

# BEAM DYNAMICS PROBLEMS FOR A $\mu^+\mu^-$ COLLIDER

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## Abstract

A  $\mu^+\mu^-$  collider requires a high-intensity proton source for  $\pi$ -production, a high-acceptance  $\pi$ - $\mu$  decay channel, a  $\mu$ -cooling system, a rapid acceleration system, and a high-luminosity collider ring for the collision of short, intense  $\mu^+\mu^-$  bunches. Significant beam-dynamics problems exist in each of these systems. These problems and some paths to solutions are discussed in this paper.

## 1. INTRODUCTION

Recently considerable interest has developed in the possibility of a high-energy high-luminosity  $\mu^+\mu^-$  Collider.[1,2,3] and a multi-laboratory collaboration has been formed to study this concept. Table 1 shows some possible parameters for a 4 TeV collider with a luminosity of  $L = 10^{35} \text{ cm}^{-2}\text{s}^{-1}$ , as well as parameters of a potential 400 GeV first collider, and Fig. 1 shows a conceptual view of the components of such a facility. The collider requires a high-intensity proton source for  $\pi$ -production, a high-intensity  $\pi$ -production target with a high-acceptance  $\pi$ - $\mu$  decay channel, a  $\mu$ -cooling system to cool the beams to collider requirements, a rapid acceleration system, and a high-luminosity collider ring for the collision of short, intense  $\mu^+\mu^-$  bunches. Each of these components poses significant beam-dynamics problems.

The critical property of muons in a collider is that the muons decay, with a lifetime of  $\tau_\mu = 2.2 (E_\mu/m_\mu) \mu\text{s}$ . This means that  $\mu$ -beam stability is only needed for a few hundred turns in the  $\mu^+\mu^-$  collider, but it also means that obtaining high luminosity requires compressing the muons into ultra-high intensity bunches. The expression for luminosity (for equal intensity round beams) is:

$$L = \frac{f_0 n_s n_b \gamma_\mu N_\mu^2}{4\pi \epsilon_N \beta^*},$$

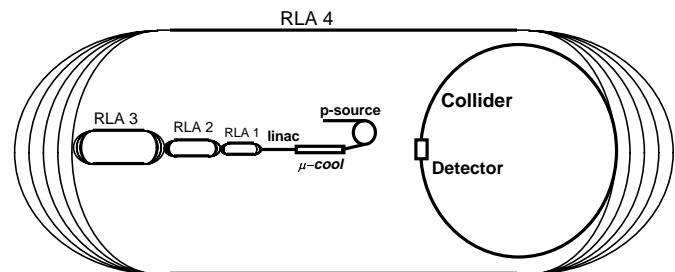
where  $n_s$  is the luminosity lifetime (in turns) in the collider,  $n_b$  is the number of colliding bunches in each beam,  $N_\mu$  is the number of muons per bunch,  $\gamma_\mu = (E_\mu/m_\mu)$ ,  $\epsilon_N$  is the normalized emittance, and  $\beta^*$  is the collider focusing parameter, with the beam size at collisions given by  $\sigma^2 = \epsilon_N \beta^* / \gamma_\mu$ . High luminosity requires maximizing  $N_\mu$  within minimal beam sizes, and that implies significant beam dynamics challenges. In the following sections we will review the beam dynamics problems in each of the successive components of the collider system, referencing more detailed studies and suggested solutions from our collaborators and indicate what we consider the most critical unresolved problems.

The primary difficulties are single-particle beam dynamics problems. However, if these single-particle problems are solved, then multiparticle problems, such as collective instabilities and beam-beam limits, will occur, and these multiparticle beam dynamics problems must also be solved to obtain high luminosity.

