# MEASUREMENT OF BETATRON FUNCTION AT THE PHOTON FACTORY

I. Honjo*, M. Katoh, A. Araki, Y. Kamiya and M. Kihara

National Laboratory for High Energy Physics
Oho-machi, Tsukuba-gun, Ibaraki-ken
305, Japan

* Fujitsu Ltd.
1015, Kamikodanaka, Nakahara-ku, Kawasaki
211, Japan

Abstract: The betatron functions (β functions) of the Photon Factory storage ring (PF-ring) at KEK have been measured in the normal operation mode and in the low emittance mode. The measured values of the β functions agree with their theoretical values very well. When the vertical wiggler (VW)[1] is excited up to 4.5 T, tune shifts are produced by about 0.04 and the β functions are distorted considerably. A simple mathematical model was made by assuming the VW to consist of three rectangular magnets with the sextupole field component. It was possible to explain the tune shifts and to give a good estimate of the distortion of the β functions. The local correction was also applied to reduce the distortions by changing the focal strengths of the quadrupole magnets adjacent to the VW.

## Principle of Measurement

The method we used for measuring the β function is to change the strength of a quadrupole magnet by shunting the current of magnet with a resistance and then to measure the shift of the betatron frequency.

When the strength of the i-th quadrupole magnet is changed by shunting the current of magnet, the tune shift can be given by the following relation,

$$\Delta\nu_i = -\frac{\beta_i}{4\pi}(kl_Q)_i \qquad (1)$$

$$k(s) = \frac{\Delta B'(s)}{B\rho}\ , \quad \Delta B'(s) = \Delta\left(\frac{\partial B(s)}{\partial x}\right)$$

where $\beta_i$ is assumed to be constant in the quadrupole magnet and $l_Q$ is the effective length of the quadrupole magnet. The value of $kl_Q$ can be evaluated by the excitation curve of the quadrupole magnet.

The schematic diagram of the measurement is shown in Fig. 1. The quadrupole magnet to be shunted can be easily chosen by the relay circuits. The shunting resistance we used in the experiment is actually the electronic load commercially available as the general purpose equipment for testing power supplies[2]. The linearity between the betatron frequencies and the shunting current was examined in advance.

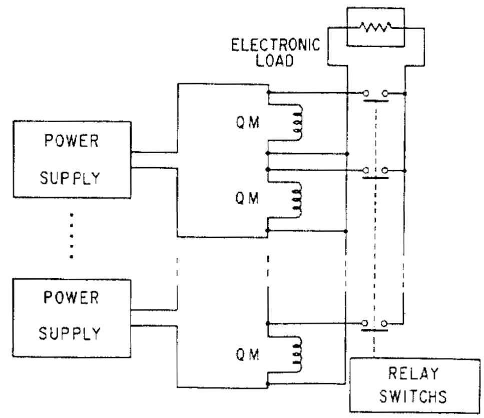


Fig. 1 Schematic diagram of shunting circuits.

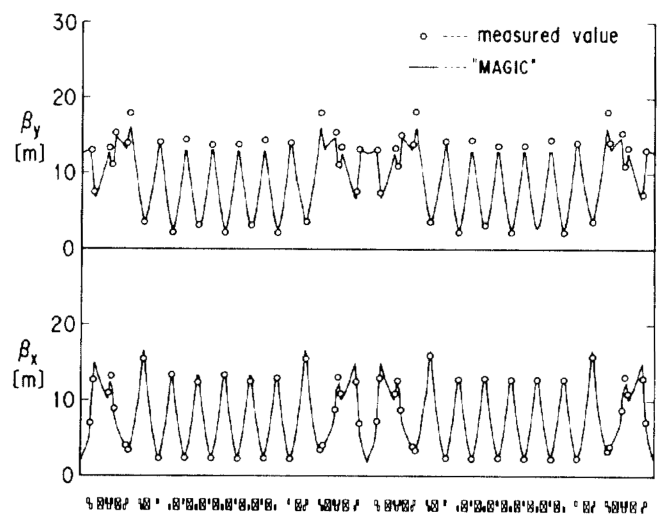


Fig. 2 Measured β functions of the normal optics.

## Experimental Results

The measured values of the β functions and the theoretical values calculated by the computer code "MAGIC" are shown in Fig. 2. As seen in the figure, They are in good agreement.

