SATURATION OF THE ION TRANSVERSE INSTABILITY *

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Abstract

Fast Ion Instability is studied in the nonlinear regime. It is shown that exponential growth of the linear regime is replaced in this case by the linear dependence on time. Numeric and analytical results are presented describing the beam profile and the beam spectrum in both regimes.

1 INTRODUCTION

The Fast Ion Instability discovered recently [1] has been studied numerically and confirmed experimentally [2]. The transverse instability is caused by the interaction of a train of bunches with the residual gas. Ions produced by transversely offset bunches in the head of a train induce oscillations of the tail of the train. The ions may be cleared out by a gap after one revolution, but the memory remains in the train. Amplitude of oscillations initially grows as $exp\sqrt{t/t_c}$ until the amplitude of a bunch centroid is of the order of the transverse rms σ of a bunch. The initial rise time of the oscillations of a bunch centroid was found to be a fraction of a millisecond, even taking into account the spread of ion frequencies [3]. This is too fast to be observed in experiments directly.

The exponential regime is limited by the nonlinearity of the beam-ion interaction. As a result, exponential growth at large amplitudes is replaced by a linear dependence of the amplitude on time [4], and only this nonlinear regime can be observed experimentally.

The dynamics of the instability in the nonlinear regime is quite complicated. Additional to the nontrivial interference of the perturbations of the beam by the ions excited by different bunches in the train, the instability in the nonlinear regime essentially depends on the feedback damping and noise in the system while experiments without feedback are hardly possible due to the adverse effects of traditional multibunch instabilities. All that make necessary numerical studies of the instability. Simulations include effects of the feedback and random noise describing the time dependence of the train profile and the beam spectrum.

2 ANALYSIS OF INSTABILITY

Vertical motion of electrons of the n-th bunch on the k-th turn is described by the equation [4]

$$\frac{\partial^2 y(t,z)}{\partial t^2} + \omega_b^2 y(s,z) = \tag{1}$$

$$-\kappa\sigma_y \int_0^z dz' f[y(t,z) - Y(t, ct-z-kC, z')] \tag{2}$$

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where ω_b is betatron frequency,

$$\kappa = \frac{4r_e}{\gamma s_b \sigma_x \sigma_y}\frac{dN_i}{ds}, \tag{3}$$

and dN_i/ds is the ion production rate per bunch proportional to pressure p.

Similarly, the motion of the ions is described by the equation

$$\frac{\partial^2 Y(t,s,z)}{\partial t^2} = \omega_i^2 \sigma_y f[y(t, ct-kC-s) - Y(t,s,z)]. \tag{4}$$

Here ω_i is ion frequency, and function $f(\xi)$ in the RHS of Eq. (2) is either $f(\xi) = \xi/\sigma_y$ for $|\xi| < 1$ (the linear regime), or $f(\xi) = \xi/|\xi|$ for $|\xi| > 1$, in saturation regime, and depends only on the sign of ξ.

In the linear regime, these equations were considered in the original paper [1]. In this case, the solution is

$$y(t,z) = a(t,z)e^{i(\omega_b \pm \omega_i)z/c - i\omega_b t} + c.c., \tag{5}$$

$$Y(t,s,z) = A(t,s,z)e^{-i\omega_b s/c \pm i\omega_i (t-s/c)} + c.c.. \tag{6}$$

The solution grows in time only for the upper sign

$$a(t,z) = a_0 e^{\sqrt{t/t_c}}, \qquad \frac{1}{ct_c} = \frac{\kappa\omega_i z^2}{4\omega_b}, \tag{7}$$

with the quasi-exponentially growth found in the original paper [1]. Correspondingly, the spectrum of the BPM signal

$$V(t) \propto \sum_{n,k} \delta(t - kC - ns_b/c) y(t, ns_b) \tag{8}$$

consists of the betatron side-bands at frequencies $\omega = l\omega_r \mp \omega_b$, $l = 0, 1..$ with the envelope centered at the ion frequency with the lower side-bands having amplitudes larger than that of the upper side-bands and growing in time.

