SIDEBAND SPECTRA OF A LONGITUDINAL INSTABILITY OBSERVED IN PHOTON FACTORY STORAGE RING

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Summary

Almost all the spectral components of the beam at high current in Photon Factory Storage Ring (PF-RING) had been explained in terms of transverse and longitudinal instabilities caused by higher modes in RF-cavities[1,2,3] or two-stream instability due to ion trapping[4]. However some spectra remained unresolved until recently.

In this paper we present a simple model to explain how these spectra appeared in the data taken with a spectrum analyzer. By this model we can qualitatively explain these as the sideband spectra of a large oscillation of the beam induced by a longitudinal instability at high current. In fact, it was observed that all of the sideband spectra disappeared at the same time as the longitudinal instability died away with the decrease of the beam current. This fact therefore implies that the spectra would not arise from any other cause.

The spectral data presented here are rather old ones and the same as that of Ref. 5, in which an analysis had been also made. However, since the data and the analysis have not been reported anywhere else, we will here reproduce these.

I. A Simple Model

We use as the mathematical model a rigid bunch approximation throughout this paper. Since this model does not take into account the distortion of the distribution function of the beam except a mere shift of the center of mass of the bunch, it clearly has limitations. Nonetheless we can still explain the spectra qualitatively by this simple model. We first describe the well-known sidebands around the fundamental RF frequency f_{RF}, the spectra being equally spaced one another by the synchrotron frequency f_s. Then we proceed to obtain the expressions of the sideband spectra induced by a large oscillation of the longitudinal instability.

Sideband of Synchrotron Oscillation around f_{RF}

The fourier component of the beam current with no synchrotron oscillation may be written as,

$$I(\omega) = Ce^{i\omega t} \tag{1}$$

for the frequency ω, here we put C as a constant, since the bunch form factor is not important to get a relevant expression. The ω may be taken as a multiple of f_{RF} for equally populated RF-buckets. When the bunches are set into motion uniformly in the longitudinal direction with the synchrotron frequency $\Omega(=2\pi f_s)$, the fourier component of the current becomes,

$$I(\omega) = Ce^{i(\omega t+\Delta\Phi\sin\Omega t)} \quad , \tag{2}$$

where $\Delta\Phi$ is $\omega\hat{\tau}$,$\hat{\tau}$ being the amplitude of oscillation in time-displacement. Eq. 2 can be rewritten as,

$$I(\omega) = C\sum_{n=-\infty}^{+\infty} e^{i(\omega+n\Omega)t}J_n(\Delta\Phi) \quad , \tag{3}$$

where we use a relation of the Bessel function J_n,

$$e^{\frac{z}{2}(t-1/t)} = \sum_{n=-\infty}^{+\infty} t^n J_n(z) \quad . \tag{4}$$

Thus we obtain the well-known relation of the power of the n-th sideband, P_n, to that of the fundamental frequency, P_0,

$$P_n/P_0 = J_n^{\,2}(\Delta\Phi)/J_0^{\,2}(\Delta\Phi) \quad . \tag{5}$$

Roughly speaking, the maximum allowed $\Delta\Phi$ for the PR-RING is estimated as,

$$\Delta\Phi_{max} \sim \begin{cases} 1.3 & \text{at 2.5 GeV} \\ 1.1 & \text{at 2.05 GeV} \end{cases} \tag{6}$$

for which the lifetime does not become much shorter, say less than a few hours. In this estimate we used as the accelerating voltage V_c the values set at the time the data of spectrum was taken,

$$V_c \sim \begin{cases} 1.85\ \text{MV} & \text{at 2.5 GeV} \\ 0.92\ \text{MV} & \text{at 2.05 GeV} \end{cases} \tag{7}$$

Sideband of a Longitudinal Instability

Here we will not describe the longitudinal instability itself; rather we suppose that the bunches are already in a stationary motion as a consequence of the instability, since a longitudinal instability does not usually give rise to the beam loss but let the beam only in a state of stationary oscillation, as observed in PF-RING or other rings. Thus we can easily obtain the expressions of the sidebands induced by a large oscillation due to the instability.

The unperturbed beam current of an electron bunch is written as,

$$I(t) = Ce^{-t^2/2\sigma^2} \quad . \tag{8}$$

Here the σ is the rms of the time-displacement of electrons in the bunch and the C a constant. From now on, the C is used to represent any constant not relevant to get the final results.

