Automatic Emittance Measurement At The ATF *

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Abstract

An automatic emittance measurement system to characterize the transverse emittance of the electron beam produced by the BNL photocathode electron gun is described. The system utilize a VAX workstation and a Spiricon beam analyzer. A operator window (created through the Vista control software package) controls the emittance measurement system and the graphic presentation of the results. Quadrupole variation method is used for the ATF automatic emittance measurement system. A simple emittance formula was derived to study the performance of the quadrupole variation method, and compared with the ATF experimental data is also presented.

I. VARIABLE QUADRUPOLE METHOD

The experimental program at the Brookhaven National Laboratory Accelerator Test Facility (ATF) requires rapid and accurate characterizing the electron beam produced by the ATF photocathode RF gun. An automatic emittance measurement system was developed at ATF to characterization of the electron beam transverse emittance at both 50 MeV and 4.5 MeV.

Using TRANSPORT notation, a 2-dimension transverse phase space of the uncoupled particle beam is described by the beam matrix σ,

$$\sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix}. \tag{1}$$

where $\sigma_{12} = \sigma_{21}$, and σ is a positive definite matrix. The one-σ geometric emittance is,

$$\epsilon = \left(\sigma_{11}\sigma_{22} - \sigma_{12}^2\right)^{1/2}. \tag{2}$$

It can be seen from Eq. (2) that the emittance measurement usually involves determining three parameters. Several techniques [1] have been widely used to measure the transverse emittance. The variable quadrupole method is one of them. It obtains the emittance by measuring beam sizes while varying an up-stream quadrupole magnet. The measured beam sizes ($\sqrt{\sigma_{11}^M}$) are connected to the beam matrix at the quadruple through the transfer matrix R,

$$R = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix}. \tag{3}$$

and

$$\sigma_{11}^M = R_{11}^2\sigma_{11}^Q + 2R_{11}R_{12}\sigma_{12}^Q + R_{12}^2\sigma_{22}^Q. \tag{4}$$

where σ_{ij}^Q are elements of the beam matrix at the quadrupole.

For n measurements ($n > 3$),[2] the Eq. (4) can be expressed,

$$\Sigma^M = A\Sigma^Q. \tag{5}$$

where

$$\Sigma^M = \begin{pmatrix} \sigma_{11}^{1M} \\ \vdots \\ \sigma_{11}^{nM} \end{pmatrix} \quad and \quad \Sigma^Q = \begin{pmatrix} \sigma_{11}^Q \\ \sigma_{12}^Q \\ \sigma_{11}^Q \end{pmatrix}$$

and

$$A = \begin{pmatrix} R_{11}^2(1) & 2R_{11}(1)R_{12}(1) & R_{12}^2(1) \\ \vdots & \vdots & \vdots \\ R_{11}^2(n) & 2R_{11}(n)R_{12}(n) & R_{12}^2(n) \end{pmatrix}.$$

Using the linear least squares fitting method, the solution of the beam matrix Σ^Q is,

$$\Sigma^Q = \left(A^T A\right)^{-1} A^T \Sigma^M. \tag{6}$$

II. ERROR ANALYSIS

Eq. (6) shows that the calculation of the beam matrix in the variable quadrupole method is fairly straight forward, it involves only matrix inversion and multiplication. We will study the possible sources which may affect the emittance obtained using the variable quadrupole method.

Error caused by the resolution of the beam width measurement. For a Gaussian approximation of the beam width, the measured beam width σ_m can be expressed, [3]

$$\sigma_m^2 = \sigma_r^2 + \sigma_{sys}^2. \tag{7}$$

where σ_r is the ideal beam width while σ_{sys} is the system error of the beam width which includes optics and electronic errors. Then the measured emittance using the variable quadrupole method is:

$$\epsilon_m = \sqrt{\frac{\sigma_{11}^Q\sigma_{sys}^2}{R_{12}^2} + \epsilon_r^2}. \tag{8}$$

The above equation shows that the emittance error can be reduced either by decreasing the systematic error σ_{sys}, or increasing the the R_{12}, which is the drift distance after the quadrupole for a simple variable quadrupole method.

Error contributed by the transfer matrix. To simplify the calculation, the thin lens approximation will be used for the quadrupole. Then the transfer matrix R for the focusing plane is:

$$R = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix} = \begin{pmatrix} 1-\frac{L}{f} & L \\ -\frac{1}{f} & 1 \end{pmatrix}. \tag{9}$$

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where f is the focal length of the quadrupole, and L is the drift distance between the exit of the quadrupole and the screen of the beam profile monitor.