In the nonlinear regime, the RHS in the Eqs. (2) and (4) depends on the function $f(\xi)$. We can expect that the variation of the argument ξ in time is similar to variation of the RHS in the linear regime, that is proportional to $e^{i(\omega_b \pm \omega_i)z/c - i\omega_b t}$ in the equation for $y(t,z)$ and $e^{-i\omega_b s/c \pm i\omega_i (t-s/c)}$ in the equation for $Y(t,s,z)$. In the strongly nonlinear regime, the spectrum of the RHS is a spectrum of a step-function which changes sign with the periods of betatron or ion oscillations. The spectrum of $f(\xi)$ in Eq. (1) and Eq. (4)) contains in this case harmonics of ω_b and ω_i correspondingly. The amplitudes of the harmonics roll off slowly as $1/n$ for the n-th harmonic. In the nonlinear regime, the ions motion is a superposition of ion frequency harmonics. The amplitudes of harmonics don't grow in time but, without the feedback system, their number does. The RHS of Eq. (2) has always a harmonic oscillating with the betatron frequency. As the result, the amplitude of the bunch centroid motion linearly increases in time.

$$A(n_b) \simeq \frac{\kappa s_b^2 n_b n_t \beta_y}{2} \frac{t}{T_r}. \qquad (9)$$

The linear growth described by Eq. (14) replaces the quasi-exponential growth of the linear regime, see Eq. (11), when amplitude is of the order of transverse beam size rms.

3 MODEL FOR SIMULATIONS

To simulate the instability we use a simplified model describing each bunch in a train of n_b bunches as a single macroparticle which goes around the ring in steps equal to s_b. All bunches get a kick from each group of ions at the new location of individual bunches

$$\bar{y}_b' = y_b' - \kappa s_b^2 \beta_y f(y_b - Y_i) \qquad (10)$$

and each group of ions gets a kick

$$\bar{Y}_i' = Y_i' + (\omega_i \tau_b)^2 f(y_b - Y_i). \qquad (11)$$

Each bunch generates an ion macroparticle with the offset equal to the offset of a bunch, and all ions are killed at the location of the ring just left by the last bunch in the train. To model variation of the rms beam size around the ring, the ion frequencies and the kicks to the bunches are periodically modulated with the period equal to $1/12$-th of the circumference of the ring (periodicity of the ALS lattice). The feedback was modeled as a single additional kick for each bunch per turn

$$\bar{y}_b'(s) = y_b'(s) + g y_b(s - \pi\beta_y/2). \qquad (12)$$

The gain g defines amplitude damping time $\tau_d = 2T_r/g$. Random kick uniformly distributed within the range $\pm a_{ns}$ was added to the RHS of Eq. (17) to simulate noise.

Most of the simulations were carried out for the ALS-like ring with the revolution period $T_r = n_t \tau_b$, $n_t = 328$, $\tau_b = 2$ ns, for the bunch train of $n_b = 50$ bunches, and He gas ($A = 4$). The bunch parameters were: $N_b = 4 \times 10^9$, $\sigma_x = 165\mu$, and $\sigma_y = 27\mu$. The betatron tune was $\nu_y = 8.18$, and the ion frequency with these parameters was 50.8 MHz. The pressure was increased to 2μTorr and the damping time of the feedback system to $\tau_d = 0.1$ ms to speed up simulations. Initially there were no ions in the ring, and initial conditions were $y_b = y_b' = 0$ for all but the first bunch, for which initial offset of $y_b = 1.0 \times 10^{-4}$ (in units of σ_y) and $y_b' = 0$ were taken. Results for different amplitude of the noise a_{ns} and modulation $\Delta\omega_i/\omega_i$ are described below.