**Table 1: Parameter list for  $\mu^+\mu^-$  Colliders**

Parameter	Symbol	Top Demo	4TeV
Collision Energy	$2 E_\mu$	400	4000 GeV
Energy per beam	$E_\mu$	200	2000 GeV
Luminosity	$L=f_0 n_s n_b N_\mu^2 / 4\pi\sigma^2$	$5 \times 10^{32}$	$10^{35} \text{ cm}^{-2} \text{ s}^{-1}$
<i>Source Parameters</i>			
Proton energy	$E_p$	10	30 GeV
Protons/pulse	$N_p$	$2 \times 2.5 \times 10^{13}$	$4 \times 3 \times 10^{13}$
Pulse rate	$f_0$	15	15Hz
$\mu$ acceptance	$\mu/p$	0.2	.2
$\mu$ -survival	$N_\mu/N_{\text{source}}$	0.3	.33
<i>Collider Parameters</i>			
Collider radius	R	120	1200m
$\mu$ /bunch	$N_{\mu\pm}$	$1.5 \times 10^{12}$	$2 \times 10^{12}$
Number of bunches	$n_b$	1	2
Storage turns	$2n_s$	1800	1800
Norm. emittance	$\epsilon_N$	$10^{-4}$	$5 \times 10^{-5} \text{ m-rad}$
$\mu$ -beam emittance	$\epsilon_t = \epsilon_N / \gamma$	$5.3 \times 10^{-8}$	$2.5 \times 10^{-9} \text{ m-rad}$
Interaction focus	$\beta_0$	1	0.3 cm
Beam size at IR	$\sigma = (\epsilon \beta_0)^{1/2}$	23	2.1 $\mu\text{m}$

$\mu^+\mu^-$  Collider Facility



**Figure 1** Overview of a  $\mu^+\mu^-$  Collider facility showing p-source,  $\mu$ -Cooling, recirculating-linac acceleration (RLA) and collider.

## 2. PROTON SOURCE

The collider requires an intense source of protons for  $\pi \Rightarrow \mu$  production. The baseline design requires intensity comparable to that proposed for a KAON factory, but with the significant difference that the beam is bunched to short bunch lengths when extracted onto targets for  $\pi$ -production (4 bunches of  $2.5 \times 10^{13}$  at  $\sim 1 \text{ ns}$  or  $I =$

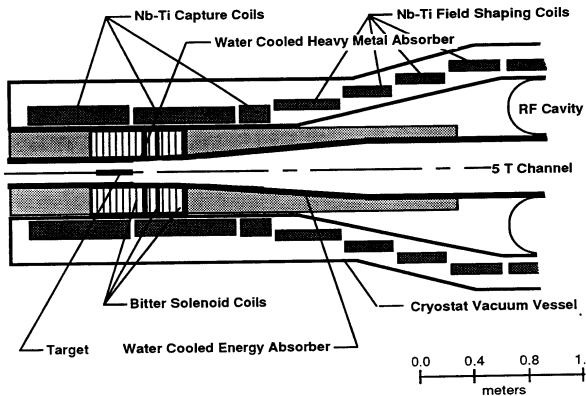
$eNc/(2\pi)^{1/2}\sigma_z \cong 1600$  A peak current). This short bunch would certainly surpass various instability thresholds and strategies to reach this intensity are being developed.[4]

At injection, the key limitation is transverse space charge and the beam would be injected within a long bunch so that the peak current is small enough to keep the total space charge tune shift less than 0.25—0.5. The beam then bunches as it is accelerated, and coherent transverse and longitudinal instabilities become a concern. Longitudinal instability is avoided by placing the beam energy always “below transition”, which means using a flexible-momentum compaction lattice.[5] It may also be possible to cancel space-charge impedances by adding inductive elements to the transport.

Compression to peak current is obtained by bunching at the end of the acceleration cycle or after extraction, with possible combination of separate bunches on the target. Simulations which include space charge and impedance effects are being developed to test these compression conditions. Also experiments on the BNL AGS are being developed in which the limits of bunch compression in an existing ring are explored at parameters near  $\mu^+ - \mu^-$  driver conditions.[6]

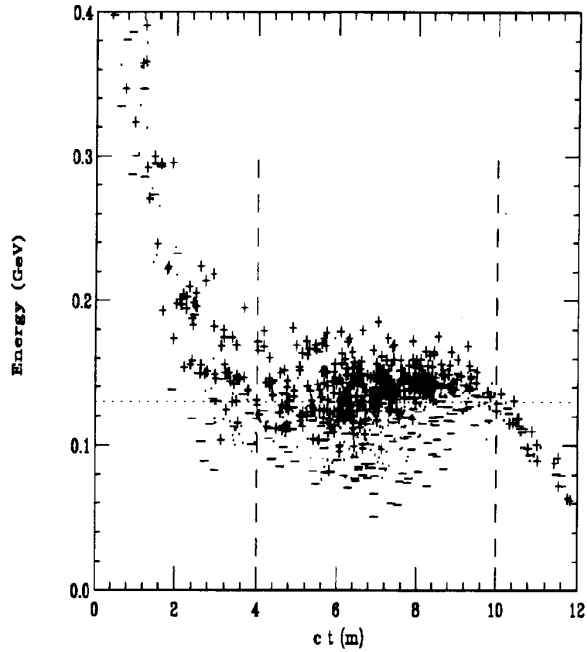
### 3. $\pi$ -PRODUCTION AND $\mu$ -COLLECTION