Since the accuracy of this tune measurement is less than 0.001, the estimated error in the β function is at most 1 m. The measurement has been done at rather low beam current of 20 mA, because at higher current the tune spread becomes wider and makes the fine tune measurement difficult. Sextupole magnet currents were set zero during the experiment. Octupole magnets were slightly excited for the Landau damping against the vertical instability.

When the VW is excited, many dangerous resonances appear in the vicinity of the operating point and their stopbands become wider[3]. Therefore the strengths of quadrupole magnets QF's and QD's have been slightly changed to keep the tunes constant. The measured β functions with the VW excited to 4.5 T are shown in Fig. 3 together with the measured values with the VW not excited. The β functions are distorted by about 45 % in the horizontal direction and 30 % in the vertical direction.

The distortion of β functions caused only by the change of the strengths of QF's and QD's was calculated to be within a few percents and this is much smaller than the observed distortion. Consequently, the distortion of β functions can be considered as mainly due to the excitation of the VW by itself.

## Mathematical Model

We first estimate tune shifts by making a mathematical model and then apply it to the estimate of the distortion of β functions.

In our mathematical model it is assumed that the horizontal tune shift originates both from the edge

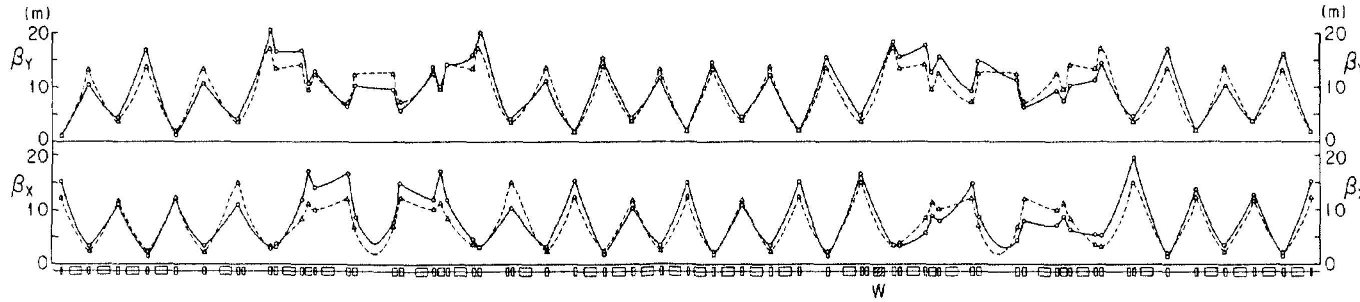


Fig. 3 Measured β functions in the normal optics with the VW excited (——) and not excited (----).

focusing of the VW and from the sextupole field component produced by the superconducting coils of the VW. The vertical tune shift is assumed to be caused only by the sextupole field component.

In order to estimate the tune shift, we rewrite Eq (1) as,

$$\Delta\nu = -\frac{1}{4\pi}\{\sum_i K_i \cdot \beta_i\} \qquad (2)$$

where K is the integrated value, "K-value", of the perturbation in the focal strength.

The K-value of the edge focusing is given by,

$$K = -\int\frac{1}{\rho^2}\,ds \qquad (3)$$

We simply approximate the magnetic field distribution of the VW along the orbit (Fig. 4) with sinusoidal functions. The K-value's of the edge focusing for each coil are listed in Table 1.

In the vertical direction, since the edge focusing at the entrance and the exit of the rectangular magnet cancel each other, the K-value due to the edge focusing in the vertical direction is zero.

In the VW, the orbit is wiggled vertically by the horizontal magnetic fields (Fig. 5), so that it has some deviations from the magnet center line and electrons feel the effective gradient field produced by the sextupole field component of the VW.

The horizontal magnetic field distribution in the vertical direction are given as follows;

$$B_x = B_0 - \frac{1}{2}B_0''Y^2 - \frac{1}{24}B_0^{(4)}Y^4 + \cdots \qquad (4)$$

Taking the derivative of Bx, we get,

$$\frac{\partial Bx}{\partial Y} = -B_0''\cdot Y \quad , \quad B_0'' = \frac{\partial^2 Bx}{\partial Y^2} \qquad (5)$$

The K-value of the gradient field is given by,

$$K = \frac{1}{B\rho}\int\left(\frac{\partial Bx}{\partial Y}\right)ds \qquad (6)$$

The measured values of $B_0''(s)$ are shown in Fig. 6. We approximate $B_0''(s)$ with simple functions in the same manner as in the edge focusing effect then obtain the K-value's of the sextupole field component (Table 1).

