# THREE-DIMENSIONAL CORRECTIONS TO THE LONGITUDINAL DYNAMICS OF A LONG BEAM IN A PERIODICALLY FOCUSED TRANSPORT SYSTEM*

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## Abstract

A series of "quasi three-dimensional" computer simulations is presented which examines the consequences of the transverse variation of the longitudinal self-electric-field on the dynamics of a coasting beam whose current is assumed to vary slowly over a distance long compared with the transverse dimensions of the beam system. In this approximation, the local longitudinal field is proportional to the product of the derivative of the line charge density and the local electrostatic potential. The longitudinal dynamics are examined by integrating the orbits of an ensemble of simulation particles in their transverse fields, and using the self-consistent potential generated during the simulation, along with an assumed value for the derivative of line charge density, to calculate the detailed longitudinal fields for integration of the longitudinal orbits.

## Introduction

Considerable progress has been made in transporting high current low emittance beams of charged particles for applications such as heavy ion beam ignited inertial fusion. Since it is necessary, in order to predict the evolution of beam emittance in these very low emittance beams, to follow the full self-consistent nonlinear dynamics of the beam, numerical simulations[1] have become a primary theoretical tool. Both simulations[2] and experiments[3,4] have now demonstrated the possibility of transporting intense beams in alternating gradient transport systems with no significant emittance growth. Studies of the evolution of beam emittance have, for the most part, been strictly valid primarily in the center of a long beam, because the longitudinal dynamics can then be neglected. The longitudinal dynamics of a long beam can, in turn, be separately examined if the longitudinal variation of the beam is assumed to occur on a scale-length long when compared with the transverse dimensions of the beam system. Even this simplified model is not strictly valid, however, because individual beam particles are subject to a longitudinal force which depends on their transverse position within the beam. R-Z simulations have shown[5], for example, that this transverse variation of the longitudinal force can be particularly significant when a beam is being slowly bunched. For an alternating-gradient transport system, however, the longitudinal dynamics of an intense beam are inherently three-dimensional. The purpose of this work is to present simulations which examine some of the details of these dynamics without resorting to full three-dimensional simulations.

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## Simulation Method

The electrostatic fields inside a perfectly conducting cylinder with constant cross section can be represented as the sum of a complete set of transverse eigenfunctions each with exponential longitudinal dependence. Calculation of the longitudinal field associated with each of these transverse modes is then straightforward, and the details will be omitted here. If the longitudinal current density is assumed to vary slowly over a longitudinal distance of the order of the transverse dimensions of the beam system, and it is further assumed that the shape of the transverse current distribution is unchanged, so that the coefficients of the various transverse eigenfunctions remain in the same ratio, then the local longitudinal electric field can be found from:

$$E_z = \frac{\partial\lambda}{\partial z}\,\Phi \qquad (1)$$

where $\Phi$ is the local electrostatic potential, and $\lambda$ is the line density, so that the current is $I = e\beta c\lambda$. It is often convenient to express the longitudinal field at the center of the beam as:

$$E_z = -eg\,\frac{\partial\lambda}{\partial z} \qquad (2)$$

so that the "g factor" becomes $g = -\Phi/e$. In the case of the center of a beam with radius a, in a pipe of radius b, $g = 1 - r^2/a^2 + 2\ln(b/a)$. This reduces to the familiar $g = 1 + 2\ln(b/a)$ at the center of the beam. Any particles not at the center therefore see a reduced longitudinal electric field. This reduction can be substantial for small b/a. As particles execute their betatron orbits, it is the average radius integrated over the orbit which determines the longitudinal acceleration in this model. Since calculating this average can be quite complex when the self consistent nonlinear fields of an intense propagating beam are considered, simulations have been employed to study the details of this process.

The numerical method is simple. A two dimensional transverse simulation program, SHIFT-XY has been modified so that in addition to integrating orbits in the transverse plane, the local potential, which is obtained at each time step, is multiplied by the appropriate constant and used to integrate the fields in the longitudinal, z, direction. The primary diagnostic which will be presented here, is the distribution of particles in longitudinal velocity, $n(v_z)$. This distribution is calculated by accumulating particles in the range of $v_z$ to $v_z+\delta v_z$ in a series of bins. One hundred bins are linearly distributed in the interval between zero and the velocity of a test particle held at the center of the pipe. The velocity of this test particle, which is the maximum velocity a particle can attain, is considered a measure of the g-factor of the beam system.

