

# MEASURING THE RMS LENGTH OF SHORT ELECTRON PULSES WITH AN RF CAVITY AND A BEAM POSITION MONITOR

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## Abstract

The Sub-picosecond Accelerator at Los Alamos National Laboratory is a 1300 MHz, 8 MeV photoinjector. Its beam can be bunched to sub-picosecond lengths using a magnetic chicane. To observe this, we use an rf cavity operating in a  $TM_{110}$  mode to generate a transverse magnetic dipole field that “streaks” each bunch as it passes. Inserting a screen downstream, we determine the pulse length by measuring the increase in the beam spot width. To achieve sufficient resolution two things must happen: first, the unstreaked beam must be well focused at the screen, and, second, the drift after the cavity must be long enough. At high charge and short pulse lengths, these goals become mutually exclusive. To eliminate this problem, we propose using a beam position monitor instead of a screen. The second moment of the beam position monitor signals determines the difference between the x and y rms widths of the beam. With a properly orientated cavity, the resulting change in this quantity also gives the pulse length. However, the BPM does not require a well focused beam at its position, eliminating that constraint on resolution.

## 1 INTRODUCTION

The Sub-picosecond Accelerator (SPA) has compressed electron bunches containing 1 nC of charge to sub-picosecond lengths[1]. However, in those experiments the length of the electron bunch was inferred by measuring the increase in the beam energy spread after it had drifted some distance[1]. Although effective, this technique is not particularly satisfying because it is not a direct measurement of the bunch length.

For low charge beams, 0.1 nC per beam bunch, we found that a very effective technique for measuring the

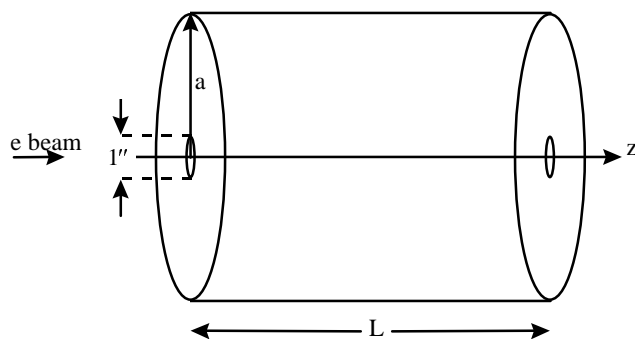


Figure 1: Schematic of fast deflector cavity.

pulse length was to utilize a simple cylindrical cavity like that shown in Figure 1[1]. By operating the cavity in its  $TM_{110}$  mode, the dominate field in the region defined by the 1 inch aperture is a time varying magnetic dipole field. When the timing between the beam pulse and the cavity fields is correct, the head and tail of the pulse are directed in opposite directions, streaking the beam. (Figure 2) Measuring the increase in beam width due to the fast deflector determines the beam bunch length.

The change in the beam spot width that results from the action of the fast deflector cavity depends upon how far the beam drifts after being streaked. For very short bunch lengths, the drift must be quite long to achieve sufficient resolution. However, the beam must also be focused tightly to get a good image for the camera. At low charge (0.1 nC) we found no conflict between these two requirements. However, at high charge (1 nC) we found that the defocusing due to space charge and the higher beam emittance limited this measurement to resolutions that were not adequate for our purposes[1].

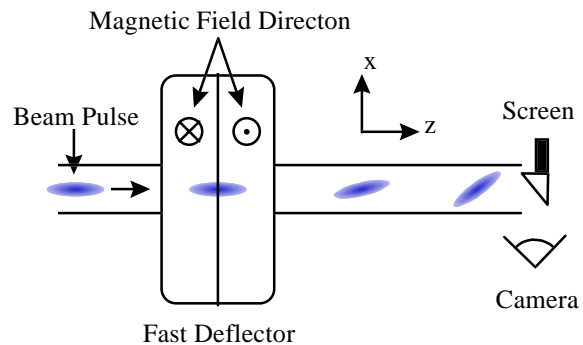


Figure 2: Illustration of fast deflector streaking beam bunch with screen.

To avoid the resolution problem, we intend to replace the screen with a beam position monitor (BPM). (Figure 3) With the fast deflector off, the second moment of the BPM signal determines  $\langle x^2 \rangle - \langle y^2 \rangle$  at the BPM location[2], [3], [4]. The angled brackets indicate an ensemble average. What I will show is that, when the fast deflector is on, this quantity will change to  $\langle x^2 \rangle - \langle y^2 \rangle + a_1^2 \langle \phi_z^2 \rangle$ , where  $a_1^2$  is a constant and  $\langle \phi_z^2 \rangle$  is the ensemble average of the beam bunch length referenced to the phase of the rf. Therefore, the change in the second moment of the BPM signal determines the rms length of the beam. Unlike the screen, however, the BPM does not require a focus at its position. Therefore, as long as the beam does not intercept the pipe walls, the

drift length after the fast deflector can be any desired length.