Another key difficulty occurs in the targetry and collection of secondary  $\pi$  and  $\mu$  beams, where we require collection of  $\sim 0.1$   $\mu$  per primary proton.[7] The current strategy is to immerse the production target in a 20-T solenoid, so that most  $\pi$ 's are trapped. This is followed by a 5T solenoid transport which accepts most of the low energy  $\mu$ 's (100—600 MeV/c) produced by  $\pi$ -decay. (see fig. 2) An rf system within that decay transport reduces the energy spread by “rf rotation”, in which the faster particles decelerate while slower ones accelerate. This transforms the short-bunch beam on target producing a large momentum spread in  $\mu$ 's to a longer  $\mu$ -bunch with reduced  $\delta p/p$ . [1] The key beam dynamics problems here are in developing a high-acceptance transport for the  $\mu$ -beam, both for large  $\delta p/p$  and transverse  $\epsilon_T$ , and in obtaining an appropriate bunch-rotation rf system.



**Figure 2** Capture solenoid and match to transport for  $\pi \rightarrow \mu$  decay + rf rotation (from ref. 1).

Solutions have been found in using a multiharmonic 30—150 MHz rf system embedded in a 5—8T short-period solenoid transport, and verified by simulations. Energy selection in the  $\mu$ -decay can be used to select a relatively high polarization in the  $\mu$ -beams (see fig. 3).[8]



**Figure 3**  $\mu$ -beam at end of rf rotation. + and - signs refer to + and - polarization. (from ref. 8)

### 4. $\mu$ -COOLING

After rf rotation the beam still has both a large momentum spread ( $\delta p/p \cong 10\%$ ) and transverse phase space  $\epsilon_T \cong 0.015$  m-rad). The  $\mu^+ - \mu^-$  Collider concept relies on ionization cooling to compress the beam phase-space volume to obtain high luminosity. This cooling method has been described by Skrinsky et al.[2] and by Neuffer.[3] In ionization cooling, the beam loses transverse and longitudinal momentum while passing through a material medium, and regains only longitudinal momentum in acceleration cavities. Cooling by large factors requires successive stages of energy loss and reacceleration (20 to 50 stages).[1,8] Since ionization cooling does not directly cool the beam longitudinally, these stages must include wedge absorbers at non-zero dispersion to exchange longitudinal and (cooled) transverse phase-space.

The differential equation for rms transverse cooling is:

$$\frac{d\epsilon_T}{ds} = -\frac{1}{\beta^2 E} \frac{dE}{ds} \epsilon_T + \frac{\beta_{\perp} E_s^2}{2\beta^3 m_{\mu} c^2 L_R E} \quad (1)$$

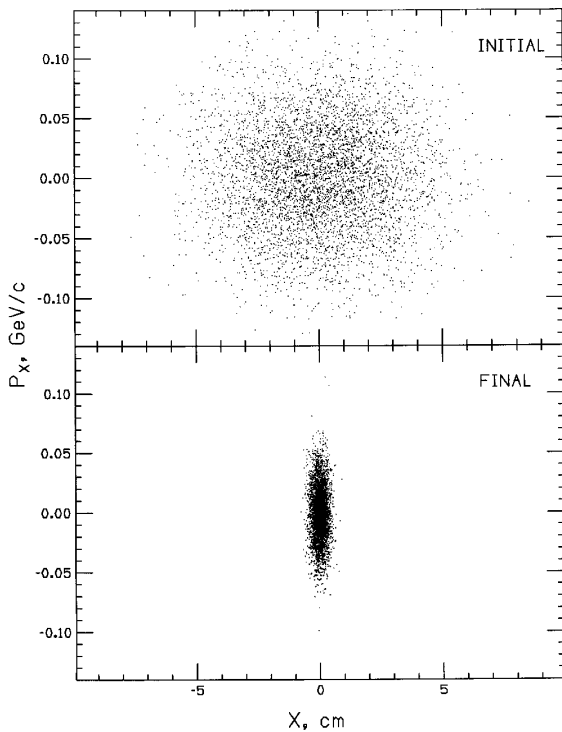
where the first term is the frictional cooling effect and the second is the multiple scattering heating term. Minimal heating requires that  $\beta_{\perp}$ , the betatron focusing amplitude at the absorber, be small, and that  $L_R$ , the absorber radiation length, be large (light elements; i.e. Li or Be). The energy loss mechanism also causes energy-loss straggling, which

naturally sets rms  $\delta p/p$  at the  $\sim 4\%$  level, even with longitudinal cooling.

