CAVITY-INDUCED INSTABILITIES IN KAON FACTORY SYNCHROTRONS

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Summary

Synchrotrons for kaon factories will not necessarily break new ground in terms of instabilities: high average beam intensity is attained by rapid cycling rather than by a high circulating current. As compared with the CPS or the AGS, the rapid-cycling nature means that there is more total impedance due to rf cavities. Both the number of cavities and their individual impedances should be minimized.

Introduction

The TRIUMF,[1] LAMPF,[2] and EHF[3] kaon factory or hadron facility proposals each consist of an injector, a booster synchrotron and a main synchrotron to reach a beam energy of 30 GeV or 45 GeV. The general characteristics of the proposed synchrotrons are summarized in Table 1. All proposals specify an rf frequency in both synchrotrons of close to 50 MHz with bucket-to-bucket transfer between booster and main rings. To allow for kicker rise time, a ~100 nsec gap of empty buckets is proposed for each case. Furthermore, in all proposals, the dc circulating current in the synchrotron is between 2 and 3 A. Because of these common features, all the proposed synchrotrons are in the same regime as far as coupled bunch instabilities are concerned.

Sacherer[4] has shown that a resonator with shunt impedance R_s and frequency (f_{res}) n times the particle revolution frequency causes longitudinal instability with growth rate $1/\tau$ given by

$$\frac{1}{\tau_m \omega_s} = \frac{4}{\pi^2 B^2}\,\frac{m}{m+1}\,\frac{hI}{V\cos\phi_s}\,\frac{R_s}{n}\,F_m D \qquad (1)$$

ω_s = angular synchrotron frequency
B = bunch length/bunch centre spacing
h = harmonic number ($f_{rf} = hf_o$)
I = dc component of circulating current
$eV\sin\phi_s$ = energy gain per turn .

The form function F_m is shown in Fig. 1 for different within bunch mode numbers m. (m=1 is the dipole mode, m=2 the quadrupole, and so on.) F_m is a function of $x = Bf_{res}/f_{rf} = Bn/h$, the number of oscillations that the resonator makes during the passage of one bunch. The factor D takes into account two effects: the attenuation of the beam-induced signal between bunches, and the cancellation between upper and lower sidebands which occurs for resonant frequencies very close to f_{rf}. The first effect is not important as long as the resonator Q is larger than $\pi f_{res}/f_{rf}$: this condition is always met for rf cavity modes. The second effect will be discussed again when the fundamental mode of the rf cavities is considered.

Table 1. The Proposed Kaon Factory Synchrotrons.

Name	Energy (GeV)	R (m)	h	I_{dc}/β (A)	t_{acc} (ms)
TRIUMF	0.44- 3.0	34.0	45	2.8	15.0
TRIUMF	3.0 -30.0	170.0	225	2.8	75.0
EHF	1.2 - 9.0	76.0	90	2.5	27.0
EHF	9.0 -30.0	152.0	180	2.5	53.0
LAMPF II	0.8 - 6.0	53.0	66	2.2	11.0
LAMPF II	6.0 -45.0	212.0	266	2.2	50.0

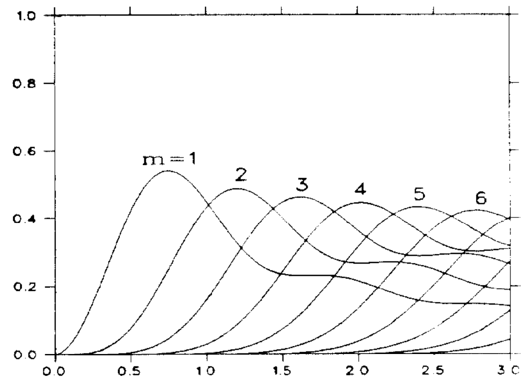

Fig. 1. Form function $F_m(x)$ for the longitudinal case.

We can see from Fig. 1 that there are two regimes: if $x > 1/2$ there is always some mode m which is close to being optimally driven, but if $x < 1/2$, only m=1 is significantly excited and less and less efficiently so as $x \to 0$. In the former case, m=2x (to the nearest integer of course) and F_m is always around 0.4. Then (1) can be written simply as

$$\frac{1}{\tau\omega_s} \approx \frac{1}{3B}\,\frac{1}{m+1}\,\frac{R_s I}{V\cos\phi_s} \quad \text{when} \quad 2B\,\frac{f_{res}}{f_{rf}} > 1 \ . \qquad (2)$$