Considering only three different quadrupole settings, Eq. (5) can be rewritten,

$$\left(1-\frac{L}{f_1}\right)^2 \sigma_{11}^Q + 2L\left(1-\frac{L}{f_1}\right)\sigma_{12}^Q + L^2\sigma_{22}^Q = b_1$$
$$\left(1-\frac{L}{f_2}\right)^2 \sigma_{11}^Q + 2L\left(1-\frac{L}{f_2}\right)\sigma_{12}^Q + L^2\sigma_{22}^Q = b_2$$
$$\left(1-\frac{L}{f_3}\right)^2 \sigma_{11}^Q + 2L\left(1-\frac{L}{f_3}\right)\sigma_{12}^Q + L^2\sigma_{22}^Q = b_3.$$

where b_i are squares of the measured beam sizes. Subtracting the third equation from the first and second equations eliminates the coefficients of σ_{22} from those two equations. The above equations can be written in matrix form $A\sigma = B$, where $\sigma^T = (\sigma_{11}, \sigma_{12}, \sigma_{22}), B^T = (b_1 - b_3, b_2 - b_3, b_3)$, and,

$$A = \begin{pmatrix} \left(2 - L\frac{f_1+f_3}{f_1 f_3}\right)\frac{f_1-f_3}{f_1 f_3}L & 2L^2\frac{f_1-f_3}{f_1 f_3} & 0 \\ \left(2 - L\frac{f_2+f_3}{f_2 f_3}\right)\frac{f_2-f_3}{f_2 f_3}L & 2L^2\frac{f_2-f_3}{f_2 f_3} & 0 \\ \left(1-\frac{L}{f_3}\right)^2 & 2L\left(1-\frac{L}{f_3}\right) & L^2 \end{pmatrix}.$$

Inverting the matrix A, the beam matrix σ_{ij} can be found by the matrix multiplication. The general expression is long and tedious. But If we choose three special points such that $f_1 = f_3 - \Delta f, f_2 = f_3 + \Delta f$, and $b_1 - b_3 = b_2 - b_3 = \Delta b$, then the emittance can be obtained by,

$$\epsilon = \frac{\sigma_{min}}{L^2}\left(\Delta b \frac{f_1 f_2 f_3}{\Delta f}\right)^{\frac{1}{2}}. \quad (10)$$

where $\sigma_{min} = \sqrt{b_3}$. In choosing three points satisfy above condition, b_3 is the minimum spot size when scanning the quadrupole. Eq. (10) shows that,

1. The measured emittance is linearly proportional to the measured minimum spot size. Fig. 1 was generated by changing only the minimum spot size using experimental data from ATF (courtesy of D.P. Russell). It agrees with the prediction of Eq. (10).
2. Eq. (10) shows the measured emittance also strongly depends on the focal lengths of the quadrupole. This dependency will affect mainly the accuracy of the measurement in the low energy situation. Our experience showed that the residual field of the magnet could also affect the experiment result.
3. Eq. (10) shows that the space charge could make the measured emittance smaller since stronger quadrupole (shorter focal length) will be used to compensate the defocusing force of the space charge.

Chromatic effect. The chromatic effect will also contribute to the error in the emittance analysis. Following simple formula [4] can be used to estimate the chromatic effect:

$$\delta\epsilon = \frac{\sigma_{11}^Q}{f}\left(\frac{\delta p}{p}\right)_{rms}. \quad (11)$$

where $\delta p/p$ is the relative momentum spread. It can be seen from Eq. (11) that the chromatic effect will be smaller if a weaker quadrupole is used.

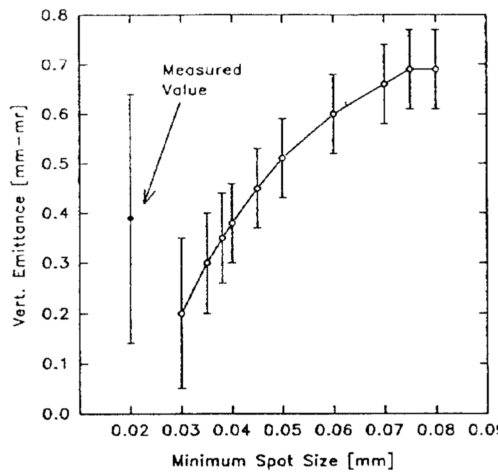

Figure 1: The relation between the emittance and minimum spot size.

III. THE ATF AUTOMATIC EMITTANCE MEASUREMENT SYSTEM

The variable quadrupole method was used for the ATF automatic emittance measurement system because its simplicity and versatility. The ATF automatic emittance measurement system is part of the ATF computer control complex. It is not only capable of processing the data and compute the emittance, but also monitor both the laser and electron beams.

Fig. 2 shows a schematic representation of the ATF emittance measurement system. The ATF control system was built around a VAX 4200 computer, which is connected CAMAC data acquisition hardware by serial highway. The Spiricon beam analyzer was connected to the VAX through a standard IEEE 488 interface card. Spiricon beam analyzer was designed for laser beam applications, it can also be used to process the electron beam images from the beam profile monitor(BPM). The ATF BPM[5] has a resolution of 40 μm. The Spiricon beam analyzer is pre-triggered by the ATF master timing system, the video camera of the BPM (PULNiX TM-745E) is synchronized with the electron beam by the sync. signals generated by the Spiricon beam analyzer. The automatic emittance measurement system is controlled by an