When the bunch is time-displaced by τ, the beam current becomes

$$I(t+\tau) = Ce^{-\frac{(t+\tau)^2}{2\sigma^2}} \quad . \tag{9}$$

If the τ is a slowly varying term compared with the frequencies of interest, the fourier component of the current can be written as,

$$I(n\omega_0) = Cf_n e^{-in\omega_0\tau} \quad , \tag{10}$$

where the ω_0 is the revolution frequency of the ring and $\omega_0=2\pi\times1.6$ MHz for the PF-RING, and the f_n is the bunch form factor defined as,

$$f_n = e^{-\frac{\sigma^2 n^2\omega_0^2}{2}} \quad . \tag{11}$$

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We now assume that all of the RF-buckets are uniformly filled with electrons. Then we have the k-th bunch current,

$$I_k(t+\tau_k) = C \sum_{n=-\infty}^{+\infty} f_n e^{-in(\omega_0 t-\frac{2\pi}{h}k)} e^{-in\omega_0\tau_k}, \qquad (12)$$

and further if the τ_k stands for the μ-th mode of the coupled-bunch oscillation, the τ_k is written as,

$$\tau_k = \hat{\tau}\cos\left(\Omega t + \frac{2\pi}{h} k\mu\right) \quad , \qquad (13)$$

where the h denotes the harmonic number and the $\hat{\tau}$ the amplitude of the μ-th mode. Substituting the τ_k into Eq. (12) and then summing up over all the bunches, we can obtain the current of the μ-th mode of the coupled-bunch oscillation,

$$I_\mu(t) = C \sum_{\ell=-\infty}^{+\infty} (-i)^\ell \sum_{m=-\infty}^{+\infty} J_\ell\left[(hm-\mu\ell)\omega_0\hat{\tau}\right] f_{hm-\mu\ell} \cdot e^{-i\left[(hm-\mu\ell)\omega_0-\ell\Omega\right]t} \quad , \qquad (14)$$

where the relation of (4) is again used.

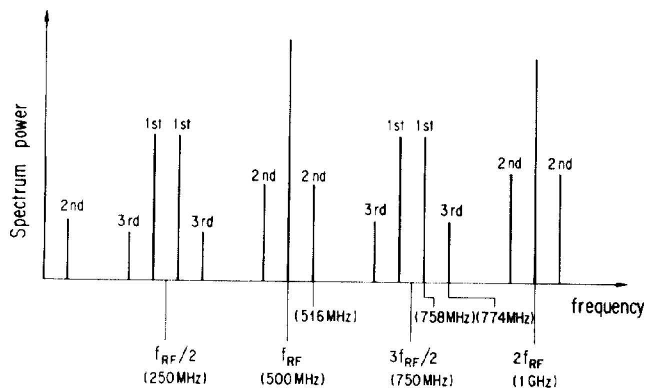

Fig. 1 Schematic diagram of the sideband spectra. The frequency of 758 MHz is one of the higher-modes of the RF-cavities at the RF-RING.

From Eq. (14), we can see the following properties;

(1) when the μ-th mode is excited by a longitudinal instability and it has a finite amplitude of oscillation, its sideband spectra could be observed as in the case of the usual sideband, each sideband spectrum being designated by ℓ in Eq. (14).

(2) Differing from the usual sideband spectra, however, each spectrum would be far apart from one another in frequency domain, because of the term of $\mu\ell\omega_0$ in the exponent in the rhs of Eq. (14).

(3) The argument in the Bessel function would be relatively small, so that the spectrum with a larger ℓ has a smaller amplitude in the low-frequency region and the peak of its spectrum power tends to move to the region with a higher frequency, which corresponds to a larger m, the m being a harmonic number of f_{RF}.

