EFFECTS OF MULTIPOLES AND ORBIT DISTORTIONS ON THE DYNAMIC APERTURE OF THE LHC

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1. Abstract

A Large Hadron Collider (LHC) in the LEP tunnel requires superconducting magnets, which contain strong multipole components up to relatively high order. In this study the dynamic aperture of the LHC is evaluated when both multipoles and orbit distortions are included. In order to get some insight into the specific effects of the different types of errors, we evaluate the dynamic aperture for three distinct cases : (1) we only consider the effect of the sextupolar component in the dipoles, (2) for the same random conditions, we introduce all the higher-order multipoles and (3) we add the misalignment and field errors. For each step of the calculation, we present both the corresponding dynamic aperture as a function of momentum and conclusions on the relative contribution of the effect considered.

2. Introduction

The present report is part of the work performed to demonstrate the feasibility of a Large Hadron Collider (LHC) in the LEP tunnel [1]. It is the logical continuation of different studies related to the optimisation of both the cell-length [2] and the insertions [3]. In order to evaluate the effects of multipole components and misalignment errors on the dynamic aperture, it is worthwhile to briefly describe the reference machine which we shall consider for these simulations : A 2 in 1 machine composed of 4 super-periods each containing 2 slightly different low-β insertions (due to the separation of the beams) and 2 arcs. An arc is a sequence of 20.5 regular cells (90^0 phase advance, l = 119.463 m) with 5 dipoles per half-cell, its length is therefore identical to that of LEP (2449 m). Starting from the interaction point (IP), the insertions are composed of a low-β triplet immediately followed by a doublet of dipoles with opposite signs to separate the beams. Then there are 4 quadrupoles used for detuning, β-matching and cancellation of the additional dispersion induced by the separation-magnets. Finally one has the dispersion suppressor itself where 4 quadrupoles alternate with 4 blocks of dipoles of an equivalent length of 30.6 m each. These insertions are tunable from β* = 1 m (top energy) up to β* = 3.5 m (injection). The β-values at the IP are the same in both planes. With the present design our machine reaches 8.5 TeV but for our purposes, we shall concentrate on its dynamical behaviour at injection, namely 450 GeV. At this energy, we shall consider a normalised emittance of 20 π μm and the simulated tunes will be Q_x = 58.2715 and Q_y = 58.285.

3. Multipoles and errors in the simulation

The evaluation of the dynamic aperture is performed by means of the DIMAD [4] program. The simulated multipole values (from 6-poles to 18-poles) at injection are listed in Table 1 (expressed in units of 10^{-4} and normalized for a radius R = 1 cm), and correspond to those defined in Ref. 5.

Table 1 - Multipole components used in the simulation

	b_3	b_4	b_5	b_7	b_9
Systematic	-3.7	-0.05	0.45	-0.31	-0.16
Random	1.60	0.15	0.20	0.02	0.005

The multipole field components are defined by :

$$B_y + iB_x = B_0 \sum_n (b_n + ia_n)\,(z/R)^{n-1}$$

Thus b_3 and b_9 refer respectively to the sextupole and the 18-poles components.

For the simulations, these multipoles will be inserted only in the middle of the superconducting dipoles (dispersion suppressor and arc) and not in the quadrupoles. In addition, for the sake of comparison with Ref. [2], the skew multipole components are not included. Furthermore, we fixed ourselves the following boundary conditions for the tracking :

- To be considered as acceptable, our machines (including all errors) should have at least a dynamic aperture of 4σ over the full range of momentum considered (0 to 0.14%). However, taking into account a reasonable additional loss in aperture of 1-2σ due to the omission of the multipoles in the quadrupoles and a further reduction of 1σ for the misalignment and field errors, it follows that, for the case where one only considers multipoles, one should require a dynamic aperture of the order 6-7σ.

- The aperture is limited by circular collimators in the quadrupoles at 20 mm.

- Tracking for off-momentum particles is performed at fixed Δp (no synchrotron oscillations simulated). Consequently, it then makes sense to only track the particles over 100 turns which, in the case of LHC at injection, corresponds to a synchrotron period.

For the misalignment and field errors, we shall take into account the same type of errors as those foreseen for LEP. The corresponding values are listed in Table 2, where **x** stands for the r.m.s. horizontal displacement, **y** for the r.m.s. vertical displacement, **a** for the r.m.s. tilt angle deviation and **s** for the r.m.s. relative strength deviation (<ΔB/B> for the dipoles and <ΔK/K> for the quadrupoles).

Table 2 - Misalignment and field errors as used in the simulation.

	Dipoles	Quadrupoles	Monitor
x (mm)	0.140	0.140	0.600
y (mm)	-	0.140	0.600
a (mrad)	0.240	0.240	-
s (%)	0.050	0.050	-

4. Only sextupole components

In this first step, we evaluate the dynamic aperture for the case where one has only sextupole components (b_3) in the dipoles. This should simply demonstrate that our reference machine exhibits the same dynamical behaviour as those previously obtained for optimization purposes [2]. The additional chromaticity induced by these sextupole components (Q'_x = -652, Q'_y = 549) is corrected before tracking. For this case, the limit in aperture is dominated by the systematic components so that it is sufficient to present the results for one of the 10 machines studied. As can be seen from Table 3, the corresponding dynamic aperture fulfills our requirement over the whole range of Δp. In addition, the behaviour

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of our 10 machines is in very good agreement with that observed in Ref. 2.

