THEORY OF SINGLE BUNCH TRANSVERSE COLLECTIVE INSTABILITIES IN ELECTRON STORAGE RINGS

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Abstract

In this paper, concerning the fast single bunch transverse instability, it is believed that the elimination of the Landau damping is the cause of this type of instability different from the mode-coupling theory. An analytical expression for the threshold current is established.

For a storage ring collider, it is analytically proved that there exists a maximum beam-beam tune shift $\xi_{y,max}$. The analytical expression of $\xi_{y,max}$ is established.

1 INTRODUCTION

The single bunch collective effects in the transverse planes of an electron storage ring can cause fast transverse instabilities when the bunch current surpasses a distinct threshold value. In section 2, the mechanism of this instability is shown to be the elimination of the Landau damping and the corresponding analytical expression of the bunch current threshold is established. In section 3, we will show why there exist a maximum beam-beam tune shift in an electron storage ring collider, and an analytical expression for the maximum beam-beam tune shift is given.

2 SINGLE BUNCH FAST TRANSVERSE COLLECTIVE INSTABILITY

In an electron storage ring the maximum single bunch current is usually limited by a fast transverse bunch size blow-up in the vertical plane when the single bunch current passes an obvious threshold as was observed in PETRA [1] and the other machines. Nowadays, the theoretical explanation to this phenomenon is based on the so-called transverse mode coupling theory originally proposed by Kohaupt [1] and enriched by many others [2][3][4]. However, the stability criterion is very empirical and there are still some ambiguities on the description of the physical pictures. In this section we try to give another explanation to this single bunch transverse collective instability based on the principle of Landau damping.

In a real particle accelerator the mechanism of Landau damping guarantees the stability of the coherent motion of the ensemble of particles in a bunch. The mechanism of Landau damping, however, can be destroyed if the coherent oscillation frequency is shifted outside of the spectrum of the incoherent oscillation frequencies of the ensemble of particles, which is believed to be just the case for the single bunch transverse collective instability. To describe mathematically the process let's look at the vertical betatron oscillation which is more problematic than that in the horizontal plane. Ignoring the variation of β_y with s, one may write

$$y = Bcos\phi \tag{1}$$

$$y' = -\frac{B}{\beta_y}sin\phi \tag{2}$$

where $\phi = s/\beta_y$ and $y' = P_\perp/P_0$. The energy of the coherent vertical betatron oscillation is therefore expressed as

$$E_\perp = \frac{y_c}{\beta_y}E_0 \tag{3}$$

were y_c denotes the amplitude of the collective betatron oscillation and E_0 is the particle energy. The shift of the coherent betatron oscillation frequency can be calculated by using Boltzmann and Ehrenfest theorem which states that for a periodical and linear working lossless engine, the product of energy and the period time is invariant for adiabatic deformation [5], and one has

$$\frac{\Delta\nu_{y,c}}{\nu_y} = \frac{\Delta E_\perp}{E_\perp} \tag{4}$$

where $\nu_y = f_y/f_0$, f_y and f_0 are the vertical betatron and the revolution frequency, respectively. The energy variation in eq. 4 can be easily calculated by using the concept of transverse loss factor $\mathcal{K}_\perp^{tot}$ of the storage ring over one turn, and

$$\Delta E_\perp = \frac{e^2 N_e \mathcal{K}_\perp^{tot}(\sigma_z) y_c}{\nu_y} \tag{5}$$

where N_e is the particle number in the bunch and σ_z is the bunch length. Combining eqs. 3, 4, and 5, one has

$$\Delta\nu_{y,c} = -\frac{e^2 N_e \mathcal{K}_\perp^{tot}(\sigma_z) < \beta_{y,c} >}{E_0} \tag{6}$$

where β_y has been replaced by $< \beta_{y,c} >$ which is the average beta function in the rf cavity region where the transverse wakefield is more important. The dispersion of the incoherent vertical betatron oscillation frequency is expressed as

