second one has an arbitrary value. It will be assumed further that $\Delta x'[n,s_K]$ is proportional to the beam deviation $x[n,s_P]$ in the pick-up:

$$\Delta\widehat{X}[n,s_K] = \frac{\mathbf{K}}{\sqrt{\beta_P\beta_K}}\widehat{X}[n,s_P], \tag{2}$$

where β_P and β_K are the transverse betatron amplitude functions in the PU and DK locations, and $\mathbf{K}$ is the gain of the feedback loop.

Let us introduce the unperturbed revolution matrix $\widehat{M}_0$ from point s_{P1} of the PU1 location to point $s_{P1}+C_0$, the transfer matrix $\widehat{M}_P$ from point s_{P1} to point s_{P2} of the PU2 location, the matrix $\widehat{M}_1$ from point s_{P2} to point s_{K1} of the DK1 location, the matrix $\widehat{M}_K$ from point s_{K1} to point s_{K2} of the DK2 location, and the transfer matrix $\widehat{M}_2$ from point s_{K2} to point $s_P + C_0$. By using these matrices it is not difficult to calculate the transformation of the bunch state matrix $\widehat{X}[n, s_{P1}]$ after the circumference pass with two angle corrections. For each kick the relations (1) and (2) should be used. Putting together the terms, the beam state at the PU1 location at the $(n+1)$-th turn is then

$$\begin{aligned}\widehat{X}[n+1, s_{P1}] &= \widehat{M}_0 \widehat{X}[n, s_{P1}] + \\ &+ \frac{\mathbf{K}_1}{\sqrt{\beta_{P1}\beta_{K1}}} \widehat{M}_2 \widehat{M}_K \widehat{T} \widehat{X}[n, s_{P1}] + \\ &+ \frac{\mathbf{K}_2}{\sqrt{\beta_{P2}\beta_{K2}}} \widehat{M}_2 \widehat{T} \widehat{M}_P \widehat{X}[n, s_{P1}] . \end{aligned} \quad (3)$$

Eq.(3) fully describes the beam dynamics in an accelerator with two feedback subsystems considered.

2.2 General Solution

Equation (3) is solved using Z-transform [3, 4] for $\widehat{X}[n, s]$:

$$\begin{aligned}\widehat{\mathbf{X}}(z) &= \sum_{n=0}^{\infty} \widehat{X}[n, s] z^{-n} ; \\ \widehat{X}[n, s] &= \sum_k \text{Res}\left[\widehat{\mathbf{X}}(z_k) z_k^{n-1}\right] , \end{aligned} \quad (4)$$

where z_k are the singular points of $\widehat{\mathbf{X}}(z)$. The motion of the particles will be stable if $|z_k| < 1$.

The beam motion parameters are fully determined by the singular points z_k : the number of oscillations per turn $\{\text{Re}Q_k\}$ equals $\arg(z_k)/2\pi$, the damping factor D_k equals $|z_k|$, and the damping time τ_D is

$$\frac{T_0}{\tau_D} = -\ln |z_k|. \quad (5)$$

Using Z–transform for Eq.(3) we get

$$\widehat{\mathbf{X}}(z) = \frac{z\widehat{I} - \widehat{\mathbf{M}}^{-1}(z) \det \widehat{\mathbf{M}}(z)}{\det\left(z\widehat{I} - \widehat{\mathbf{M}}(z)\right)}\, z\widehat{X}[0, s_{P1}], \quad (6)$$

where

$$\begin{aligned}\widehat{\mathbf{M}}(z) = \widehat{M}_0 &+ \frac{\mathbf{K}_1}{\sqrt{\beta_{P1}\beta_{K1}}} \widehat{M}_2 \widehat{M}_K \widehat{T} + \\ &+ \frac{\mathbf{K}_2}{\sqrt{\beta_{P2}\beta_{K2}}} \widehat{M}_2 \widehat{T} \widehat{M}_P . \end{aligned} \quad (7)$$

$\widehat{I}$ is the unit matrix; $\widehat{X}[0, s_{P1}]$ is the initial beam state matrix. The singular points z_k in (6) are found from the equation [3, 5]:

$$\begin{aligned}&\det\left(z_k\widehat{I} - \widehat{\mathbf{M}}(z_k)\right) = \\ &= z_k^2 - z_k \text{Tr}\widehat{\mathbf{M}}(z_k) + \det \widehat{\mathbf{M}}(z_k) = 0. \end{aligned} \quad (8)$$

For damper system with one correction at every turn (for example, $\mathbf{K}_2 = 0$), Eq. (8) for z_k becomes [5]

$$\begin{aligned}&z^2 - (2\cos(2\pi Q_0) + \mathbf{K}_1 \sin(2\pi Q_0 - \psi_{PK}))\, z + \\ &+1 - \mathbf{K}_1 \sin\psi_{PK} = 0, \end{aligned} \quad (9)$$

where Q_0 is the number of unperturbed betatron oscillations per revolution in the transverse plane, and ψ_{PK} is the betatron phase advance from the PU1 to the DK1. The damper with one correction at every turn is known as a classical feedback system that has been used widely in synchrotrons (see, for example, [6]). It is easy to find the roots of Eq.(9). They correspond to the eigen frequencies with the number of oscillations per turn in the neighbourhood of ReQ_0. If $|\mathbf{K}_1| \ll 1$, then in linear approximation the damping time is

$$\frac{T_0}{\tau_D} = \frac{1}{2} |\mathbf{K}_1 \sin\psi_{PK}| .$$

This decrement formula is well known. The best damping will be for PU and DK locations such that

$$|\sin\psi_{PK}| = 1 , \quad (10)$$

i.e. if the phase advance ψ_{PK} from PU to DK equals an odd number of $\pi/2$ radians.