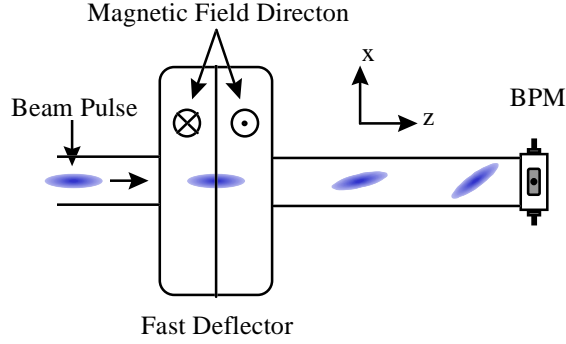


Figure 3: Illustration of fast deflector streaking beam bunch with beam position monitor (BPM).

## 2 CAVITY FIELDS

In this section, I start with the ideal electric and magnetic fields for a cylindrical cavity operating in its  $TM_{110}$  mode. I then change to Cartesian coordinates and make a first order approximation so that the calculation of the motion of an electron through the fast deflector cavity is more manageable.

### 2.1 Ideal fields

The ideal fields for a cylindrical cavity operating in its  $TM_{110}$  mode are

$$E_z = E_0 J_1(k_{11}r) \cos \theta \cos(\omega t + \phi), \quad (1)$$

$$B_r = \omega \frac{a^2}{x_{11}^2 r c^2} E_0 J_1(k_{11}r) \sin \theta \sin(\omega t + \phi) \quad (2)$$

and

$$B_\theta = \omega \frac{a}{x_{11} c^2} E_0 J_1'(k_{11}r) \cos \theta \sin(\omega t + \phi). \quad (3)$$

All other field components are zero. The angular frequency is  $\omega = 2\pi f$  where  $f$  is 1300 MHz. The constant  $k_{11}$  is defined as

$$k_{11} \equiv \frac{x_{11}}{a}$$

where  $x_{11}$ , equal to 3.8317, is the first zero of the first Bessel function. The radius of the cavity,  $a$ , is 14 cm. The length of the cavity,  $L$ , is 14.48 cm. The maximum value of the electric field,  $E_0$ , is 24 MV/m, from measurement. The kinetic energy of the electrons as they enter the cavity will be on the order of 8 MeV. The aperture of the cavity, the opening that the beam travels through, is one inch in diameter, or 1.27 cm in radius.

### 2.2 First order approximation in Cartesian coordinates

Using the relationships

$$\hat{r} = \hat{x} \cos \theta + \hat{y} \sin \theta$$

and

$$\hat{\theta} = -\hat{x} \sin \theta + \hat{y} \cos \theta,$$

I can convert (1), (2) and (3) to Cartesian coordinates. Then, in the 1.27 cm radius aperture region, I can show that the cavity fields are well approximated by

$$E_z \cong E_0 \frac{k_{11}}{2} x \cos(\omega t + \phi), \quad (4)$$

$$B_x \cong \frac{\omega x_{11}}{4ac^2} E_0 xy \sin(\omega t + \phi) \quad (5)$$

and

$$B_y \cong \frac{\omega a}{2x_{11}c^2} E_0 \sin(\omega t + \phi). \quad (6)$$

The magnitude of the ratio of  $B_x$  to  $B_y$  is

$$\left| \frac{B_x}{B_y} \right| \leq \frac{x_{11}^2 r_{\max}^2}{2a^2} = 0.03.$$

It can be shown that the electric field will produce a relative change in an 8 MeV electron's energy of less than four percent. In most cases it will be much smaller than this. Therefore, the only significant field in the aperture region is given by equation (6).

## 3 SOLUTION TO EQUATIONS OF MOTION

In this section, the change in trajectory of a single electron due to the fast deflector is derived. This result is then extended to show the effect of the fast deflector on the value of  $\langle x^2 \rangle - \langle y^2 \rangle$  at the BPM position.

### 3.1 Trajectory change of single electron

Using equation (6), the equation of motion for an electron through the fast deflector is

$$x'' = \frac{e}{\beta \gamma m c} \frac{\omega a}{2x_{11}c^2} E_0 \sin\left(\frac{\omega z}{\beta c} + \phi\right)$$

where  $z=0$  is defined as the beginning of the cavity. This equation is easily integrated, resulting in

$$x_L \equiv x(L) = -\frac{e}{\gamma m} \frac{a}{2x_{11}c^2} E_0 \frac{\beta c}{\omega} \sin\left(\frac{\omega L}{\beta c} + \phi\right) + c_1 L + c_2 \quad (7)$$

and

$$x'_L \equiv x'(L) = -\frac{e}{\gamma m} \frac{a}{2x_{11}c^2} E_0 \cos\left(\frac{\omega L}{\beta c} + \phi\right) + c_1 \quad (8)$$

where

$$c_1 = x'_i + \frac{e}{\gamma m} \frac{a}{2x_{11}c^2} E_0 \cos \phi \quad (9)$$

and

$$c_2 = x_i + \frac{e}{\gamma m} \frac{a}{2x_{11}c^2} E_0 \frac{\beta c}{\omega} \sin \phi. \quad (10)$$

The initial values of  $x$  and  $x'$  at the entrance to the fast deflector are  $x_i$  and  $x'_i$ , respectively. The values of  $x$  and  $x'$  at the fast deflector exit are  $x_L$  and  $x'_L$ .