## Simulation Results

Probably the simplest case which can be discussed is a tenuous beam with a Kapchinskij-Vladimirskij (K-V) distribution propagating in a uniform focusing system. Because the local potential and the density distribution both vary quadratically with radius, the plot of the fraction of beam particles in any interval of longitudinal velocity, $n(v_z)$, will initially be uniform between the maximum and minimum beam velocities, at r=0 and r=a respectively. This is indeed observed in the simulations. As the system evolves and particles execute their betatron orbits each particle samples an average longitudinal field which depends on its average radius. If the longitudinal field is constant in time, a K-V distribution will tend to a delta function in velocity, since all particles have the same average radius. The simulation does in fact show a very rapid narrowing after only a few periods. After 25 betatron periods, the particles have a velocity distribution concentrated in two bins, or a velocity spread of order one percent.

When an intense beam with an initial K-V distribution is simulated, however, the beam rapidly goes unstable. After about 15 Larmor periods the beam has reached a steady state and the resulting $n(v_z)$ is almost identical to what is obtained from an initial semi-Gaussian (uniform in configuration space, Gaussian in velocity) distribution with similar initial rms parameters. While some further narrowing of the velocity spread seems to still be occurring after the initial rapid transient, this narrowing is on a much slower time scale, and is probably not appropriately followed in this nonself-consistent model.

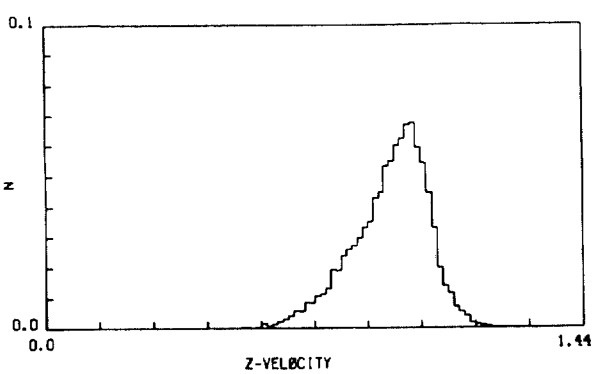


Fig. 1. Plot of $n(v_z)$, the number of beam particles in a given range of z-velocities, after 25 betatron periods, for an initially semi-Gaussian beam occupying 0.8 of the radius of a uniformly focused circular pipe. The horizontal axis is calibrated in effective g-factor. The vertical axis is the fraction of the beam in each of the hundred bins shown.

Figure 1, is a plot of the distribution of longitudinal velocities, after 25 Larmor periods, of an initially semi-Gaussian distribution in a uniform focusing system and in a circular conducting pipe. The horizontal scale is normalized to the velocity of a test particle which is held at the center of the system and is therefore a measure of the effective g-factor. For this case, the beam radius is 0.8 of the pipe radius, and the g factor measured this way is 1.445 compared with a calculated 1.446. The vertical axis is labeled with the fraction of the beam in each of the hundred bins used to accumulate the number density, so that a vertical value of unity would mean that all of the beam particles are in that bin. The beam intensity can be characterized by a tune depression of approximately a factor of ten.

Details of the longitudinal behavior, even in the simplified case under consideration here, can get considerably more complex when alternating gradient systems are considered. The beam is generally elliptical and varies during the focusing period, making even the behavior of a particle at the center tedious to calculate. A tenuous K-V beam in a thin lens focusing system with a phase advance of $60^\circ$ per cell, and with the major axis of the ellipse at lens center (the maximum extent of the beam) of 0.8 of the radius of a circular conducting pipe, is found to have a g-factor, for a particle held at the center, of 1.97. This would correspond to a circular beam of radius about 0.6 times radius of the pipe, which is between the major and minor radii of the elliptical cross section at the center of a lens. Otherwise the behavior of this beam is very similar to the circular beam in the circular pipe. All of the beam particles rapidly acquire velocities in a very narrow range.

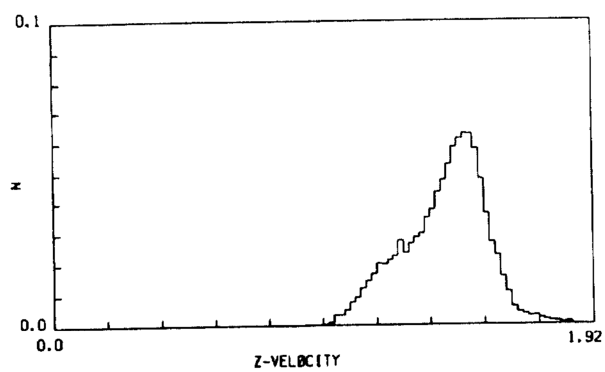


Fig. 2. The distribution $n(v_z)$, after 50 magnet periods, for an initially semi-Gaussian beam in an alternating-gradient transport system with $60^\circ$ phase advance depressed to $6^\circ$ by space charge. The major axis of the ellipse is 0.8 of the circular pipe at the center of a lens.