$$\sigma_{\nu_{y,inc}} = |\xi_{c,y}|\nu_y \frac{\sigma_{\varepsilon 0}\mathbf{R}_\varepsilon}{E_0} \tag{7}$$

where $\sigma_{\varepsilon 0}$ is the natural energy spread, $\mathbf{R}_\varepsilon = \sigma_\varepsilon/\sigma_{\varepsilon 0}$ and $\xi_{c,y}$ is the chromaticity in the vertical plane (usually positive to control the head-tail instability). To shift the coherent frequency totally out of the incoherent frequency spectrum, one needs

$$\Delta\nu_{y,c} = -4\sigma_{\nu_{y,inc}} \tag{8}$$

and one gets finally the instability threshold current

$$I_{b,fast}^{th} = \frac{4 f_y \sigma_{\varepsilon 0}\mathbf{R}_\varepsilon |\xi_{c,y}|}{e < \beta_{y,c} > \mathcal{K}_\perp^{tot}(\sigma_z)} \tag{9}$$

In fact $\mathbf{R}_\varepsilon$ and $\mathcal{K}_\perp^{tot}(\sigma_z)$ are the functions of I_b, eq. 9, therefore, should be solved in a consistent way. It is useful to express $\mathcal{K}_\perp^{tot}(\sigma_z)$ as $\mathcal{K}_\perp^{tot}(\sigma_z) = \mathcal{K}_{\perp,0}^{tot}/\mathbf{R}_z^\Theta$, where $\mathcal{K}_{\perp,0}^{tot}$ is the value at the natural bunch length, $\mathbf{R}_z = \sigma_z/\sigma_{z0}$, and Θ is a constant. The variations of $\mathbf{R}_\varepsilon$ and $\mathbf{R}_z$ with respect to the bunch current I_b can be obtained by solving longitudinal single bunch motion as shown in ref. 6. It is important to note that by increasing the chromaticity one can push the threshold bunch current upwards.

The threshold current from the coupling mode theory is shown as follows [3][4]

$$I_{b,coupling}^{th} = \frac{F f_s E_0}{e < \beta_{y,c} > \mathcal{K}_\perp^{tot}(\sigma_z)} \tag{10}$$

The difference between eq. 9 and eq. 10 comes from that the mode coupling theory requires $\Delta\nu_{y,c} = F\nu_s$ instead of $\Delta\nu_{y,c} = -4\sigma_{\nu_{y,inc}}$, where F is a variable depending on the bunch length.

Now we take the parameters of Super-ACO for example. Using $< \beta_{y,c} >= 5$ m, $E_0 = 800$ MeV, $\nu_y = 1.7$, $\xi_{c,y} = 1$ and $\mathcal{K}_{\perp,0}^{tot} = 303$(V/pC/m)$/\mathbf{R}^\Theta$ with $\sigma_{z0} = 2.4$ cm and $\Theta = 0.7$ [7], it is found that $I_{b,fast}^{th} = 140$ mA as shown in Fig. 1.

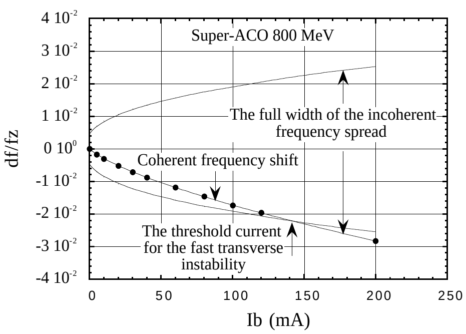

Figure 1: The fast transverse instability is caused by the elimination of the Landau damping (the coherent oscillation frequency is shifted out of the spectrum of the incoherent oscillations).