5. All multipole components included

We shall now evaluate the dynamic aperture of our reference machine by keeping the same random distribution for the b_3 but also including the higher-order components described in Section 3. For the sake of consistency with the subsequent calculations, we will now concentrate ourselves on two machines, which we consider to be representative of our first set of results. The corresponding results are illustrated in Table 3 with a direct comparison with the machine only affected by sextupole components in the dipoles.

Table 3 - Dynamic aperture (expressed in number of stable sigmas) in presence of only sextupoles and all multipoles included (# 1, # 2).

$\Delta p/p$ (%)	Only b_3	# 1	# 2
0.0	10	9	9
0.02	9	8	7
0.04	9	7	6
0.06	9	6	7
0.08	8	6	6
0.10	8	5	5
0.12	8	6	5
0.14	8	4	6

Contrary to all previous expectations, one observes that the higher-order multipoles are far from being negligible since our basic requirement of 6σ is not fulfilled anymore over the whole momentum-range.

At this point, it seems necessary to investigate this surprising behaviour in more detail, especially by trying to evaluate the effects of each single component. To do this, we shall consider machine # 1, and re-evaluate the aperture by removing one single component at a time. The corresponding results are listed in Table 4 where one clearly observes that the loss in aperture is basically not related to one specific component, but much more to the combination of them. Nevertheless, such a detailed investigation enabled us to underline a few outstanding features :

a) The systematic components of the higher-order multipoles are responsible for the loss in aperture. This is illustrated by the last column of Table 4 where NOSYST indicates that - apart from the b_3 components - we only considered the random part for the multipoles $b_4 \rightarrow b_9$.

b) By studying the detuning on amplitude, one clearly observed that the decapole component b_5 introduced a strong coupling in the motion.

c) The 18-pole component b_9, which was expected to be absolutely harmless, appears to be strongly related to the loss in aperture.

d) A simulation over a few turns demonstrated that the strong effect of b_9 was reinforced by the presence of the b_5 component.

In addition to these purely dynamical considerations, it seems unavoidable to look for the physical consistency of these results. A careful study demonstrated that in fact, the whole problem was linked to the effect of multipoles at large amplitudes. As mentioned in Table 1, the latter are evaluated at a radius R = 1 cm. Consequently, for particles with an amplitude lower than this value, the multipoles do indeed behave like a converging series, and the usual assertion that the higher the multipole, the weaker

Table 4 - Dynamic aperture obtained by removing single multipole components.

$\Delta p/p$(%)	Only b_3	No b_4	No b_5	No b_7	No b_9	NOSYST
0.0	10	9	10	8	9	10
0.02	9	8	8	9	9	9
0.04	9	7	7	7	7	9
0.06	9	6	6	6	6	9
0.08	8	5	6	6	6	9
0.10	8	5	5	5	7	8
0.12	8	5	5	5	6	8
0.14	8	4	5	4	6	8

the expected effect is then certainly justified. However, in our case, one has to deal with particles circulating at much larger amplitudes (up to 20 mm at the collimators), for which these convergence properties do not hold anymore. To illustrate this, we shall now compare the kicks affecting the particles as a function of their amplitudes. To ease the interpretation, the results presented in Tables 5 (systematic) and 6 (random) are normalized to the sextupole components.

Table 5 - Comparison of the normalized kicks due to multipoles as a function of amplitude (systematic components).

bn	Kn (1 cm)	Kn (1.5 cm)	Kn (2 cm)
b_3	1.000	1.00	1.00
b_4	0.015	0.02	0.03
b_5	0.120	0.30	0.50
b_7	0.084	0.42	1.34
b_9	0.043	0.50	2.80

Table 6 - Comparison of the normalized kicks due to multipoles as a function of amplitude (random components).

bn	Kn (1 cm)	Kn (1.5 cm)	Kn (2 cm)
b_3	1.00	1.00	1.00
b_4	0.09	0.15	0.19
b_5	0.13	0.30	0.50
b_7	0.01	0.06	0.20
b_9	0.003	0.04	0.20

This evaluation is felt to be useful in the sense that it finally allows a better understanding of both the results presented in Table 3 and the comments made under a) to d). Apart from confirming the consistency of the calculations it also sheds some light on the parameter directly related to our limitations, namely the large required emittances of the beam. Indeed, keeping in mind that the multipoles values are uniquely defined by the concept of the magnet, one observes that in the arcs (dispersion = 2 m, β = 200 m) - for a particle travelling at 5σ with Δp = 0.1% - the related amplitude just corresponds to the limit where the multipoles still behave like a converging series, namely 10 mm. The comparison presented in Tables 5 and 6 thus appears to be a plausible tool for obtaining easily an upper limit of the achievable aperture as a function of the emittances to be considered.

