GAMMA-RAY PRODUCTION USING MULTI-PHOTON COMPTON SCATTERING

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Abstract

The nonlinear Compton scattering cross-section for laser light is formulated by treating the electron in the laser field as a dressed electron, in which the laser field is expressed as classical field. In comparison, multi-photon Compton scattering can be represented by the Feynman diagram where the electron is treated as an undressed one; hence, this formula can be applied only to a low intensity laser. To study the effect of the coherency of the photon on Compton scattering, these two formulas were compared for the case of two-photon scattering. The difference might be due to the fact that the Feynman formula cannot take into account the coherency of the photon. To account for the coherency of a laser field in quantum mechanics, therefore, the formula based on coherent-state formalism is required.

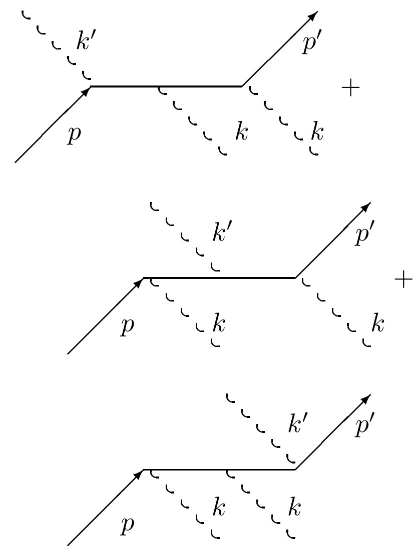
1 INTRODUCTION

There have been many investigations of multiphoton processes in strong electromagnetic fields since the invention of the laser in the early 1960s. Nikishov and Ritus derived formulae for the multiphoton absorption of an electron in a plane electromagnetic field [1]; this effect was observed recently [2]. Other theoretical studies made by the classical approach [3] - [7] and also by the semiclassical scattering theory, as reviewed by Ehlotzky [8], all of which describe multiphoton absorption processes, have contributed to great advances in our understanding. Both the classical approach and the semiclassical theory use the classical way to describe the field of laser beam; therefore, they yield the same results for multiphoton processes [3].

In using quantum theory to describe multiphoton processes in the intense electromagnetic fields of laser beam, one encounters difficulties. The laser beam is a kind of electromagnetic wave which has high density of photons that may be in coherent states; therefore, the usual perturbation theory of Quantum electrodynamics(QED) may be not appropriate for such a case. In the present paper, we calculate the cross-section of Compton scattering of the absorption of two photons by an electron from Feynman diagrams, and compare our results with the semiclassical QED theory [1].

2 FORMULAE OF CROSS-SECTION

We can illustrate the Feynman diagrams for the absorption of two photons by an electron as:



where p and p' are four-vector momentums of the initial and final states of electron, and k and k' are photons. We follow the formulae in ref. [9] to derive the differential cross-section of an unpolarized electron scattering with two unpolarized photons, as shown in the Feynman diagrams, above. The formulae will much simplified if we use the invariant $u = \frac{(kk')}{(kp')}$ and $u_1 = \frac{2(kp)}{m^2}$, then:

$$d\sigma_a = \pi {r_0}^2 \frac{\pi e^2}{2u_1 m^2 \omega} \frac{2\pi}{(1+u)^2} X_a du \tag{1}$$

where r_0 is the classical radius of electron, and,

$$\begin{aligned} X_a = &-\frac{32}{{u_1}^4}u^4 + \frac{112-16u_1}{{u_1}^3}u^3 + \frac{-116+36u_1}{{u_1}^2}u^2 \\ &+\frac{-16+36u_1}{{u_1}^2}u + \frac{16+36u_1}{{u_1}^2} - \frac{16+36u_1}{(1+u){u_1}^2} \end{aligned} \tag{2}$$

To compare our findings with the results in ref. [1], we substitute ω in equation (1) to $\frac{4\pi e^2}{m^2\xi^2}$, as was done in section 101 of ref. [10], where ξ is a parameter of the laser beam's intensity. Then equation (1) becomes:

$$d\sigma_a = \frac{\pi {r_0}^2}{8u_1}\xi^2 \frac{2\pi}{(1+u)^2} X_a du \tag{3}$$

and the total cross-section can be obtained by integral it over u,

$$\sigma_a = \frac{\pi {r_0}^2}{8u_1}\xi^2 \int_0^{2u_1} \frac{2\pi}{(1+u)^2} X_a du \tag{4}$$