If $x < 1/2$ then m=1 and $F_1(x) \approx \pi^2 x^2/4$ and (1) becomes simply

$$\frac{I}{\tau\omega_s} \approx \frac{n}{2h}\,\frac{R_s I}{V\cos\phi_s} \quad \text{when} \quad \frac{n}{h}\left(= \frac{f_{res}}{f_{rf}}\right) < \frac{1}{2B} \ . \qquad (3)$$

Expressions (2) and (3) are handy because they can be used without reference to graphs of form factors. These expressions are accurate to ~±25%.

rf Cavity Fundamental

We use expression (3) because $n/h = 1 < 1/(2B)$

$$\frac{1}{\tau\omega_s} = \frac{1}{2}\,\frac{R_s I}{V\cos\phi_s} \sim \frac{1}{2}\,\frac{I}{I_o} \ . \qquad (4)$$

For cavities in the 50 MHz range, $I_o = V/R_s$ (the generator current for no beam loading with the cavities tuned to resonance) is typically 1 A or less. With I=2 or 3 A, the formula gives $1/\tau \gtrsim \omega_s$ which seems to imply that there is no stability at all. In fact, this analysis is incorrect because the rf cavities are not passive resonators. The presence of phase, amplitude, tuning and stabilization loops drastically modify the response of the rf cavities to the beam.[5] An accurate analysis is not possible at this stage but one can make the following 'order of magnitude' argument.

All proposals specify a gap in the beam to allow for kicker magnet rise time for lossless injection and extraction. In the TRIUMF proposal, this gap consists of 5 empty rf buckets out of the 45 in the booster. For LAMPF II, the gap is 6 out of 66. The required overall gain of the cavity stabilization loop(s) will probably be dominated by the transients due to this beam gap. Assuming 90% of the buckets are filled and 10% empty, one can show that to reduce the voltages induced by the beam's n=h-1 and n=h+1 Fourier harmonics to less than 1% of the fundamental (n=h) rf 1% of the

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voltage, a gain of at least 40 I/I_0 is required.[6] Feedback reduces the shunt impedance apparent to the beam by a factor equal to the gain. Therefore, in all cases,

$$1/\tau \sim \omega_s/100 \ . \tag{5}$$

It should be pointed out that this result does not apply to the n=h coupled bunch mode. This mode is stabilized by detuning the rf cavity in accordance with the Robinson criterion.[7] What happens is that for impedances around the rf frequency, upper and lower synchrotron sidebands (nf_0+mf_s and nf_0-mf_s) can cancel each other because they belong to the same coupled bunch mode. By tuning the cavity to be slightly above the rf frequency (when below transition and vice versa above transition), the stabilizing upper sideband dominates over the destabilizing lower sideband for the mode n=h. But then the upper stabilizing sideband of the n=h-1 mode has less overlap with the driving impedance and the lower sideband has more. Hence, the n=h-1 mode is driven (below transition). Expression (5) is for the worst case, where the upper sideband, $f=(h-1)f_0+f_s$, has no overlap with the driving impedance and the lower sideband, $f=(h+1)f_0-f_s$ coincides with the cavity resonant frequency. Such large detunings ($\Delta f \sim f_0$) can actually occur under conditions of high beam loading; specifically, when

$$\frac{I}{I_0} \geqslant \frac{Q}{h} \tag{6}$$

(Q is the cavity quality factor.) In any case, (5) gives e-folding times of $\tau \sim 1$ ms for the booster synchrotrons and $\tau \sim 10$ ms in the main synchrotrons. These are dangerous but not catastrophic. An active damping system for the n=h-1 mode and possibly the n=h-2 mode will have to be included in the design of the rf system.

rf Cavity Parasitics

The lowest frequency modes are worst because they have the largest impedances and because they couple best to the beam. We assume that the lowest parasitic has a frequency of about twice the fundamental. Then $x=Bf_{res}/f_{rf}=2B$. This can be less than 1/2 ($B < 1/4$). Again we use expression (3)

$$\frac{1}{\tau\omega_s} = \frac{R_s I}{V \cos\phi_s} \ . \tag{7}$$

For now, we assume that the parasitic resonances of different cavities do not overlap. Hence, R_s in (7) refers to a single cavity while V is the total voltage per turn. For a typical R_s of a few 100 kΩ,[8] $1/\tau \sim \omega_s$ in the boosters. For such large growth rates we expect the simple instability theory to break down. For detailed calculations the more sophisticated mode-coupling theory[4b] should be used but an upper bound on $1/\tau$ can be obtained from coasting beam theory.[4c]

$$\frac{1}{\tau} = \frac{c}{R}\sqrt{\frac{nInR_s}{4\pi E}} \qquad \begin{aligned} E &= \gamma\, m_0 c^2/e \\ \eta &= |\gamma_t^{-2} - \gamma^{-2}| \end{aligned} \ . \tag{8}$$

Dividing by ω_s yields

$$\frac{1}{\tau\omega_s} = \sqrt{\frac{n}{2h}\frac{IR_s}{V\cos\phi_s}} \tag{9}$$

i.e. just the square root of the expression (3). Hence, for $1/\tau \sim \omega_s$, the two approaches give approximately the same result and so (7) should be fairly good all the way up to $1/\tau \sim \omega_s$.