The beam dynamics problems in  $\mu$ -cooling include the beam-material interactions intrinsic to the cooling process, the single-particle beam transport problems associated with obtaining strong foci at the absorbers, the chromatic effects of  $\sim 4\%$   $\delta p/p$ , dispersion and transverse matching at wedge absorbers, as well as longitudinal motion control with rf reacceleration, and the multiparticle constraints imposed by space-charge and wake-fields in the short intense bunches, where the beam intensifies as it is cooled.

Lattices for cooling have been developed and a favored design includes sequences of solenoid "FOFO" cells with rf cavities and LiH absorbers at low- $\beta$  foci of the lattice. Another desirable focusing situation is obtained by confining the cooling beam within a high-current Li (or Be) rod which both focuses and cools the beam. The transport must include arc segments with wedges for cooling longitudinally; obtaining large  $\delta p/p$  acceptance configurations with cooling and transport stability is nontrivial.[9]

An outline design scenario for  $\mu$ -cooling has been developed, and critical sections of the cooling section have been simulated.[10,11] Figure 4 displays transverse phase space before and after a cooling section which cools transverse phase space by  $25\times$ . [12] However an integrated design including the full complexity of the beam transports, reacceleration and bunching, and including nonlinear beam



**Figure 4** Transverse phase space ( $p_x$ - $x$ ) before and after a cooling channel which reduces  $\epsilon_x$  from 0.01 to 0.0004 m-rad.

dynamics coupled with the ionization interactions has not yet been developed and simulated. Initial cooling experiments verifying cooling efficiency must also be developed.

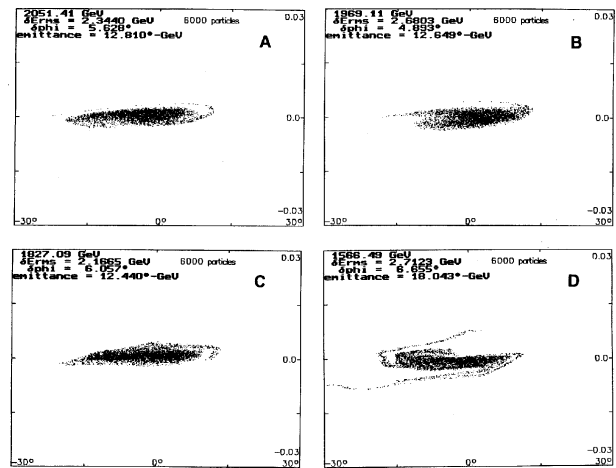
## 5. $\mu$ -ACCELERATION

Acceleration must be completed before  $\mu$ -decay. This constraint can be written as the equation:

$$eV_{rf}' \gg \frac{m_\mu c^2}{L_\mu} \cong 0.16 \text{ MeV/m},$$

where  $eV_{rf}'$  is the acceleration rate, and  $L_\mu$  is the  $\mu$  decay length (660m). Relatively fast acceleration is required, and two alternatives have been developed: recirculating linacs (RLAs) or very rapid-cycling synchrotrons (RCS).[13] In both cases significant challenges exist in obtaining acceleration without phase-space dilution. Simulations show that longitudinal matching is relatively straightforward, and transverse matching is possible.[14] However precise matching in rapid-cycling systems may be difficult, and beam decay within the transport and acceleration must be tolerated. Collimation for  $e$ 's from  $\mu$ -decay will be required, particularly at the entrances of arcs.