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The formulae in ref. [1] are probabilities dw rather than cross-sections $d\sigma$, and the initial photon is linearly polarized. Therefore, by integrating the initial polarization angle ϕ, and dividing the incident flux density, then we obtain:

$$d\sigma_b = \frac{16\pi {r_0}^2}{u_1\xi^2}\frac{du}{(1+u)^2}\int_0^{2\pi} d\phi X_b \tag{5}$$

where,

$$X_b = -{A_0}^2 + \xi^2(2+\frac{u^2}{1+u})({A_1}^2 - A_0A_2) \tag{6}$$

where the functions A_0, A_1, and A_2 are those found in APPENDIX of ref. [1]; here, we consider only the case of the absorption of two photon, in this case, we have,

$$A_0(\alpha,\beta) = \sum_{l=-\infty}^{\infty} J_{2+2l}(\alpha)J_l(\beta) \tag{7}$$

where $J_l()$ is Bessel function, A_1 and A_2 can be calculated by formulae in ref. [1]. The total cross-section of the absorption of two photon can be obtained by integral over u and azimuthal angle ϕ,

$$\sigma_b = \frac{16\pi {r_0}^2}{u_1\xi^2}\int_0^{2\pi} d\phi \int_0^{2u_1} \frac{du}{(1+u)^2} X_b \tag{8}$$

The X_b in equation (5) can be expanded by powers of ξ, and the term of ξ^4 order corresponds to two-photon absorption [11]. Because the formula is for linearly polarized plane waves, therefore we should integrate over azimuthal angle ϕ, then we get the term of the ξ^4 order of equation (6):

$$\begin{aligned} X_c = & -3\frac{u^4}{{u_1}^4} + 10\frac{u^3}{{u_1}^3} - \frac{21u^2}{2{u_1}^2} + 5\frac{u}{u_1} \\ & + \frac{u^2(u+4)}{2u_1(1+u)} - \frac{u^4}{{u_1}^2(1+u)} \end{aligned} \tag{9}$$

and the differential cross-section is:

$$d\sigma_c = \frac{2\pi {r_0}^2}{u_1}\xi^2\frac{2\pi du}{(1+u)^2}X_c \tag{10}$$

the total cross-section can obtained by integral (10):

$$\sigma_c = \frac{2\pi {r_0}^2}{u_1}\xi^2\int_0^{2u_1} \frac{2\pi du}{(1+u)^2}X_c \tag{11}$$

The cross-sections given in above formulae are easy to calculate by computer.

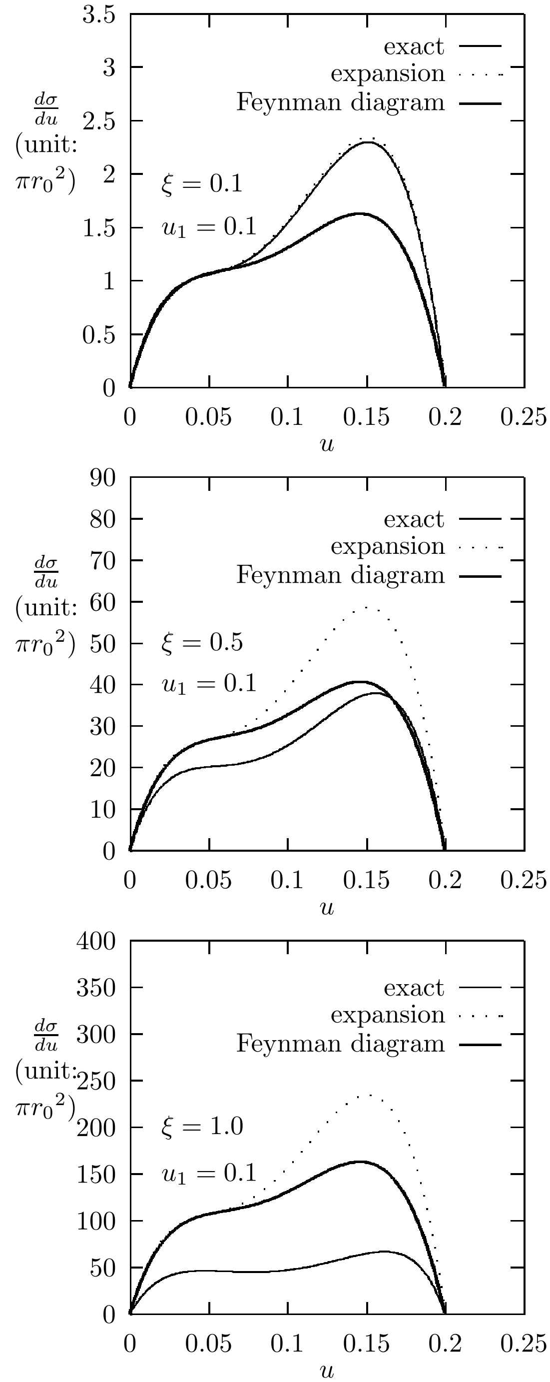

Figure 1: The differnetial cross-sections of an electron scattered by two photons via u.