The theoretical values of tune shifts are compared with the measured ones in Table 2.

This mathematical model is applied to a rough estimate of the distortion of β functions. The distortion of β functions is generally given by the following relations;

$$\frac{\Delta\beta}{\beta 1} = \frac{1}{2\sin(2\pi\nu)}\{\sum_i K_i\cdot\beta_i\}\cos 2\nu(\pi+\phi_w-\phi_1) \qquad (7)$$

$$K_i = k(\phi)\cdot\Delta s$$

where $k(\phi)$ is the gradient errors in the VW and $\phi_w$ is the phase at the center of the VW.

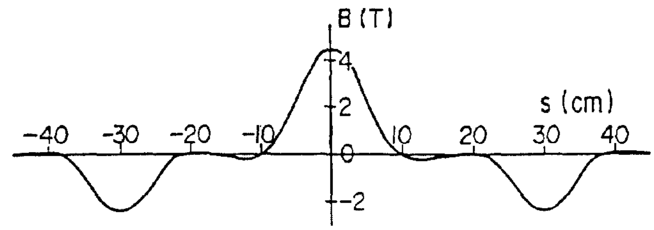


Fig. 4 Horizontal magnetic field in the VW.

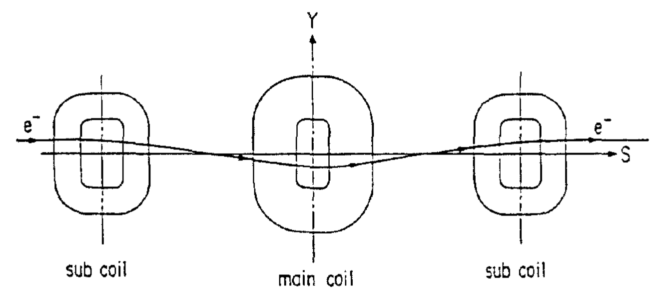


Fig. 5 Orbit in the VW.

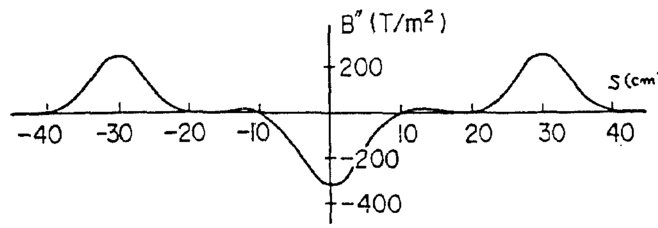


Fig. 6 Sextupole field component in the VW.

Using the K-value's in Table 2, we obtained the theoretical expressions of the distortion of β functions as follows,

$$\left.\frac{\Delta\beta}{\beta}\right)_x = -0.40\cos 2\nu_x(\pi+\phi_{wx}-\phi_x)$$

$$\left.\frac{\Delta\beta}{\beta}\right)_y = 0.29\cos 2\nu_y(\pi+\phi_{wy}-\phi_y) \qquad (8)$$

where $\nu_x = 5.39$ and $\nu_y = 4.15$ are used.

These theoretical values are shown in Fig. 7 together with the measured data.

## The Low Emittance Optics

The β functions in the low emittance optics were measured after the fractions of betatron tunes were made almost the same as in the normal optics. Since the tunes were adjusted only by QF's and QD's at first, the β functions were fairly distorted especially in the vertical direction. The correction has been made by re-adjusting the currents of all families of quadrupole magnets. The measured β functions are shown in Fig. 8.

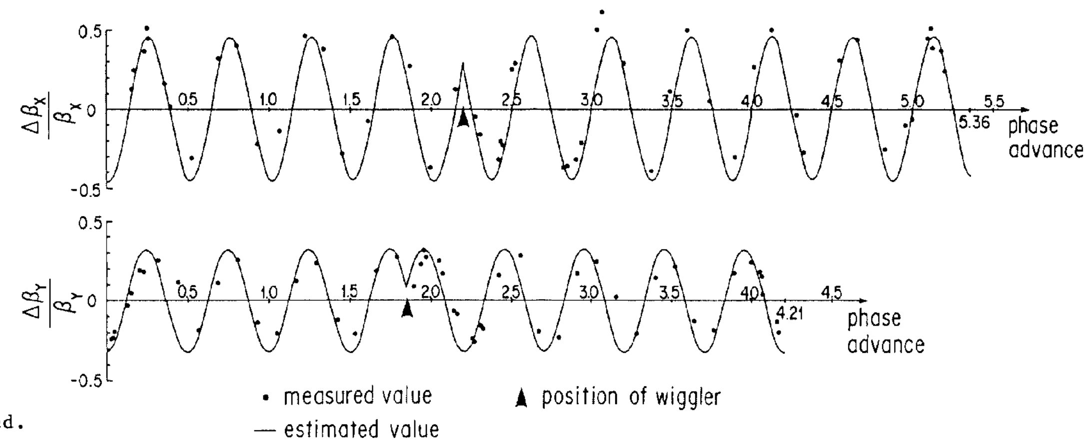


Fig. 7

Measured and estimated distortion of β functions with the VW excited.