Acceleration of high intensity bunches is required. Wake fields from short, high-intensity bunches can be large. From TESLA 1300 MHz cavity calculations, wake fields at the level of  $k_{||} \cong 10 \text{ V/pC/m}$  may be expected.  $k_{||}$  is expected to vary as  $1/a^2$  and  $1/\lambda^2$  and  $1/\sigma^{1/2}$ , where  $a$  is the cavity aperture,  $\lambda$  is the acceleration wavelength and  $\sigma$  is the bunch length. [15,16] Calculations indicate that the wakefield would limit bunch intensities to  $\sim 2 \times 10^{12}$  with 1300 MHz rf in a RLA scenario. The longitudinal dynamics is microtron-like and off-crest acceleration enables compensation of the linear part of the wakefields, with synchrotron-like phase stability.[14] Longer wavelength and large aperture rf systems are favored for maximal bunch intensities.



**Figure 5** Simulation results of RLA acceleration of beam to 2 TeV with wakefields, for  $N_\mu = 0, 1, 2,$  and  $4 \times 10^{12}$  (A, B, C and D, respectively) (from ref. 14).

## 6. $\mu$ -COLLIDER

After acceleration to full energy, the  $\mu^+$ - $\mu^-$  beams are inserted into a storage ring for multiturn collisions at full energy until  $\mu$ -decay. The number of storage turns before decay is  $\sim 300B$ , where  $B$  is the mean ring bending field in T, or  $\sim 2000$  turns at  $B=6.7$  T. High luminosity requires that the beams be focussed to small spots and short bunches at the interaction points (IPs). It also implies high beam densities and that could allow multiparticle instabilities.

The short-bunch requirement ( $\sim 3\text{mm}$ ) implies a nearly isochronous ring to avoid bunch-lengthening and that is obtained by using a flexible momentum compaction lattice for the Collider arcs. In these arcs the dispersion oscillates with an average value near zero, so that the momentum compaction (the variation of path length with momentum)  $\alpha_p = 1/\gamma_T^2 = (\delta C/C)/(\delta P/P)$  is near zero ( $\alpha_p \sim 10^{-5} - 10^{-8}$  in recent designs).

The small focus at the IP ( $\sigma=2\mu$ ,  $\beta=0.003\text{m}$ ) with the geometric and chromatic acceptance requirements is a significant design challenge. A design has been developed which uses final focusing triplets of 10 T quads (10 cm radius), where  $\beta_{\text{max}} = 100\text{km}$ , and chromaticity correction inserts of  $\sim 300\text{m}$  (containing quads, dipoles and sextupoles) on each side of the IP. [17,18] The lattice has adequate dynamic apertures of  $\delta p/p \sim \pm 0.15\%$  with  $5\sigma$  beam amplitudes.

The peak currents associated with 3mm bunches of  $2 \times 10^{12}$   $\mu$  (13000A) pose the possibility of coherent instabilities. For  $\alpha_p \cong 10^{-8}$ , there is no longitudinal motion within the  $\mu$  lifetime and the beam motion is  $e^-$ -linac-like, with the linac length given by the  $\mu$  decay length, with instability modes such as the possibility of head-tail beam breakup, and with possible solutions such as BNS damping.[19]

For  $\alpha_p \cong 10^{-5}$ , there are  $\sim 10$  synchrotron oscillations/ $\mu$ -lifetime. The usual synchrotron stability criteria for longitudinal and transverse impedances  $Z_L$ ,  $Z_\perp$  may be applied:

$$\left| \frac{Z_L}{n} \right| < F \frac{E|\eta|}{e\beta^2 I_{\text{peak}}} \left( \frac{\Delta E}{E} \right)^2 ; \quad Z_\perp < \frac{4\pi m\gamma \beta \omega_\beta c}{e I_{\text{peak}}} \Delta v ,$$

obtaining  $Z_L/n < \sim 0.022\Omega$ , but these are moderated by the finite  $\mu$ -lifetime. [20]