Growth rates therefore appear to be horrendous but we are saved by the fact that these impedances are very narrow and in fact $Q \gg h$. This means that in accumulator rings parasitics can be tuned to have no overlap with revolution harmonics. In proton synchrotrons, the frequencies of the revolution harmonics change during acceleration. If

$$\Delta\beta > 1/n \left(\approx 1/(2h) \text{ for } f_{res} \approx 2f_{rf}\right) \tag{10}$$

then the frequency change of mode n is greater than the mode separation and the parasitic will coincide with a coupled bunch mode at least once in the acceleration ramp. Inequality (10) holds true even in the case of the EHF main ring where the injection energy is 9 GeV [$\Delta\beta = 0.004$, $1/(2h) = 0.003$].

The time that it takes for a coupled bunch mode to cross a parasitic resonance is

$$\Delta t = \frac{1}{Q}\frac{\beta}{\dot\beta} \tag{11}$$

so the number of e-folding times during crossing is

$$\frac{\Delta t}{\tau} = \frac{I(R_s/Q)}{V\cos\phi_s}\frac{\beta}{\dot\beta}\,\omega_s \ . \tag{12}$$

Two general points can be made. (1) Passive damping is not directly effective in reducing $\Delta t/\tau$ because both R_s and Q will decrease i.e. the parasitic is weaker but takes longer to cross. (2) Parasitics should be tuned so that crossing occurs at times when $\dot\beta$ is large. Obviously, then, crossing is to be avoided near injection and extraction. Also, since $\dot\beta \sim \gamma^{-3}$, crossing should occur as early in the ramp as possible. In Table 2, $\Delta t/\tau$ has been calculated for each of the synchrotrons under consideration for the earliest possible crossing:

$$\beta \text{ (at crossing)} = \beta_{final} - 1/n \text{ (for } n = 2h) \ . \tag{13}$$

We assume $R_s/Q = 20\ \Omega$; this should be fairly close for the first parasitic for any realistic cavity design. Also calculated are $1/\tau\omega_s$ assuming $R_s = 10^5\ \Omega$.

Table 2. Growth rates due to cavity parasitics.

Machine	Energy (GeV)	$\Delta t/\tau$	$1/\tau\omega_s$
TRIUMF 'B'	2.5	1.5	0.60
TRIUMF 'D'	14.0	2.0	0.16
EHF booster	6.8	1.3	0.41
EHF main	12.0	2.6	0.18
LAMPF II booster	4.5	0.5	0.17
LAMPF II main	16.0	1.0	0.044

These results are for only one parasitic in one cavity. Higher parasitic modes will have smaller growth rates but by (13) will be crossed later in the cycle when $\dot\beta$ is smaller. The parasitics from the different cavities should be tuned to lie sufficiently far apart; preferably as far as one revolution frequency apart. This may not even be possible if the number of cavities is too large. Clearly, it is advantageous to minimize the number of cavities by maximizing the voltage per cavity. In this regard, the Los Alamos cavity design[2] with perpendicularly biased ferrite is favored for booster synchrotrons.

When the amplitude of a coupled bunch mode starts from the noise level, up to 4 or 5 e-folding times is generally safe. However, in our case, the presence of the kicker gap in the beam means that effectively the mode amplitude does not start from zero. A circulating beam without gaps has Fourier components only at harmonics of the rf frequency. With a kicker gap, there are Fourier components at all harmonics of the revolution frequency. The worst of these components will have amplitude of about $I_n \approx 2rI$ where r is the ratio of empty buckets to full buckets. For example, in the case of the TRIUMF main ring, $I_n \approx 0.7$ A. At some point, a cavity parasitic coincides with this harmonic ($\omega = n\omega_c$) inducing a voltage of 70 kV (for $R_s = 10^5\ \Omega$). Subsequently, when the coupled bunch mode $\omega = n\omega_0 - \omega_s$ crosses the parasitic, this voltage is multiplied by a factor of $\exp(\Delta t/\tau) = \exp(2.0)$ yielding 500 kV. The beam would instantly be lost. Similarly, still using $R_s = 10^5\ \Omega$, for the EHF and LAMPF II main rings the induced voltages would be 400 kV and 100 kV respectively and for the TRIUMF, EHF, and LAMPF II boosters, we get respectively, 300, 100, and 70 kV. Parasitic mode impedances must be reduced to the level of 10 kΩ or lower (1 kΩ in the case of the TRIUMF booster). At this level, coupled bunch modes will be almost continually driven because the Q's would also be lower and the parasitics begin to overlap each other. The growth rates would then be in the hundreds of inverse seconds; low enough to be dealt with by active damping.