A schematic diagram is shown in Fig. 1 to illustrate the above properties for a few lowest-order sideband spectra, the amplitudes of which are proportional to,

$$\begin{aligned} &J_1\left[(hm\pm\mu)\omega_0\hat{\tau}\right]f_{hm\pm\mu} && \text{at } \omega = (hm\pm\mu)\omega_0\pm\Omega \\ &J_2\left[(hm\pm2\mu)\omega_0\hat{\tau}\right]f_{hm\pm2\mu} && \text{at } \omega = (hm\pm2\mu)\omega_0\pm2\Omega \qquad (15) \\ &J_3\left[(hm\pm3\mu)\omega_0\hat{\tau}\right]f_{hm\pm3\mu} && \text{at } \omega = (hm\pm3\mu)\omega_0\pm3\Omega \ . \end{aligned}$$

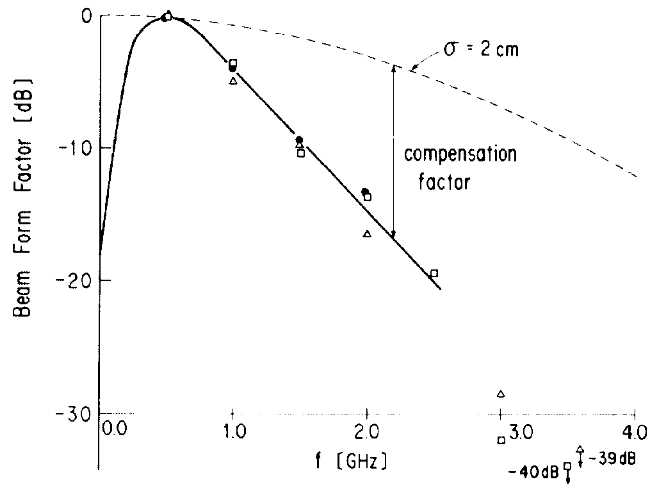

Fig. 2 Beam form factor versus frequency. The broken line shows the theoretical value for the bunch length of 2 cm. The solid line is the measured spectrum powers for a single-bunch mode at 5 mA. The squares indicate the data taken at the harmonics of f_{RF} for a multi-bunch mode, and the triangles and closed circles indicate the data at the harmonics of the revolution frequency near the 2nd-order sidebands due to the instability of 758 MHz.

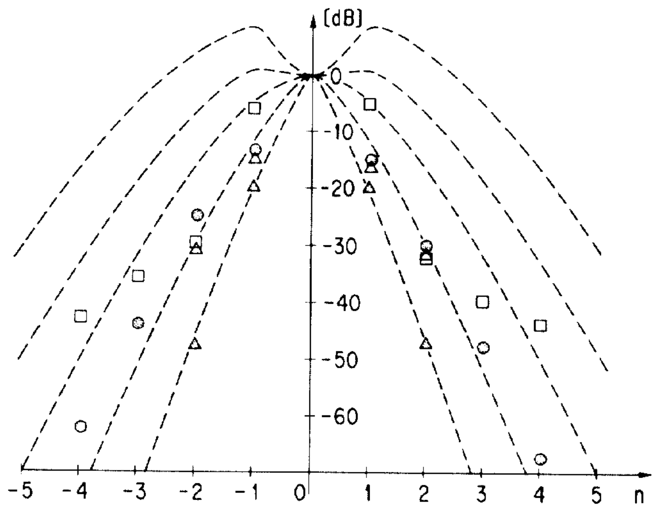

Fig. 3 Sidebands of f_{RF}. The abscissa, n, denotes the order of the sidebands. The broken lines are the values calculated by Eq. (5). From the inside of the figure toward the outside, ΔΦ = 0.2, 0.5, 1.0, 1.5, and 2.0.
closed circle ---- f_s-modulation of the f_{RF}.
triangle & square ---- Robinson instability.

Remark: closed triangles are not the sidebands of f_{RF}.

II. Measurements

Since the signal from one of the electrodes of the position monitors is fed into a spectrum analyzer at the control room through a long coaxial cable, its spectrum powers at higher frequencies are decreased due to the loss in the cable as seen in Fig. 2. The decrease at lower frequencies in the figure is due to the frequency response of the the position monitor, $(\omega/\omega_0)/(1+(\omega/\omega_0))$ with $\omega_0 = \sim 2\pi\times 320$ MHz.

Fig. 3 shows the sidebands of f_{RF} except the closed triangles in the figure, the data of which were taken during a machine study. The closed triangles denote the sidebands around one of the spectra induced by the longitudinal instability of 758 MHz, but not the sidebands indicated in (14). Though these sidebands can not be explained by the model described in Sec. I, we can easily identify them as the spectra associated with the longitudinal instability, since the spectra are lumped together in a narrow frequency span.