4 RESULTS OF THE SIMULATIONS

Fig. 1 shows growth of the amplitude of the last bunch in the train with number of turns. Dependence is shown in logarithmic and natural scales. In the left hand side, results are shown with the feedback system turned down ($1/\tau_d = 0$), without noise (amplitude of the noise $a_{ns} = 0$) and with the amplitude of modulation of ion frequencies $\mathrm{mod} = \Delta\omega_i/\omega_i = 0$ or $\mathrm{mod} = 0.5$. Initially, result clearly corresponds to the quasi-exponential growth of the linear regime with the parameter $t_c = 0.41\mu s$ in accordance with Eq. (7. Later, the growth of the dimensionless amplitude is only linear with time and in agreement with Eq. (17), which gives the rate $dA/dn = 0.05$.

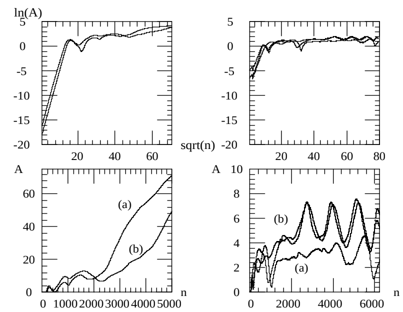

Figure 1: Amplitude of the last bunch vs number of turns in regular (above) and logarithmic (below) scales. Note transition from exponential to linear growth. Left: noise and feedback turned off. (a) mod=0, (b) mod=0.5. Right: (a) $\tau_{fdb} = 0.1$ ms, $a_{ns} = 0.01$, mod=0.5; (b) $\tau_{fdb} = 0.1$ ms, $a_{ns} = 0.002$, mod=0.5; (dots) $\tau_{fdb} = 0.1$ ms, $a_{ns} = 0.002$, mod=0.

Results with the feedback turned on ($\tau_d = 0.1$ ms) are shown in the right hand side of Fig. 1. After initial growth, the amplitude of the last bunch oscillates around some steady level. Effect of the ion frequency modulation in the saturation is small, see two curves without modulation and with the amplitude of modulation $\mathrm{mod} = 0.5$.

The variation of the beam profile can be understood from the following. Initially, the amplitude of a bunch grows according to the linear theory and much faster for the bunches in the tail of the train then in the head. Later, however, the feedback takes over and suppresses oscillations of the bunches in the head of the train to zero amplitudes. As a result, the growth rate and the amplitudes of the following bunches decrease and the bunch number with the amplitude $A = 1$ increases in time. Oscillations with large amplitudes retain only in the very tail of the train and, eventually, all oscillations are damped out.

If we now, additionally to the feedback, turn on the noise, the beam profile goes to a steady-state, see Fig. 2. Without the instability, the equilibrium amplitude of a bunch in units of σ would be

$$A_\infty = \sqrt{y_b^2 + y_b'^2} = \sqrt{a_{ns}^2 \tau_d / 6T_r}. \qquad (13)$$

For the parameters used in simulations, $T_r = 0.656\mu s$ and $\tau_d = 0.1$ ms, this amplitude corresponds to the nonlinear regime $A_\infty > 1$ for the amplitude of the noise $a_{ns} > 0.2$ If the amplitude is smaller than that, the head of the train oscillates in the linear regime, and the transition to the nonlinear regime takes place somewhere closer to the train tail.

With the instability, the beam profile oscillates around almost triangular shape with amplitudes larger in the tail

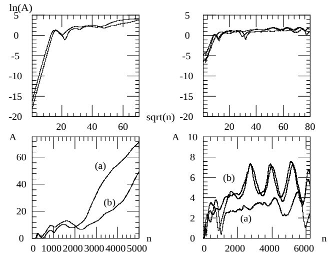

Figure 2: Snap-shot of the beam profile. Amplitude vs bunch number. Vertical scale is blown up 12.5 times. $\tau_{fdb} = 0.1$ ms, mod=0. Case (a) $a_{ns} = 0.01$, case (b) $a_{ns} = 0.002$. Curve $a_{ns} = 0.002$, mod=0 is shown in two cases and is basically the same as in the case (b).

of the train. This beam profile was observed experimentally [5]. The steady-state amplitudes depend on the feedback and are smaller for smaller τ_d. Comparison of the beam profile with different level of the noise shows that the maximum excitation of the beam is not monotonic function of the amplitude of the noise a_{ns} and may be larger for smaller noise although it goes down again at larger a_{ns}. Possible explanation is mentioned above.

