

# ACCELERATION FOR THE $\mu^+\mu^-$ COLLIDER

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## Abstract

We discuss possible acceleration scenarios and methods for a  $\mu^+ - \mu^-$  collider. The accelerator must take the beams from  $\sim 100$  MeV to 2 TeV energies within the muon lifetime ( $2.2 \times 10^{-6} E_\mu / m_\mu \mu\text{S}$ ), while compressing bunches of  $\sim 10^{12}$  muons from m to cm bunch lengths. Linac, recirculating linac, and very rapid-cycling synchrotron approaches are studied. Multiple recirculating linac approaches are matched to the muon lifetime and appear readily feasible. Rapid-cycling approaches require innovations in magnet designs and layouts, but could be much more affordable.

## 1 INTRODUCTION

For a  $\mu^+\mu^-$  collider [1], muons must be rapidly accelerated to high energies while minimizing the kilometers of radio frequency (RF) cavities and magnet bores. Cost must be moderate. Some muons may be lost to decay but not too many. As the muon energy increases and the bunch length decreases, higher frequency, higher gradient RF cavities may be used to reduce cost.

### 2 100 MEV $\rightarrow$ 2 GEV USING RF = 2 GV

This is the initial acceleration of cooled muons. The bunch length decreases from 2 m to 20 cm. A single pass 2 GV linac is used. The RF frequency increases from 10 to 100 MHz from entrance to exit. 93% of the muons survive.

### 3 2 GEV $\rightarrow$ 25 GEV USING RF = 2.5 GV

This is the first recirculating ring and has 2.5 GV of 100 MHz RF [2]. A superconducting magnet with 10 bores, each with a different fixed field, is used to pass the muons through a pair of linacs 10 times. The design is similar to the TJNAF in Virginia. 92% of the muons survive.

### 4 25 GEV $\rightarrow$ 250 GEV USING RF = 6 GV

This stage uses a single ring of fast ramping  $\cos \theta$  dipoles [3]. Thin stranded copper conductor is used at room temperature to achieve a 4 Tesla field. The low duty cycle is exploited to keep the  $I^2R$  losses reasonable. 6 GV of 350 MHz RF is distributed around the ring and accelerates the muons from 25 GeV to 250 GeV in 40 orbits. 85% of the muons survive.

### 5 250 GEV $\rightarrow$ 2 TEV USING RF = 25 GV

For the final stage we consider two 2200m radius hybrid rings [4] of fixed superconducting magnets alternating with

Table 1: Fast ramping  $\cos \theta$  dipole parameters.

Coil inner radius	2 cm
Magnet length	10 m
Field	4 Tesla
Current	29.5 kA
Stored Energy	160 kJ
Inductance	370 $\mu\text{H}$
Coil Resistance	19000 $\mu\Omega$
Ramp time, 10% to 90%	360 $\mu\text{S}$
Power Supply Voltage	31.2 kV
Storage Capacitance	340 $\mu\text{F}$
$I^2R$ magnet heat per cycle	9400 J
Magnet temperature rise per cycle	0.13 $^\circ\text{C}$
Power into magnet @ 15 Hz	141 kW
Number of Dipoles for a ring	144
Total power @ 15 Hz	20 MW

iron magnets ramping at 200 Hz and 330 Hz between full negative and full positive field. Muons are given 25 GV of RF energy (800 MHz) per orbit. The RF is divided into multiple sections as at LEP, so that magnetic fields and energies will match around the rings. The first ring has 25% 8T magnets and 75%  $\pm 2\text{T}$  magnets and ramps from 0.5T to 3.5T during 54 orbits. The second has 55% 8T magnets and 45%  $\pm 2\text{T}$  magnets and ramps from 3.5T to 5.3T during 32 orbits. The packing fraction is taken as 70% in each ring. Acceleration is from 250 GeV/c to 2400 GeV/c and requires a total of 86 orbits in both rings; 82% of the muons survive.

$$\text{SURVIVAL} = \prod_{N=1}^{86} \exp \left[ \frac{-2\pi R m}{[250 + (25 N)] c\tau} \right] = 82\% \quad (1)$$

Consider the power consumption of an iron magnet which cycles from a full -2T to a full +2T. First calculate the energy,  $W$ , stored in a 2T field in a volume 6 m long, .03 m high, and .08 m wide.  $\mu_0$  is  $4\pi \times 10^{-7}$ .

