

# VACUUM SYSTEM DESIGN USING SYMBOLIC NUMERIC PROCESSORS

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## Abstract

The widespread availability of software for the personal computer (Maple, MathCAD, Mathematica, etc.) provides a convenient means of analyzing the design of vacuum systems for large and small facilities. The overall vacuum system is easily expressed as a matrix equation that includes surface outgassing and vacuum pumping rates and can include such effects as opening and closing of valves. Once the matrix equation has been defined, it is a simple matter to derive pressure solutions as a function of the different variables and to determine the effects of opening or closing valves. Thus one can derive the vacuum pressure in each section of the entire system as a function of these variables and quickly optimize the overall vacuum system design.

## 1. INTRODUCTION

Because of the relative uniqueness of each accelerator facility, the designs of their associated vacuum systems also tend to be unique. Often a great deal of effort goes into the overall system design and relatively little of this accumulated analysis effort can be applied to the following generations of facilities, in some cases because the technology has changed and in other cases because the facilities are not compatible. Some of these vacuum design efforts have been archived into relatively crude vacuum-design programs that are generally too narrowly focused or are difficult to adapt and are not widely used within the community. The purpose of this paper is to introduce a mathematical formulation for the design of a vacuum system that is compatible with analysis using one of the symbolic numeric processors now available for the personal computer.

In the past, this author has been involved in the design of several vacuum systems. Virtually all of these designs were started from scratch because no coherent design framework survived the gap between projects. The vacuum design system was reinvented with each new project, usually in the form of rough calculations performed with the help of a calculator. More recently, some assistance in the form of computer-assisted characterization of various components has helped the design process [1], but these codes are generally limited in their flexibility and the relatively small number of allowed elements is particularly inconvenient for the analysis of the large and complicated systems generally associated with accelerator facilities.

The widespread availability of symbolic numeric processor environments for the personal computer (such as MathCAD [1], Maple [2], Mathematica [3], Macsyma [4], etc.) now provides a convenient framework to support the design of large and complicated vacuum systems. The mathematical model introduced below provides a framework for using symbolic processors to design both simple and complex vacuum systems.

## 2. MATHEMATICAL FRAMEWORK

All vacuum systems can be modeled as a series of chambers or segments comprised of outgassing surfaces with mutual conductances between adjacent segments. In each segment,  $n$ , the mass flow of gas,  $Q_n$  (Torr-liters/sec) is equal to the outgassing from the surfaces of that segment,  $R_n$ , plus the gas conducted in from the adjacent segments minus the gas conducted out into the adjacent segments and minus any vacuum pumping,  $V$ , connected to that segment:

$$Q_n = R_n + (P_{n-1} - P_n) \cdot C_{n-1,n} + (P_{n+1} - P_n) \cdot C_{n+1,n} - V_n \cdot P_n,$$

where  $P_n$  is the pressure in segment  $n$  and  $C_{n,m}$  is the conductance (liters/sec) between segment  $n$  and segment  $m$ . Since  $C_{n,m} = C_{m,n}$ , we can write:

$$Q_n = R_n + C_{n-1,n} \cdot P_{n-1} - (C_{n-1,n} + C_{n+1,n}) \cdot P_n + C_{n+1,n} \cdot P_{n+1} - V_n \cdot P_n.$$

Conservation of mass ensures that these flow rates balance and the net mass flow is zero,  $Q_n \rightarrow 0$ . Therefore the vacuum system can be modeled as the sum of a tri-diagonal conductance matrix,  $\mathbf{C}$ , and a sparse diagonal vacuum pumping matrix,  $\mathbf{V}$ , multiplying a pressure vector,  $\mathbf{P}$ , being equal to a gas-effusion rate vector  $\mathbf{R}$ :

$$(\hat{\mathbf{C}} + \hat{\mathbf{V}}) \cdot \bar{\mathbf{P}} = \bar{\mathbf{R}}. \quad (1)$$

The vacuum pressure vector, and hence the vacuum pressure in each segment of the system, is given by the solution of Equation 1. Conservation of mass also shows up in the conductance matrix by requiring the sum of all elements in a row to be zero. This feature is a useful cross-check to ensure that the conductance matrix has been properly defined.