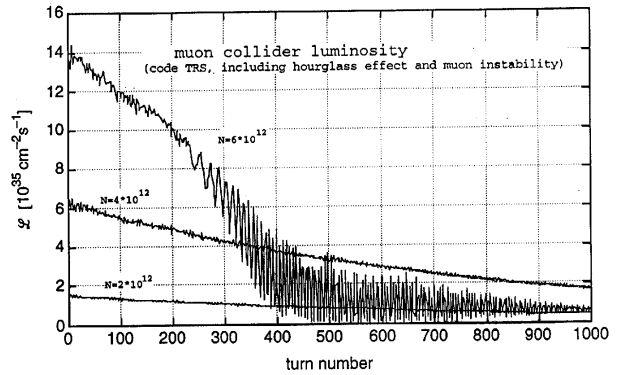
Cheng et al. [21,22] have analyzed potential longitudinal and transverse instabilities in a tracking code that includes linear and non-linear  $\alpha_p$ , wakefields (including resistive-wall, rf cavities and broad-band resonators), and  $\mu$ -decay. They find acceptable dynamics at  $Z_L/n = 0.1\Omega$  with  $\alpha_p = -10^{-5}$ ,  $\delta p/p = 0.15\%$ , and 1GV of 3000MHz rf. (Synchrotron dynamics with negative  $\alpha_p$  was preferable to isochronous motion.) The simulations did not include advanced stabilization methods such as BNS damping, multiharmonic rf, alternating chromaticity, etc., although these may added in the future.

Another critical limitation in collider rings is set by the beam-beam interaction, the nonlinear interaction at the collision points. The beam-beam tune shift  $\Delta v_{\text{BB}}$  is given by:

$$\Delta v_{\text{BB}} = \frac{N_\mu r_\mu}{4\pi\epsilon_N} ,$$

and is chosen to be  $\sim 0.05$  in the parameters of Table 1. Simulations by P. Chen[23] and by M. Furman[24] have shown some hour-glass and disruption effects but general stability with the beam-beam interaction for the  $\mu$ -lifetime. Somewhat larger  $\Delta v_{\text{BB}}$  can be tolerated; figure 6 shows simulation results with  $\Delta v_{\text{BB}}=0.05, 0.10$  and  $0.15$  and only the largest value shows luminosity loss.

Stupakov and Chen[25] and Skrinky[26] have suggested that even larger  $\Delta v_{\text{BB}}$  could be tolerated if the collision points were immersed in a plasma or in solid Li, which would neutralize the beam-beam force. The  $\mu$ -material interaction rates and multiple scattering in the plasma or Li are tolerably small within the  $\mu$ -lifetime; however, detector backgrounds may increase.



**Figure 6** Simulation results of strong-strong beam-beam interactions with  $\mu$ -decay at  $\Delta v_{\text{BB}} = 0.05, 0.10,$  and  $0.15$ . Luminosity is lost only at  $\Delta v_{\text{BB}} = 0.15$ . [24]

## 7. THE FIRST (LOW-ENERGY) $\mu$ -COLLIDER

The first  $\mu^+$ - $\mu^-$  collider would be a lower-energy machine (possibly at  $100 \times 100$  to  $200 \times 200$  GeV), designed both to test the basic concepts as well as to provide significant physics at the Higgs or Top mass, and may be at somewhat lower intensity. Palmer has extrapolated from the high-energy case to set low-energy collider parameters.[27] Assuming equal  $\Delta v_{\text{BB}}$ , equal-aperture IP quads and equal 6-D emittances, it is found that  $\sigma$ ,  $\beta^*$  and  $\epsilon_T$  should vary as  $E_\mu^{-1/3}$ . Instability constraints are relaxed by almost an order of magnitude,[21] and the lower energy makes strong focusing (to a larger  $\beta^*$ ) somewhat easier.  $\beta_{\text{max}}$  at the IP quads is reduced by an order of magnitude to  $< 10\text{km}$ ; which makes the chromaticity correction easier and a special insertion may not be needed. Proton driver and beam-cooling requirements would be somewhat similar for lower and high energy machines, except that the transverse cooling may be reduced by a factor of 2—4. The beam dynamics of lower-energy machines is

overall somewhat easier but still quite difficult, and a low-energy machine would be suitable as an initial research machine.

## 8. DISCUSSION

We have discussed some of the beam dynamics problems associated with the challenge of a  $\mu^+\mu^-$  Collider, and outlined some of the recent research toward solving these. (In this review we have not included discussion of other problems - detector problems, cost issues and radiation control, etc. These are discussed by Palmer.[27]) However the beam dynamics problems are not yet completely defined, and complete solutions are not yet obtained. Much research and invention is needed toward obtaining complete and optimal solutions, and we hope the present discussion will assist in stimulating that research.[28]

We acknowledge the assistance of the many contributors to the  $\mu^+\mu^-$  collider studies, based at BNL, Fermilab, LBL, and other universities and laboratories, including R. Palmer, R. Noble, A. Tollestrup, A. Sessler, J. C. Gallardo, and many others.

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