A more elegant and conceptually simpler solution would be to actively tune the parasitic to continually lie between coupled bunch modes. This may even turn out to be simpler than damping the parasitics by the required amount and would obviate the need for active damping.

Landau Damping

The longitudinal focusing force is sinusoidal rather than linear and this gives rise to a synchrotron frequency spread and, potentially, to Landau damping. In high intensity proton synchrotrons, there is a large reactive term in the coupling impedance and this gives a real part to the frequency shift of the oscillation mode. Although this tune shift does not in itself cause instability, it can cause loss of Landau damping i.e. it can shift the mode frequency outside of the incoherent band of frequencies so that any small resistive impedance will cause instability. In all the synchrotrons under consideration here, the tune shift is large enough to cause loss of Landau damping of the dipole (m=1) mode and possibly also higher modes.

The shift of the incoherent band of synchrotron frequencies due to coupling impedance $Z_\parallel$ ($Z_\parallel/n$ independent of frequency) is[9]

$$\frac{\Delta Q_s}{Q_s} = \frac{3}{2\pi^2}\,\frac{h}{B^3}\,\frac{I\ \mathrm{Im}(Z_\parallel/n)}{V\cos\phi_s}\,. \tag{14}$$

This can be negative (space charge dominated below transition or inductive wall dominated above transition) or positive (vice versa). The coherent dipole mode will lie outside the incoherent band if the dc circulating current, I, exceeds I_T where

$$I_T = -\frac{\pi^4}{30}\,\frac{B^5}{h}\,\frac{V}{\mathrm{Im}(Z_\parallel/n)\cos\phi_s} \tag{15a}$$

for $\Delta Q_s < 0$ and

$$I_T = \frac{\pi^4}{30}\,\frac{B^5}{h}\,\frac{V}{\mathrm{Im}(Z_\parallel/n)\cos\phi_s}\left(\frac{1}{4}+\frac{5}{6}\sin^2\phi_s\right) \tag{15b}$$

for $\Delta Q_s > 0$. (15a) is derived in Ref. 9 and (15b) can be shown by an extension of the same theory.

Combining (15a) and (15b) with (16) yields the following condition for Landau damping of the dipole mode:

$$-\frac{1}{2}\left(\frac{B}{\cos\phi_s}\right)^2 < \frac{\Delta Q_s}{Q_s} < \frac{1}{2}\left(\frac{B}{\cos\phi_s}\right)^2\left(\frac{1}{4}+\frac{5}{6}\sin^2\phi_s\right)\,. \tag{16}$$

For a well-designed high intensity proton machine, $|\Delta Q_s/Q_s|$ is always around 10%. The reason is an economic one: to make $\Delta Q_s/Q_s$ small, V must be made large (for a constant B), but this adds to cost both of the rf system and because $\Delta p/p$ becomes large enough to impact upon vacuum chamber size. On the other hand, if $\Delta Q_s/Q_s$ is allowed to be too large, we are in danger from the microwave instability. Specifically, Hofmann and Pedersen[9] have shown that to avoid the microwave instability, a sufficient condition is $|\Delta Q_s/Q_s| < 22\%$.

Using reasonable values of B and ϕ_s for $|\Delta Q_s/Q_s| = 0.1$, we find that (17) is almost always violated: the dipole mode is not Landau damped.

Conclusions

An rf system which is stabilized against beam loading also tends to lower the growth rates of other coupled bunch modes driven by the rf cavity fundamental. The coupled bunch modes nearest the fundamental will need to be actively damped, but no essential difficulty is foreseen.

Parasitic modes in the rf cavities will be strongly excited by Fourier harmonics due to the beam's kicker gap. The smaller the kicker gap, the less severe is the problem. Kickers with faster rise are therefore very desirable. In particular, for a rise time of ~10 ns, no gap is needed. Assuming that such kickers will not be developed, it will be necessary to reduce the impedances of the parasitic cavity modes to the level of a few kΩ.

The problem of the cavity parasitics becomes less severe as the number of cavities is reduced. Higher voltage cavities are desirable.

References

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