3 INCREASE OF THE TRANSVERSE BUNCH DIMENSIONS DUE TO BEAM-BEAM EFFECT AND THE MAXIMUM TUNE SHIFT

The luminosity of a circular collider can be expressed as

$$L = \frac{I_{beam}\gamma\xi_y}{2er_e\beta_y^*}\left(1 + \frac{\sigma_y^*}{\sigma_x^*}\right) \tag{11}$$

where r_e is the electron radius, β_y^* is the beta function value at the interaction point, γ is the normalized particle energy, σ_x^* and σ_y^* are the bunch transverse dimensions after the pinch effect, respectively, I_{beam} is the circulating current of one beam, and

$$\xi_y = \frac{N_e r_e \beta_y^*}{2\pi\gamma\sigma_y^*(\sigma_x^* + \sigma_y^*)} \tag{12}$$

is the vertical beam-beam tune shift. Experiments show that when the bunch current is larger than a certain threshold ξ_y will not be able to surpass a maximum value $\xi_{y,max}$. In the following we try to explain this phenomenon and give an anlytical expression for $\xi_{y,max}$.

At each interaction point particles in a bunch will be deflected transversely by the counter-rotating bunch. According to the linear theory of beam-beam dynamics [8], one knows that for two equal charge Gaussian bunches after each collision, the average beam-beam kicks of each particle in the horizontal and the vertical planes are expressed as follows

$$\delta x' = -\frac{2N_e r_e x}{\gamma\sigma_{x,+}^*(\sigma_{x,+}^* + \sigma_{y,+}^*)} \tag{13}$$

$$\delta y' = -\frac{2N_e r_e y}{\gamma\sigma_{y,+}^*(\sigma_{x,+}^* + \sigma_{y,+}^*)} \tag{14}$$

where $\sigma_{x,+}^*$ and $\sigma_{y,+}^*$ are the bunch transverse dimensions just before the interaction point. In fact, these kicks are random and they will move the zero-current equilibrium transverse beam sizes to the new values which depend on the bunch current.

Let's look first at the horizontal plane. Following the arguments in ref. [9], the betatron oscillation is described as follows

$$x = a\sqrt{\beta_x}\cos\phi \tag{15}$$

with

$$a^2 = \frac{1}{\beta_{x,s}}\{x_s^2 + (\beta_{x,s}x_s' - \frac{1}{2}\beta_{x,s}'x_s)^2\} \tag{16}$$

where the subscript s denotes an arbitrary longitudinal position. At the position s there are sudden changes for x_s and x_s' due to the synchrotron quantum radiation excitations:

$$\delta x_s = -D(s)\frac{u_p}{E_0} \tag{17}$$

$$\delta x_s' = -D_s'\frac{u_p}{E_0} \tag{18}$$

where $D(s)$ is the dispersion function and u_p is the energy of one synchrotron radiation photon. Summing all the excitations up, the natural horizontal emittance is found to be

$$\epsilon_{x0} = \frac{\tau_x Q_x}{4} \tag{19}$$

where

$$Q_x = \frac{< \mathcal{N}_p < u_p^2 > \mathcal{H}(s) >}{E_0^2} \tag{20}$$

$$\mathcal{H}(s) = \frac{1}{\beta_{x,s}}\{D(s)^2 + (\beta_{x,s}D_s' - \frac{1}{2}\beta_{x,s}'D(s))^2\} \tag{21}$$

and

$$\tau_x = \frac{2E_0}{J_x < \mathcal{P}_p >} \tag{22}$$

where J_x is the damping partition number, $\mathcal{N}_p$ is the local rate of synchrotron photon emission, and $< \mathcal{P}_p >$ is the average synchrotron radiation power for one turn. For the vertical plane, similarly, one gets

$$\epsilon_{y0} = \frac{\tau_y Q_y}{4} \tag{23}$$

where

$$Q_y = \frac{< \mathcal{N}_p < u_p^2 > \beta_y(s) >}{2\gamma^2 E_0^2} \tag{24}$$

and

$$\tau_y = \frac{2E_0}{< \mathcal{P}_p >} \tag{25}$$

Assuming that there are N_{IP} interaction points in the machine, if one includes in eq. 18 the N_{IP} independent random kicks $\delta x'$ expressed in eq. 13 (similarly in the vertical plane), one gets the new equilibrium horizontal and vertical emittances