3 RESULTS

In order to simplify the final expressions, all further results are shown for a damper system considered (see Fig. 1) when the relations (10) for the phase advances from PU1 to DK1 and from PU2 and DK2 are fulfilled. In this case Eq. (8) for z_k becomes

$$\begin{aligned}&z^2 - (2 - (\mathbf{K}_1 + \mathbf{K}_2))\, z\cos(2\pi Q_0) + \\ &+1 - (\mathbf{K}_1 + \mathbf{K}_2) + \mathbf{K}_1\mathbf{K}_2 \sin^2\psi_P = 0 , \end{aligned} \quad (11)$$

where ψ_P is the phase advance of the betatron oscillation of the particle on its way from PU1 to PU2. Eq. (11) coincides with the single correction equation (9) if $\mathbf{K}_1$ or $\mathbf{K}_2$ equals zero. The roots of Eq. (11) are

$$\begin{aligned}z_{1,2} &= (1 - \overline{\mathbf{K}}) \cos(2\pi Q_0) \pm \\ &\pm\ i\sqrt{\left(1 - \overline{\mathbf{K}}\right)^2 \sin^2(2\pi Q_0) - \Delta^2} , \end{aligned} \quad (12)$$

where

$$\begin{aligned}\overline{\mathbf{K}} &= \frac{1}{2}(\mathbf{K}_1 + \mathbf{K}_2) , \\ \Delta^2 &= \overline{\mathbf{K}}^2 \cos^2\psi_P + \frac{1}{4}(\mathbf{K}_1 - \mathbf{K}_2)^2 \sin^2\psi_P . \end{aligned}$$

The solutions (12) can be used to compute the damping time and the other beam motion parameters. Thus, the damping time and the number of oscillations per revolution are

$$\frac{T_0}{\tau_D} = -\frac{1}{2} \ln |(1 - \overline{\mathbf{K}})^2 - \Delta^2| , \quad (13)$$

$$Q = Q_0 - \frac{\Delta^2 \cot(2\pi Q_0)}{4\pi |(1 - \overline{\mathbf{K}})^2 - \Delta^2|} . \quad (14)$$

The relation (14) is valid if the deviation of Q from Q_0 is small ($2\pi|Q - Q_0| \ll 1$).

Note, that all basic properties of solutions (12) depend on the phase advance ψ_P and the gains $\mathbf{K}_1$, $\mathbf{K}_2$. If the phase advance ψ_P from PU1 to PU2 equals an integer number of π radians ($\psi_P = n\pi$) then we obtain the same expressions as for a classical feedback system. But the damping rate will be higher because the sum angle correction per turn is provided by two kickers. It is necessary to emphasize that for all feedback gains in such systems ($\psi_P = n\pi$) there is an additional phase advance per turn for the particle betatron oscillations. This negative effect does not allow to use such damper for a fast correction of injection errors during few revolutions. Regime of injection angle suppression per one turn can be realized only if the bunch injected crosses one of the kickers with zero position error and the kick value is strongly proportional to the bunch amplitude error. This regime is used for all injection schemes in synchrotrons but the kicker pulse amplitude is generated in accordance with calculated value by hand and not with a pick-up signal passed on to the feedback processing and power electronics which drives the kicker.

If the phase advance ψ_P from PU1 to PU2 equals an odd number of $\pi/2$ radians ($\psi_P = (2n+1)\pi/2$) then the locations of pick-ups and kickers are the same as for a fast feedback system that has been proposed for UNK-I [7]. Note, that for $\mathbf{K}_1 = \mathbf{K}_2$ the solution (12) coincides with the similar expression for a fast feedback system that has been discussed in [5]. Only for these PUs and DKs locations ($\psi_P = \pi/2$) and gains ($\mathbf{K}_1 = \mathbf{K}_2$) the Δ value equals zero. Hence, in accordance with Eq. (12) and Eq. (14) the tune Q does not depend on the gain and the damping rate will be

$$\frac{T_0}{\tau_D} = -\ln|1 - \overline{\mathbf{K}}| \ .$$

If $|\overline{\mathbf{K}}| = 1$ then injection oscillations are completely cancelled after the kickers' pass and this result does not depend on the betatron phases of a particle crossing PU1, PU2, DK1 and DK2.

The damper with $\Delta = 0$ can be used for a multiturn suppression of injection oscillations. The scheme may be the following. The bunch injected crosses DK1 and DK2 which correct partially the initial oscillation amplitude. The pulse amplitudes for DKs should be calculated by hand in accordance with the initial beam state, the kickers' locations and the degree of the initial oscillation amplitude suppression (for example, on 60%; this value depends on a dynamic aperture in an accelerator). Because two kickers are used with not 100% correction then the pulse amplitude for DKs generators is lower in comparisons with a traditional one kicker injection scheme. After kickers' pass, the feedback is turned on: the pick-ups PU1 and PU2 measure the residual bunch deviation; these values are used in the feedback loops for angle corrections by DK1 and DK2, and so on. This scheme for damping of injection oscillation can be valid for a large hadron accelerators in order to decrease the power generated for injection kickers.

4 CONCLUSION

The consideration of damping regimes allows one to maintain that every turn correction with two subsystems is preferable. The best conditions for damping are achieved for those locations of pick-ups and kickers when the phase advances of the betatron oscillation of the particle on its way between pick-ups and kickers equal an odd number of $\pi/2$ radians. In this case a special regime can be realized for suppressing as initial injection amplitude of oscillations as residual errors and transverse instabilities.

5 REFERENCES

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