The ion frequency modulation reduces the rate of the instability [3]. Effect is quite noticeable in the linear regime, but affects less the steady state amplitudes which are mostly given by the relation between the feedback and the noise.

The beam spectrum at small number of turns has all features of the linear regime: envelope is centered at the ion frequency, $f_i/f_r = 33.3$, see Fig. 3, and the upper side-bands have lower amplitudes then lower side-bands. On the longer time scale, the spectrum changes: more harmonics with frequencies $f_i \pm nf_b$ appear and the ion frequency decreases due to the increase of the amplitudes of ion. In the extreme nonlinear case, ions oscillate in a potential well $U = k_i^2|Y|$ and have frequencies depending on the amplitudes $A = max(y_i)$,

$$\frac{f}{f_r} = \frac{n_t \omega_i \tau_b}{4\sqrt{2A}}, \tag{14}$$

where ω_i is ion frequency in the linear regime. In the nonlinear regime, where the interaction between ions and bunches depend mostly on the sign of the relative position of the bunch and ion centroid, there is no reason to expect that the spectrum is centered around the ion frequency which is typical for the dipole signal of the linear regime. It should be noted, that for relatively low noise level, the head of the train can have small amplitudes corresponding to the linear regime while the tail of the train at the same time may be in the nonlinear regime.

The beam spectrum in the nonlinear regime with feedback and noise initially is much wider than that in the linear regime, Fig. 4, but with time only relatively few harmonics with low frequencies survive.

Calculations with the train of 100 bunches lead to similar results scaled correspondingly with the number of bunches.

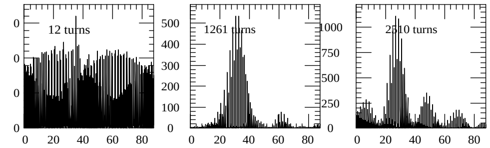

Figure 3: Snap-shot of the beam spectrum. Amplit. vs revolution harmonic number. Feedback, noise, and modulation are off.

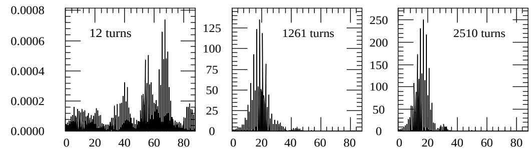

Figure 4: Beam spectrum. $\tau_{fdb} = 0.1$ ms, $a_{ns} = 0.01$, mod=0.5.

5 CONCLUSION

Ion-induced fast transverse instability is constrained by nonlinear effects. Nonlinear effects stop quasi-exponential growth of the amplitude and only the linear with time growth remains. The feedback damping suppresses the bunch oscillations first in the head of the train, effectively reducing the train length and, therefore, the growth rate of the instability. With the noise, the beam takes the typical triangular shape with the profile determined by relation between noise and the feedback. The spectrum of the beam become wider and flatter comparing to the spectrum predicted by the linear theory. Details of the spectrum again depend on the noise and feedback. This may explain unstable character of the spectrum in the experiments.

Acknowledgments

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6 REFERENCES

[1] T. Raubenheimer, F. Zimmermann. Interaction of a Charged Particle Beam with Residual Gas Ions or Electrons, SLAC, January, 1995

[2] J. Byrd, A. Chao, S. Heifets, M. Minty, T. Raubenheimer, J. Seeman, G. Stupakov, J. Thomson, F. Zimmermann, Measurements of a Fast Beam-Ion Instability at the ALS, PAC 1997, Vancouver, Canada

[3] G. Stupakov, T. Raubenheimer, F. Zimmermann. Effect of Ion Decoherence on Fast Beam-Ion Instability, SLAC, January, 1995

[4] S. Heifets, Saturation of the Ion Induced Transverse Blow-up Instability, SLAC-PUB-6959, January 1996

[5] J. Byrd, Preliminary measurements of the FII, PAC 1996