$$\epsilon_x = \epsilon_{x0}\left(1 - \frac{(e^2 N_e \mathcal{K}_{IP,BB,x})^2 N_{IP}\tau_x}{4T_0 E_0^2}\right)^{-1} \tag{26}$$

and

$$\epsilon_y = \epsilon_{y0}\left(1 - \frac{(e^2 N_e \mathcal{K}_{IP,BB,y})^2 N_{IP}\tau_y}{4T_0 E_0^2}\right)^{-1} \tag{27}$$

where T_0 is the revolution period, and

$$\mathcal{K}_{IP,BB,x} = \frac{\beta_x^*}{2\pi\epsilon_0 \sigma_{x,+}^*(\sigma_{x+}^* + \sigma_{y,+}^*)} \tag{28}$$

$$\mathcal{K}_{IP,BB,y} = \frac{\beta_y^*}{2\pi\epsilon_0 \sigma_{y,+}^*(\sigma_{x,+}^* + \sigma_{y,+}^*)} \tag{29}$$

For an isomagnetic ring, one gets

$$\epsilon_x = \epsilon_{x0}\left(1 - \frac{3\epsilon_0 R(eN_e\mathcal{K}_{IP,BB,x})^2 N_{IP}}{2m_0c^2\gamma^5 J_x}\right)^{-1} \tag{30}$$

and

$$\epsilon_y = \epsilon_{y0}\left(1 - \frac{3\epsilon_0 R(eN_e\mathcal{K}_{IP,BB,y})^2 N_{IP}}{2m_0c^2\gamma^5}\right)^{-1} \tag{31}$$

where R is the local bending radius.

For a flat bunch ($\sigma_{y,+}^* << \sigma_{x,+}^*$), from eq. 31 one knows that

$$\sigma_{x,+}^*\sigma_{y,+}^* > \left(\frac{3RN_{IP}(eN_e\beta_y^*)^2}{8\pi^2\epsilon_0 m_0 c^2\gamma^5}\right)^{1/2} \tag{32}$$

Defining

$$H = \frac{\sigma_{x,+}^*\sigma_{y,+}^*}{\sigma_x^*\sigma_y^*} \tag{33}$$

where H is a measure of the pinch effect. Keeping in mind the physics of beam-beam effect at the interaction point, one can write

$$H = \frac{H_0\sqrt{N_{IP}}}{\gamma} \tag{34}$$

Combining eqs. 12, 32 and 34 one gets finally

$$\xi_y \leq \xi_{y,max} = \frac{H_0}{2\pi\gamma}\sqrt{\frac{T_0}{\tau_y}} \tag{35}$$

or, for an isomagnetic case

$$\xi_y \leq \xi_{y,max} = H_0\sqrt{\frac{\gamma r_e}{6\pi R}} \tag{36}$$

We have therefore found the analytical expression for $\xi_{y,max}$ and explained the well-known phenomenon in circular colliders that $\xi_y \leq \xi_{y,max}$. From eq. 35 it is clear that for fixed γ and T_0, $\xi_{y,max} \propto 1/\sqrt{\tau_y}$. The experimentally reached maximum H_0 is found to be about $\frac{1}{6}\times 10^6$. In fact, eqs. 35 and 36 are valid for the round beam also.

By using eqs. 35 or 36, $\xi_{y,max}$ can be calculated instead of assumed when a new machine is designed. Taking Beijing Tau-Charm Factory parameters for example [10], one finds $\xi_{y,max} = 0.043$ with $R = 8.58$ m and $\gamma = 3914$ (2GeV).

4 CONCLUSION

A new current threshold expression for the fast transverse instability is established based on the concept of the elimination of the Landau damping which shows the dependance of the threshold current on the chromaticity. For a storage ring collider, it has been proved analytically that there exsits a maximum value for the beam-beam tune shift, and an analytical formaule is given for the $\xi_{y,max}$.

5 ACKNOWLEDGEMENTS

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