6. Aperture including misalignment and field errors

For the evaluation of the closed-orbit corrections, we shall consider again the 10 machines studied in Section 4 and evaluate both the average closed-orbit distortions and the related maximum excursions of the trajectory after correction. This yields :

$\langle r.m.s.\ x\rangle = 0.3914 \pm 0.014$ mm
$\langle r.m.s.\ y\rangle = 0.3945 \pm 0.009$ mm
$\langle\hat{x}\rangle = 1.293 \pm 0.045$ mm
$\langle\hat{y}\rangle = 1.245 \pm 0.032$ mm.

Starting from these results, we shall now concentrate on 2 dedicated machines, namely # 1 and # 2 of the preceding section. Once again, one includes all the multipoles and then tracks in presence of misalignment and field errors. The resulting apertures are listed in Table 7 where one clearly observes that the effect of residual orbits on the tracking is very weak. The fact that the strongest effect happens at low momentum is absolutely consistent with our expectations.

Table 7 - Dynamic aperture in presence of both multipoles and misalignment errors.

Δp/p (%)	# 1	# 2
0.0	7	7
0.02	9	7
0.04	6	7
0.06	7	6
0.08	6	6
0.10	6	5
0.12	5	4
0.14	4	5

A graphical summary of the numerical results obtained in the previous sections is given in Fig. 1, which illustrates the dynamic apertures obtained when considering the sextupole components (a), all multipoles (b) and both multipoles and errors respectively (c).

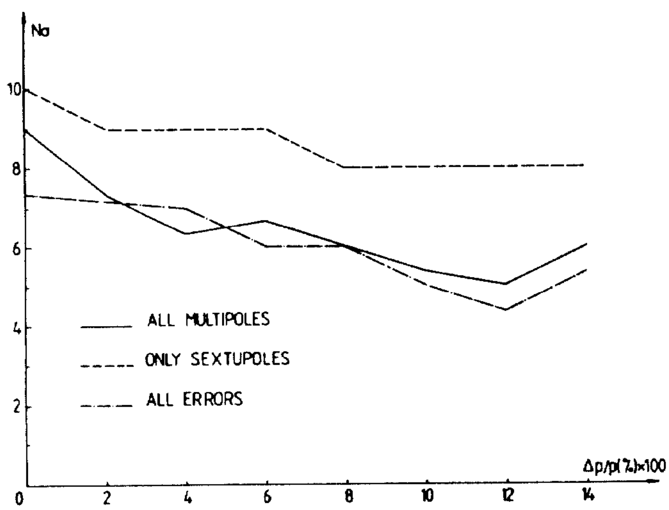

Fig.1 - Dynamic apertures for the different types of errors considered.

7. Possible Cures

Considering the results presented in Tables 3 and 7, it is quite obvious that the real limitation does not come from the presence of misalignment and field errors, but essentially from the effects of the higher-order multipole components. Keeping in mind that our final objective is to ensure an aperture of at least 5σ over the full momentum range, we could envisage the following solutions :

a) Modify the values of the tunes considered, in order to reduce coupling in the motion. First attempts did not indicate any real improvement.

b) Reduce both systematic and random b_3 by compensation and sorting of the magnets. This is certainly a helpful measure which will be retained for the future evaluations.

c) Reduce the higher-order multipole components by at least a factor of 3. For the present coil dimensions, the values considered for the multipoles are felt to be already very small, so that this does not seem to be very realistic.

d) Reduce the required emittances.

e) Reduce the cell-length.

Although first tests indicate that the counter-measures discussed under point b) indeed improved somewhat the situation, we still feel that our safety margin is not sufficient so that one should aim at a solution which definitively fulfills our requirements. A possible choice would be to reduce the required emittance as mentioned under d). This solution is not favoured, since the quoted value of 20π μm is inherent to the optimization of the electron-proton option, for which one is interested to have the maximum intensity in the proton beam. However, for the p-p operation the emittances will be 4 times smaller (5π μm) so that for the same aperture one then will obtain twice as much stable sigmas and therefore largely fullfil our basic requirements. The only reasonable choice seems to be to reduce somewhat the cell length. The actual concensus is to opt for a 100 m cell length which would yield an energy of 8 TeV. Both the corresponding cell (with 4 dipoles per half-cell) and preliminary insertions have been designed and the evaluation of the dynamic aperture in presence of both multipole components and misalignment errors is presently under study.

8. Conclusions

The aim of this study was to investigate the dynamical behaviour of the LHC machine in presence of both multipoles and misalignment errors. Contrary to the usual expectations that the sextupolar components would be the dominant terms for the limitation of the dynamic aperture, one observes that the higher-order multipoles are absolutely not negligible in the sense that they prevent us to guarantee a dynamic aperture of 6σ over the full range of momentum considered. It seems that the combination of b_5 (10-poles) and b_9 (18-poles) bears the main responsibility for these surprising results. In addition to this, the present study yielded two positive conclusions, namely that the presence of both systematic and random sextupole components could be mastered satisfactorily and secondly that the effects of misalignments and field errors had no dramatic implication on the resulting dynamic aperture. A similar study with the new 100 m cell-length should definitively confirm our expectations that the remaining limitations can be overcome.

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