Figs. 4 to 6 show some of the spectra measured by the spectrum analyzer, which seems to be the sidebands caused by a large amplitude of the beam due to the instability of 758 MHz. The spectra indicated by arrows in the figures are the examples of the data that would be explained by the simple model. In order to compare the measured data with (14), we first multiply the data by the compensation factor in Fig. 2, and then compare the spectrum power of 758 MHz with that of $2f_{RF}$ (=1 GHz) to find the value of the argument in the Bessel function in (14). Its value was found to be about 0.065. Fig. 7 shows the compensated data and the calculated values. The spectrum powers of the 1st-order sidebands fairly agree with the calculated values, while those of the 2nd-order sidebands deviated from the calculated ones at higher frequencies. For the 3rd-order sidebands, the accuracy of measurement is not so good, but the discrepancy is apparent. The rising tendency of the measured powers with frequencies, however, qualitatively agrees with the calculation. These spectra together with the other spectra as seen in Figs. 4 to 6 disappeared at the same time as the the longitudinal instability died away, as mentioned in the Summary.

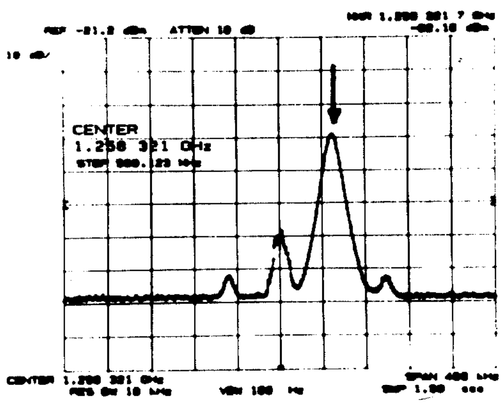

Fig. 4 The 1st-order sideband spectrum.

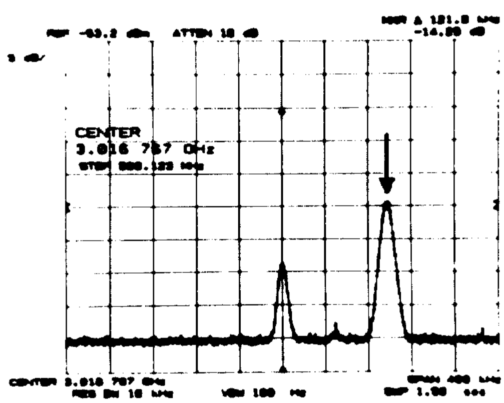

Fig. 5 The 2nd-order sideband spectrum.

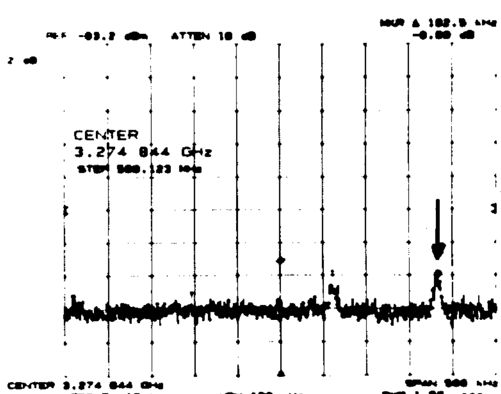

Fig. 6 The 3rd-order sideband spectrum.

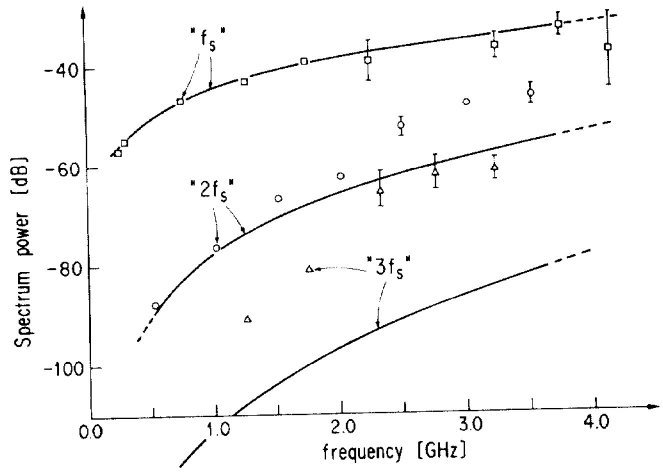

Fig. 7 Comparison of the compensated data of spectrum power with the calculation.
"f_s" : the 1st-order sidebands.
"$2f_s$": the 2nd-order sidebands.
"$3f_s$": the 3rd-order sidebands.

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