Table 1 K-value's of edge focusing and K-value's due to sextupole field for each coil.

| $(m^{-1})$ | sub coil | main coil |
|---|---|---|
| $K^H$ (edge) | -0.00676 | -0.0263 |
| $K^V$ (edge) | 0 | 0 |
| $K^H$ (sex.) | -0.0148 | -0.0212 |
| $K^V$ (sex.) | 0.0148 | 0.0212 |

Table 2 Comparison of tune shifts.

| | Theoretical value | Experimental value |
|---|---|---|
| $\Delta\nu_x$ | 0.041 | 0.045 |
| $\Delta\nu_y$ | -0.038 | -0.040 |

As we see in the previous sections, the gradient errors in the VW cause the tune shift and the large distortion of β functions. It is desirable to compensate the gradient errors locally by shunting the quadrupole magnets near the VW and then to minimize the distortions as small as possible. We used three quadrupole magnets near the VW for this local correction. Their shunting currents are analytically chosen to compensate the tune shifts and to minimize the distortions of β functions[4]. The corrected β functions with the VW excited to 5.0 T is shown in Fig. 9. As seen in the figure, the distortion of β functions is kept within about 20 % at most.

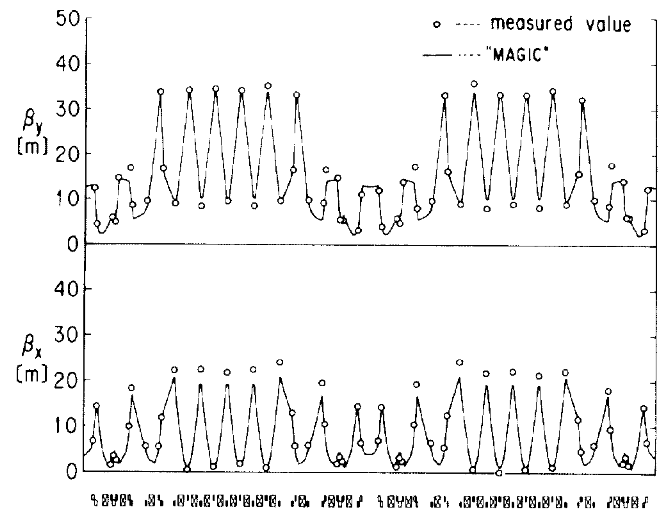


Fig. 8 Measured β functions of the low ε optics.

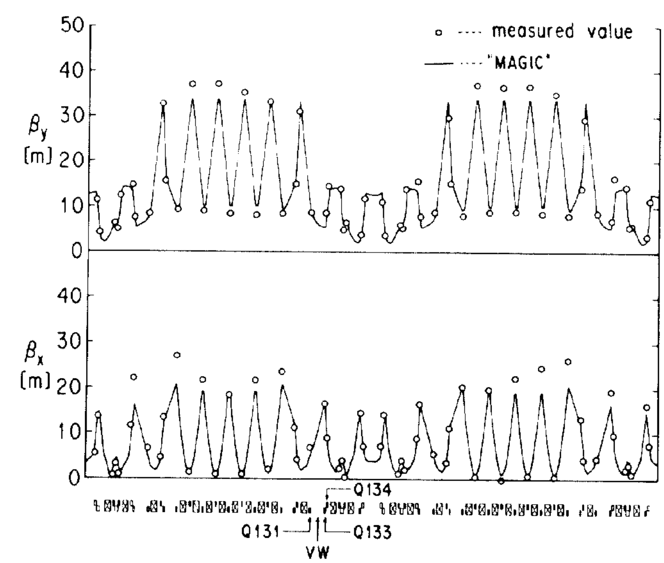


Fig. 9 Measured β functions of the low ε optics with the VW excited to 5.0 T.


## Acknowledgment

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