Since the conductance is generally a strong function of pressure, Equation 1 is non-linear except in the molecular-flow pressure region. The examples used in

this paper are restricted to that regime where the conductance is constant. Since the goal of the vacuum system design is usually to compare the ultimate pressures in the system with the vacuum requirements, the pumpdown characteristics are usually addressed after the design is more or less complete. Hence the solution of Equation 1 in the molecular flow region is the primary focus of this paper.

In principle Equation 1 can be solved in the higher pressure regions by an iterative schemes follows. The conductance at a particular pressure is used to derive an initial conductance matrix. That conductance matrix and the vacuum pumping speed at that pressure are used to derive a new pressure vector using Equation 1. That new pressure vector is then used to derive new conductances etc. This scheme can be used bootstrap the system from atmospheric pressure into the molecular flow region and hence derive the pumpdown characteristics of the system.

Deriving solutions to Equation 1 is relatively simple using one of the symbolic numeric processor packages. The examples used in this paper were produced using MAPLE V, Release 3. Although MAPLE provided quick solutions to the problems posed, Release 3 lacks the capability of saving the results in any format except the MAPLE environment. This feature is particularly inconvenient for translating results into written reports or in producing output that can be copied into other environments such as word processors or spreadsheets. Apparently more recent versions of MAPLE support cross-platform compatibility in the Windows environment.

### 3. EXAMPLES

The first example, illustrated in Figure 1, is a simple 10-element beam transport line with three potential locations for vacuum pumps. The beam line is comprised of 61 cm lengths of 10 cm ID tubing and the pump boxes are 30 cm cubes with 100 l/sec ports for attaching vacuum pumps. The outgassing rates were

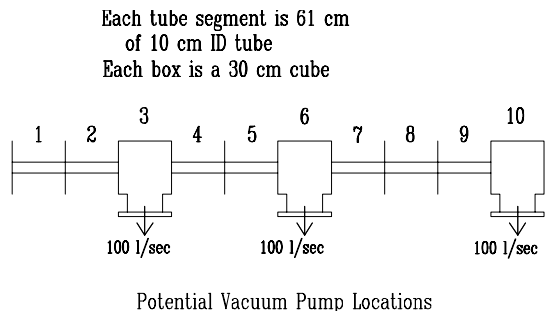


Figure 1. Example 1 beamline consisting of ten segments with potential pumping ports in segments 3, 6, and 10.

taken from Appendix C of reference 5. The pressures expected from placement of a single 100 l/sec pump in segment 3, 6, or 10 are illustrated in figure 2.

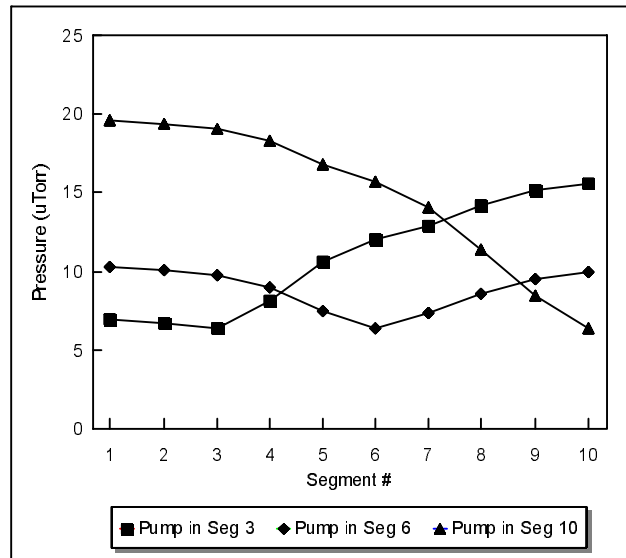


Figure 2. Plot of vacuum pressures of the Example 1 vacuum system for different pump configurations.

The second example is a 70-250 MeV proton beam transport line comprised of two sections, a switching magnet, and in-line valves (Figure 3). The beam exiting a cyclotron is directed by a pair of 30° bends towards the switchyard and proton therapy treatment gantries. The beam transport line is comprised of 6 cm ID tubing with 8 cm by 4 cm rectangular tubing in the dipole magnets. The vacuum system design model is comprised of 43 elements. Each segment in the model consists of an outgassing surface connected to adjacent regions by the effective conductances computed from formulae in reference 5. A closed valve is represented by zero conductance between segments.