3 NUMERICAL RESULTS

With the formulae discussed in previous section, we can make numerical calculations. For the sake of simplicity, in the following discussion and figures we use “exact” to denote the results with the semiclassical QED (equation 5). Similarly, we use “expansion” to denote the results from the expansion of “exact” formulae and the term “Feynman diagram” to express the results of solving equation (3). The calculated cross sections are all relative to $\pi {r_0}^2$.

The three graphs in Figure 1 correspond to differential cross-section of $\xi = 0.1$, $\xi = 0.5$, and $\xi = 1.0$ (top to bottom). When $\xi = 0.1$, the “exact” curve and “expansion” curve are very close, but as ξ increases to $\xi = 1.0$ for instant, the discrepancy between them becomes large. The results from “Feynman diagram” also differ from the “exact”; as ξ increases, at first when $\xi < 0.4$ they become close, after then gradually the difference between them also

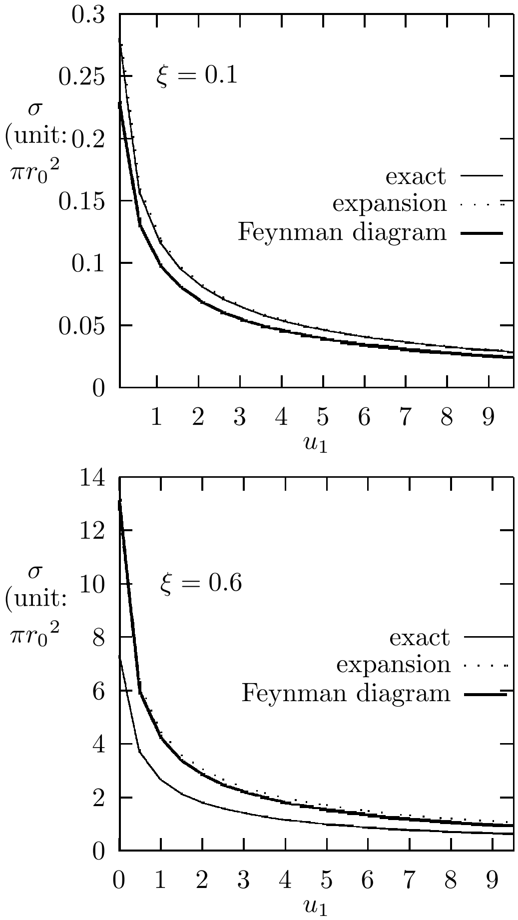

Figure 2: The total cross-sections of an electron scattered by two photons via u_1.

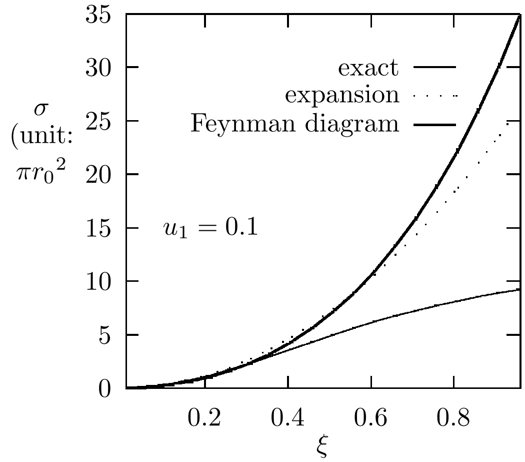

Figure 3: The total cross-sections of an electron scattered by two photons via ξ.

becomes greater. There are no such changes between "expansion" and "Feynman diagram" as ξ increases. These discrepancies supposedly arise from the approximation of the electron state's state to a free state, rather than to a "dressed" state, which means that we have neglected the coherent effects of laser beam. The large ξ signifies the strong intensity of the laser beam which may be related to its strong coherence. However, although there are so many difference between these results calculated by the three methods, they are of the same order of magnitude. Fig. 2 shows the results via another parameter u_1, which is the relative value of the incident particle's energy and the electron's mass. From the figures, we see that the differences between these different theory are not changed by that parameter.

Fig. 3 demonstrates the total cross-section calculated by different theories; their different increases as the intensity of laser beam ξ increases. In the strongest laser beam $\xi \approx 0.6$, either the "expansion" line or the "Feynman diagram" line differ from "exact" line by about 100%. Therefor, if one uses the perturbation theory to consider the intensity field, coherency should be taken into account.

4 CONCLUSION

Our numerical calculations show that the ξ^4 term of the expansion in powers of ξ differs from the results of the perturbation theory of Feynman diagram, but they are in the same order of magnitude. Both the "expansion" and "Feynman diagram" are different from the "exact" results, and the difference becomes larger as ξ increases, which means that the intense laser beam has strong coherent effects.

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