### 3.2 Effect of fast deflector on $\langle x^2 \rangle - \langle y^2 \rangle$

When the fast deflector is off, the second moment of the BPM signal determines the value of  $\langle x^2 \rangle - \langle y^2 \rangle$  for the beam at the position of the BPM. So, we can define this measurement as

$$M_{\text{FD Off}} \equiv \langle x^2 \rangle - \langle y^2 \rangle.$$

When the fast deflector is on, the value of  $\langle x^2 \rangle - \langle y^2 \rangle$  changes because the fast deflector is now streaking the beam. The goal is to calculate this change using (7), (8), (9) and (10).

When considering a beam and not a single electron, it is convenient to write the phase angle,  $\phi$ , as

$$\phi = \phi_0 + \Delta\phi + \phi_z.$$

The angle  $\phi_0 + \Delta\phi$  is the phase of the beam bunch center with respect to the rf.  $\Delta\phi$  is defined as much less than one and is included for calibration purposes. The angle  $\phi_z$  is defined as the phase of a particular electron in the bunch with respect to  $\phi_0 + \Delta\phi$ .

If the electron beam bunches are short, as is true for the SPA beam, then

$$|\Delta\phi + \phi_z| \ll 1.$$

If we assume that the beam line between the fast deflector cavity and the BPM is linear, with the transfer matrix

$$\bar{\bar{\mathbf{R}}} = \begin{bmatrix} R_{11} & R_{12} & 0 & 0 \\ R_{21} & R_{22} & 0 & 0 \\ 0 & 0 & R_{33} & R_{34} \\ 0 & 0 & R_{43} & R_{44} \end{bmatrix},$$

then it can be shown that  $\langle x^2 \rangle - \langle y^2 \rangle$  changes to

$$\langle x^2 \rangle - \langle y^2 \rangle + a_1^2 \langle \phi_z^2 \rangle$$

where

$$a_1 = \frac{e}{\gamma m} \frac{a}{2x_{11}c^2} E_0 \left\{ R_{11} \frac{\beta c}{\omega} \left[ \cos \phi_0 - \frac{\omega L}{\beta c} \sin \phi_0 - \cos \left( \frac{\omega L}{\beta c} + \phi_0 \right) \right] + R_{12} \left[ \sin \left( \frac{\omega L}{\beta c} + \phi_0 \right) - \sin \phi_0 \right] \right\}. \quad (11)$$

Therefore, the measurement with the fast deflector is on is

$$M_{\text{FD On}} \equiv \langle x^2 \rangle - \langle y^2 \rangle + a_1^2 \langle \phi_z^2 \rangle = M_{\text{FD Off}} + a_1^2 \langle \phi_z^2 \rangle.$$

So,

$$\langle \phi_z^2 \rangle = \frac{M_{\text{FD On}} - M_{\text{FD Off}}}{a_1^2}.$$

## 4 ESTIMATE OF MEASUREMENT RESOLUTION

Assume that the beam line between the fast deflector and the BPM is a drift of length  $d$ . The value of  $a_1$  can be calculated from (11). Assuming that the longitudinal distribution of the beam is Gaussian, the value of  $a_1^2 \langle \phi_z^2 \rangle$  can be estimated for different drift lengths and different pulse lengths. The results are shown in Table 1.

It is expected that we will be able to measure the value of  $M_{\text{FD Off}}$  and  $M_{\text{FD On}}$  to  $\pm 0.5 \text{ mm}^2$ . Therefore, with a drift length of 2 meters, it is not unreasonable to expect a resolution of 1 picosecond.

d	FWHM of Beam Bunch (picoseconds)			
	20 ps	10 ps	1 ps	0.1 ps
0.5 m	13.65	3.41	0.03	0.000341
1.0 m	52.67	13.17	0.1317	0.001317
1.5 m	117.1	29.28	0.2928	0.002928
2.0 m	206.9	51.73	0.5173	0.005173

Table 1: Value of  $a_1^2 \langle \phi_z^2 \rangle$  (in  $\text{mm}^2$ ) versus drift length,  $d$  (in meters), and the full width at half maximum (FWHM) pulse length of the beam.

## 5 ACKNOWLEDGEMENTS

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## REFERENCES

- [1] B. E. Carlsten and S. J. Russell, "Subpicosecond Compression of 0.1-1 nC Electron Bunches with a Magnetic Chicane at 8 MeV," *Physical Review E*, Volume 53, Number 3, p. 2072-2075.
- [2] R. H. Miller et al., "Non-intercepting Emittance Monitor," Proc. 12<sup>th</sup> Int. Conf. On High Energy Accelerators, (Fermilab, 1983), p. 602 (1983).
- [3] S. J. Russell and B. E. Carlsten, "Measuring Emittance Using Beam Position Monitors," Proc. of the 1993 Part. Accel. Conf., p. 2537, IEEE 93CH3279-7.
- [4] S. J. Russell, "Preliminary Results of RMS Emittance Measurements Performed on the Sub-picosecond Accelerator using Beam Position Monitors," (these proceedings.)