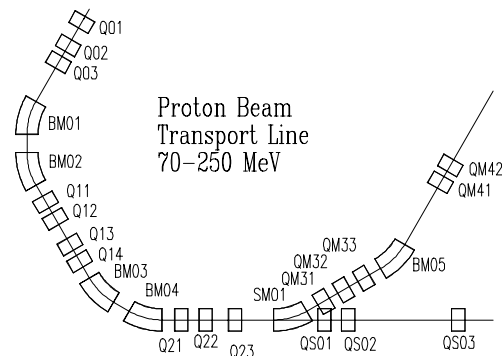


Figure 3. Beamline geometry of example 2. This beamline is comprised of two branches with in-line valves.

The goal of this particular vacuum system design was to minimize the number and size of vacuum pumps while maintaining a low level of scattering from the residual gas atoms in the vacuum.

Equation 1 was modeled in MAPLE using the various conductances, outgassing surfaces, and vacuum pump configurations. Table I provides a matrix of solution parameters and the resulting vacuum profiles are illustrated in figure 4. Figure 4 is notable not because of the information displayed, but because all nine different vacuum configurations were analyzed within a few seconds. The conclusion that can be drawn from figure 4 is that only three vacuum modes are important and the pressure profiles fall within one of these three categories. The final vacuum system design consists of two 100 l/sec vacuum pumps in the Q22 and Q34 segments and no pumping elsewhere in the system.

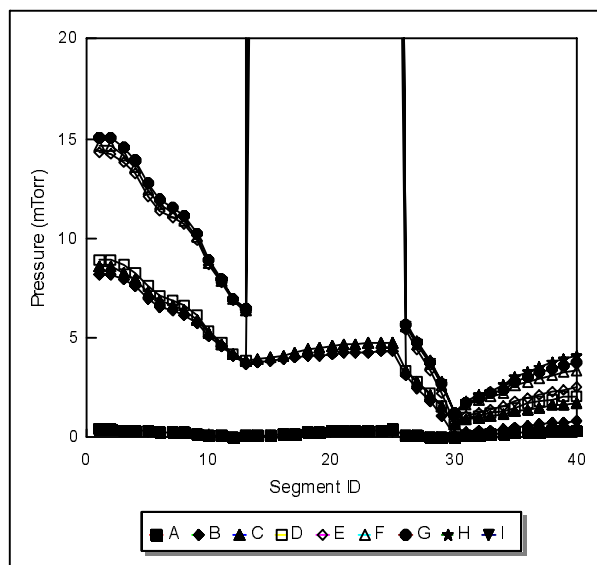


Figure 4. Vacuum pressure solutions of example 2 for the conditions listed in Table I. Segments 13-26 represent the left-hand branch of the beamline while Segments 27-43 represent the straight-through branch. The gap in the left-hand branch pressure values illustrates the result of closing the in-line valve to that branch.

#### 4. SUMMARY AND CONCLUSIONS

A mathematical description of a generic vacuum system was derived. This matrix formulation is easily implemented in one of the symbolic numeric processors available for the personal computer thus facilitating the analysis and design of simple and complex vacuum systems as illustrated in the examples. Although these examples illustrate ultimate vacuum pressures in the molecular flow pressure region, the mathematical framework can be used to derive pump-down characteristics.

Table I. The beamline vacuum system model was solved for the following configurations:

Config ID	Q22 Pump Status	Q34 Pump Status	Valve #1 Status	Valve #2 Status
A	OFF	ON	OPEN	OPEN
B	OFF	ON	OPEN	OPEN
C	ON	OFF	OPEN	OPEN
D	ON	ON	CLOSED	OPEN
E	OFF	ON	CLOSED	OPEN
F	ON	OFF	CLOSED	OPEN
G	ON	ON	OPEN	CLOSED
H	OFF	ON	OPEN	CLOSED
I	ON	OFF	OPEN	CLOSED

#### 5. REFERENCES

- [1]for example VACTRAN, a product of Professional Engineering Computations, Livermore CA
- [2]MathCAD is a product of MathSoft Inc, Cambridge MA
- [3]Maple is a product of Waterloo Maple Software, Waterloo, Ontario Canada
- [4]Mathematica is a product of Wolfram Research Inc, Champaign IL
- [5]Macysma is a product of Macysma Inc, Arlington MA