Figure 2 shows the distribution, $n(v_z)$, of an initially semi-Gaussian beam in a thin lens alternating gradient transport system with $60^\circ$ phase advance depressed by space charge to $6^\circ$. This curve is quite similar to what is obtained in a uniformly focused channel with a similar g-factor. The major consequence of the alternating gradient focusing seems, therefore, to occur as a consequence of the change in average radius resulting from the changing elliptical shape of the beam during each period.

A greater degree of complexity can be introduced if the conducting boundary consists of the electrodes in an electrostatic quadruple system. Figure 3 shows $n(v_z)$ after traversing 50 magnet periods of a $60^\circ$ alternating-gradient transport system with a factor of ten tune depression. The beam major axis is at 0.8 of the distance to the electrostatic quadruples. The quadruple electodes, which are semi-circular with radius 0.267 times the separation of their centers, were chosen to be the same as in previous simulation[6], and corresponds to the design of the LBL Single Beam Transport Experiment. Some increase in the g-factor is observed compared to the circular boundary. This increase is presumably due to the distortion of field lines near the electrodes. The primary difference in behavior, as compared with the previous case of the same focusing forces, but a circular pipe, appears to be a consequence of the distortion of the beam shape. The edges of the beam are pulled toward the electrodes, giving the beam a diamond shape and giving the beam distribution a greater spread in average radii as the outermost particles are pulled toward larger average radii and lower average g-factors.

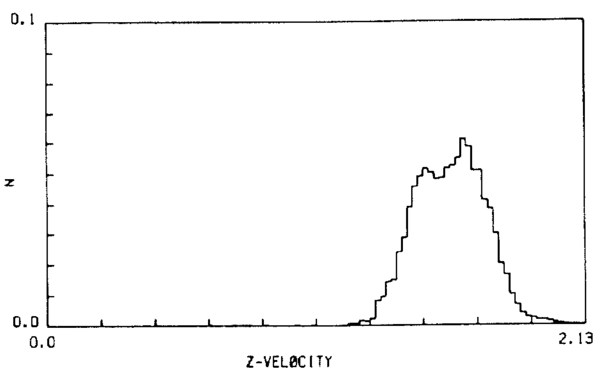


Fig. 3. $n(v_z)$ for a $60^o$ alternating-gradient transport system depressed to $6^o$, but with the major axis of the ellipse at 0.8 of the distance to a set of conducting electrostatic quadruples.

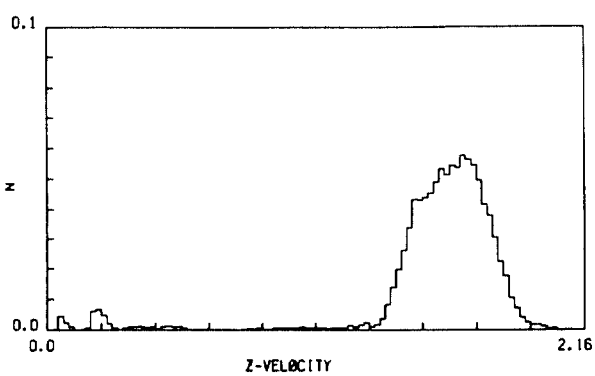


Fig. 4. $n(v_z)$, after 50 magnet periods, of an initially semi-Gaussian beam in the presence of electrostatic quadruple electrodes, and an initial offset of 0.05 times the distance between electrodes at an angle of $45^o$ relative to the direction of the electrodes.

Figure 4 is an illustration of the effect on the longitudinal dynamics of off-centering the beam. The beam in Fig. 3 is given an initial offset of magnitude of half of the distance from the edge of the beam to the electrode but at an angle of $45^o$. The primary consequence is that some of the beam particles appear to be pulled off the beam and close to the electrodes and so see a reduced longitudinal field.

## Conclusions

While it is difficult to specify precisely the extent of the validity of the "quasi three-dimensional" model used here to calculate the transverse variation of the longitudinal dynamics, some conclusions are still possible.

The method used assumes a particular model for the longitudinal fields, but whatever approximation is used for the longitudinal fields, the self-consistent transverse simulations are a good model for estimating the distribution of average radii during betatron orbits in a variety of transport system geometries. For example the similarity of behavior of the alternating-gradient and uniformly focused transport systems is another of many illustrations that have emerged that when phase advances well below $90^o$ are employed, the substitution of a smooth focusing force averaged over the magnet period appears to be a good representation of beam behavior. A consequence of this is the argument that the longitudinal behavior of an alternating-gradient transport system can be approximated by assuming an azimuthally-symmetric geometry so that r-z simulations are probably a good approximation to the longitudinal dynamics.

The large spread in average radii observed for reasonable parameters does, however, argue that those circumstances where transverse nonuniformities of the longitudinal forces can have a significant impact should be investigated further.

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