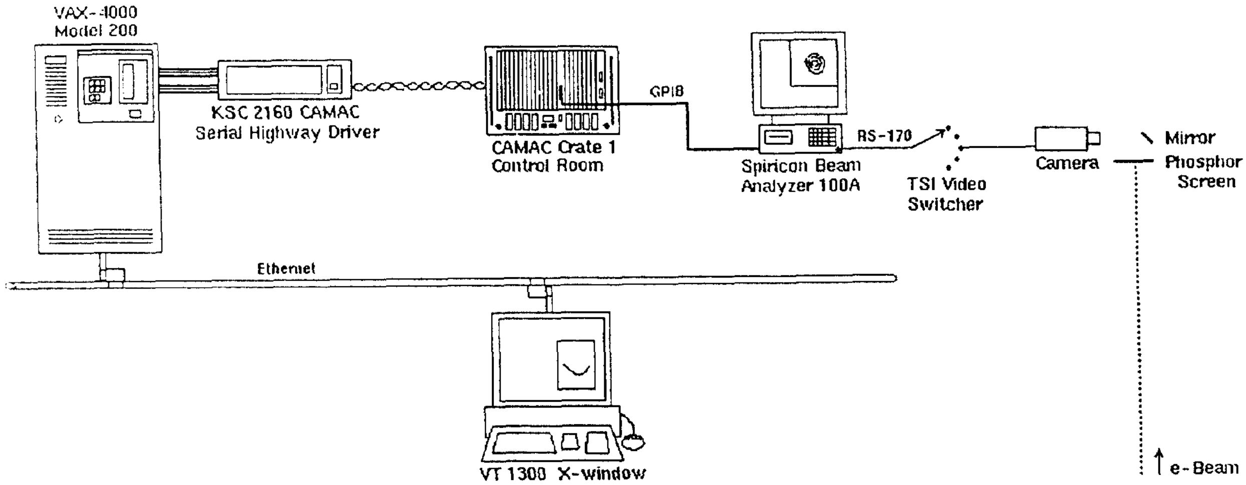

Figure 2: The schematic of the ATF emittance measurement system.

operator window, which was created using Vista control software package. For each emittance scan, the emittance measurement system step through the quadrupole magnet for a given range. It turns on and off the laser beam, acquires dark current and photoelectron beam images. The images are sent to the Spiricon beam analyzer to obtain photoelectron image only by subtraction. Then the Spiricon calculates the elliptical beam parameters for specified portion of the beam image. The Spiricon send the beam information to the VAX by hardware interruption. When data acquisition finished, the VAX calculates the beam matrix and emittance using the algorithm presented in the previous sections. The emittance measurement system also plots the data in real time (Fig. 3), and the beam matrix ellipse in the end.

The Spiricon can calculate the image in several manners, and performs Gauss fit.[6] The calculation method used in the ATF emittance measurement system is Energy Method. It allows us to calculate elliptical beam parameters for different portions of the beam, and compare with the measured emittances with an ideal gaussian beam. For a gaussian distribution beam, the beam fraction F is given by,[7]

$$F = 1 - EXP\left(-\frac{\epsilon}{2\epsilon_{rms}}\right). \qquad (12)$$

and the emittance of a fraction of a gaussian beam is,

$$\frac{\epsilon_{rms}(F)}{\epsilon_{rms}} = 1 + (1-F)\,ln(1-F)/F \qquad (13)$$

Using Spiricon beam analyzer's energy method, we can calculate the beam emittance of the different fractions of the electron beam, and determine the beam distribution by using Eq. (13).

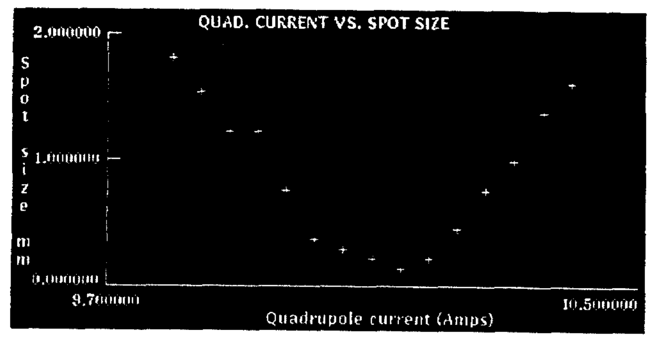

Figure 3: The beam sizes as function of the quadrupole strength.

IV. REFERENCES

1. K.T. Mcdonald and D.P. Russell, "Methods of Emittance Measurement", p. 122 - 132, Proceeding of the Joint US-CERN School on Observation, Diagnosis and Corrections in Particle beams(1988).
2. M.C. Ross *st. al.*, p.725, Proceeding of Particle Accelerator Conference (1983).
3. M.C. Ross *et. al.*, SLAC-PUB-4278.
4. X.J. Wang *et al.*, "THE BROOKHAVEN ACCELERATOR TEST FACILITY INJECTION SYSTEM", p. 307, Proceeding of Particle Accelerator Conference (1989).
5. D.P. Russell and K.T. McDonald, p. 1510, Proceeding of Particle Accelerator Conf.(1989).
6. Beam Analyzer Operator's Manual, Spiricon Inc., Logan, Utah.
7. O. Sander, "Transverse Emittance: Its definition, applications, and Measurement", AIP Conf. Proc. No.212, 127-155 (1989).