$$W = \frac{B^2}{2\mu_0} [\text{Volume}] = 23\,000 \text{ Joules} \quad (2)$$

Next given 6 turns, an LC circuit capacitor, and a 250 Hz frequency; estimate current, voltage, inductance, and capacitance. The height,  $h$ , of the aperture is .03 m. The top and bottom coils may be connected as two separate circuits to halve the switching voltage.

$$B = \frac{\mu_0 NI}{h} \rightarrow I = \frac{Bh}{\mu_0 N} = 8000 \text{ Amps} \quad (3)$$

$$W = .5 L I^2 \rightarrow L = \frac{2W}{I^2} = 720 \mu\text{H} \quad (4)$$

$$f = \frac{1}{2\pi} \sqrt{\frac{1}{LC}} \rightarrow C = \frac{1}{L(2\pi f)^2} = 560 \mu\text{F} \quad (5)$$

$$W = .5 C V^2 \rightarrow V = \sqrt{\frac{2W}{C}} = 9000 \text{ Volts} \quad (6)$$

Now calculate the resistive energy loss, which over time is equal to 1/2 the loss at the maximum current of 8000 Amps. The 1/2 comes from the integral of cosine squared. A six-turn copper conductor 3 cm thick, 10 cm high, and 7800 cm long has an  $I^2 R$  power dissipation of 15 kilowatts.

$$R = \frac{7800 (1.8 \mu\Omega\text{-cm})}{(3)(10)} = 470 \mu\Omega \quad (7)$$

Now calculate the dissipation due to eddy currents in this conductor, which will consist of transposed strands to reduce this loss [5–7]. To get an idea, take the maximum B-field during a cycle to be that generated by a 0.05m radius conductor carrying 24000 amps. This ignores fringe fields from the gap which will make the real answer higher. The eddy current loss in a rectangular conductor made of square wires 1/2 mm wide with a perpendicular magnetic field is as follows. The width of the wire is  $w$ .

$$B = \frac{\mu_0 I}{2\pi r} = 0.096 \text{ Tesla} \quad (8)$$

$$P = [\text{Volume}] \frac{(2\pi f B w)^2}{24\rho} \quad (9)$$

$$= [.03 .10 78] \frac{(2\pi 250 .096 .0005)^2}{(24) 1.8 \times 10^{-8}} = 3000 \text{ watts}$$

A similar calculation shows that the cooling water tube losses due to eddy currents can be held to 1200 watts. The tubes must be made of a high resistivity material such as 316L stainless steel.

Table 2: Soft magnetic material properties [8].

Material	Composition	B		
		$\rho$ $\mu\Omega\text{-cm}$	Max $H_c$ T	$H_c$ Oe
Pure Iron [9]	Fe 99.95, C .005	10	2.16	.05
1008 Steel	Fe 99, C .08	12	2.09	0.8
Grain-Oriented	Si 3, Fe 97	47	1.95	.1
NKK Super E-Core	Si 6.5, Fe 93.5	82	1.8	
Supermendur [10]	V 2, Fe 49, Co 49	26	2.4	.2
Hiperco 27 [11]	Co 27, Fe 71	19	2.36	1.7
Metglas 2605SC	Fe 81, B 14, Si 3	135	1.6	.03

Eddy currents must be reduced in the iron not only because of the increase in power consumption and cooling,

but also because they introduce multipole moments which destabilize beams. If the laminations are longitudinal, it is hard to force the magnetic field to be parallel to the laminations near the gap. This leads to additional eddy current gap losses [12]. So consider a magnet with transverse laminations as sketched in Fig. 1 and calculate the eddy current losses. The yoke is either 0.28 mm thick 3% grain oriented silicon steel [13] or 0.025 mm thick Metglas 2605SC [14, 15]. The pole tips are 0.1 mm thick Supermendur [10] to increase the field in the gap [16].

$$P(3\% \text{ Si-Fe}) = [\text{Volume}] \frac{(2\pi f B t)^2}{24\rho} = 27 \text{ kw} \quad (10)$$

$$= [6 ((.42 .35) - (.20 .23))] \frac{(2\pi 250 1.6 .00028)^2}{(24) 47 \times 10^{-8}}$$

Similar calculations for the eddy current losses in a Metglas yoke and in Supermendur pole tips yield much lower values, 75 and 210 watts, respectively.

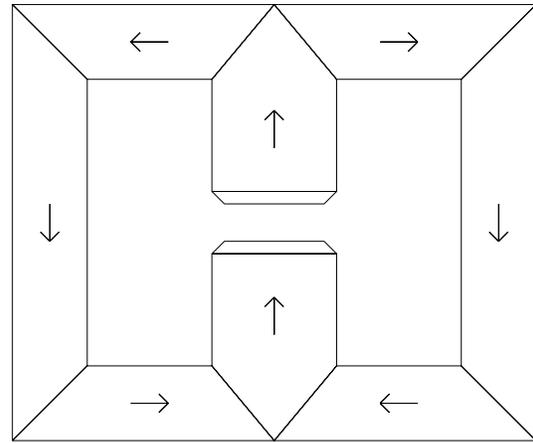


Figure 1: A two dimensional picture of an H frame magnet lamination with grain oriented 3% Si-Fe steel. The arrows show both the magnetic field direction and the grain direction of the steel. Multiple pieces are used to exploit the high permeability and low hysteresis in the grain direction [17]. If Metglas 2605SC is used for the yoke, multiple pieces are not needed, except for the poles. The pole tips are an iron-cobalt alloy for flux concentration exceeding 2 Tesla.

Eddy currents are not the only losses in the iron. Hysteresis losses,  $\int \mathbf{H} \cdot d\mathbf{B}$ , scale with the coercive force,  $H_c$ , and increase linearly with frequency. Anomalous loss [9] which is difficult to calculate theoretically must be included. Thus I now use functions fitted to experimental measurements of 0.28 mm thick 3% grain oriented silicon steel [18], 0.025 mm thick Metglas 2605SC [14], and 0.1 mm thick Supermendur [18].

$$P(3\% \text{ Si-Fe}) = 4.38 \times 10^{-4} f^{1.67} B^{1.87} \quad (11)$$

$$= 4.38 \times 10^{-4} 250^{1.67} 1.6^{1.87}$$

$$= 10.7 \text{ w/kg} = 49 \text{ kw/magnet}$$

Table 3: Magnet core materials.

Material	Thickness (mm)	Density (kg/m <sup>3</sup> )	Volume (m <sup>3</sup> )	Mass (kg)
3% Si-Fe	0.28	7650	0.6	4600
Metglas	0.025	7320	0.6	4400
Supermendur	0.1	8150	0.01	90

$$\begin{aligned}
 P(\text{Metglas}) &= 1.9 \times 10^{-4} f^{1.51} B^{1.74} \quad (12) \\
 &= 1.9 \times 10^{-4} 250^{1.51} 1.6^{1.74} \\
 &= 1.8 \text{ w/kg} = 7.9 \text{ kw/magnet}
 \end{aligned}$$

$$\begin{aligned}
 P(\text{Supermendur}) &= 5.64 \times 10^{-3} f^{1.27} B^{1.36} \quad (13) \\
 &= 5.64 \times 10^{-3} 250^{1.27} 2.2^{1.36} \\
 &= 18 \text{ w/kg} = 1.6 \text{ kw/magnet}
 \end{aligned}$$

Table 4: Power consumption for a 250 Hz dipole magnet.

Material	3% Si-Fe	Metglas
Coil Resistive Loss	15 000 watts	15 000 watts
Coil Eddy Current Loss	4200 watts	4200 watts
Total Core Loss	50 600 watts	9500 watts
Total Loss	69 800 watts	28 700 watts

In summary, a 250 Hz dipole magnet close to 2 Tesla looks possible as long as the field volume is limited and one is willing to deal with stranded copper and thin, low hysteresis laminations. Total losses can be held to twice the I<sup>2</sup>R loss in the copper alone, using Metglas.

The 1925 ramping dipoles which are required consume 56 megawatts when running. Given a 15 Hz refresh rate for the final muon storage ring [1], the average duty cycle for the 250 → 2400 GeV/c acceleration rings is 6%. So the power falls to 4 megawatts, which is small.

## 6 REFERENCES

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Table 5: 4 acceleration stages; 60%  $\mu^\pm$  survival overall.

	Linac	10 Bore SC Magnet	Cu $\cos\theta$ Magnet	Iron & SC Magnets
E(GeV)	0.1→2	2→25	25→250	250→2000
RF(MHz)	10→100	100	350	800
N(turns)	1	10	40	86
RF(GV)	2	2.5	6	25
Length(km)	0.4	0.36	2.3	14
$\tau$ (ms)	0.0013	0.012	0.307	4.0
$\mu$ Bunch(cm)	200→20	6	2	1
$\mu$ Survival	93%	92%	85%	82%

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