BEAM DYNAMICS IN HIGH-CURRENT RECIRCULATING ACCELERATORS WITH STELLARATOR/TORSATRON FIELDS

A. Prakash
U.S. Army Ballistic Research Laboratory
Aberdeen Proving Ground, MD 21005-5066

Abstract

In order to study the beam dynamics of high current beams in strong focusing fields used in new concepts of compact recirculating accelerators, we have derived a beam envelope equation for beam transport in vacuum, in a straight section which has an external $\ell = 2$ stellarator field applied in addition to a longitudinal magnetic field. We do not assume that the helical field is small compared to the longitudinal field, nor do we limit our treatment to the lower powers in r.

Introduction

High-current charged particle accelerators are an active area of research because of their various possible applications, including national defense. Many innovative recirculation concepts have been studied. We are here concerned with schemes that rely solely on magnetic fields for beam confinement.[1-10] High-current recirculating accelerators in which there is a rapid acceleration of charged particles as the beam repeatedly passes through a high-gradient electric field produced in a localized region by a convoluted parallel transmission line[1,2] or a dielectric cavity,[3] with the beam confined in a toroidal pipe by using (helical) torsatron fields, in addition to a (toroidal) solenoid field, combined with vertical magnetic fields in the curved sections, have been referred to as "rebatrons." In the "stellatron" accelerator concept,[4] there is a stellarator field added to the "modified betatron"[6,7] so that the beam undergoes a slow and continuous acceleration in the closed toroidal beam line. The race-track accelerator concept which has been referred to as "RIA,"[8] also relies on the stellarator field (together with vertical and toroidal fields) for beam confinement in the curved sections but uses ferromagnetic or ferrite cores in accelerating modules. All the above acceleration schemes can be lumped together under the generic name of "strong focused recirculating accelerators" (SFRA). The advantage of recirculating the beam in the same toroidal pipe until the acceleration is completed is not only the achievement of compactness in size but also compactness in weight of the beam line. However, in order to avoid the need for an extraction scheme, a strong-focused spiral line induction accelerator[9] has been studied conceptually and is undergoing experimental development. Rather than recirculating the beam in a closed loop in a single toroid, this scheme envisages recirculation of the beam through a ferrite or ferromagnetic induction module in off-centered beam line segments of the multiple turns of an open-ended spiral. It also relies on a $\ell = 2$ stellarator field in addition to vertical fields at the bends. Since there is recirculation through an accelerating module, although not in a single toroidal pipe, this concept can also be included in the generic name SFRA. In order to study the beam dynamics in such devices, one needs the beam envelope equation without any parameter restrictions.

Beam Envelope Equation

Thus, for the analysis and development of strong-focused high current accelerators, it is of great interest to obtain a differential equation of the radial beam envelope for vacuum transport of a high current charged particle beam in strong focusing (torsatron/stellarator) magnetic fields. We have derived a beam envelope equation for beam transport, in vacuum, in a straight pipe which has a stellarator field (produced by 2ℓ helically wound wires with currents flowing in opposite directions in the neighboring wires) in addition to a longitudinal magnetic field, without assuming that the helical magnetic field is small compared to the longitudinal field.

For a continuously rotating quadrupole field, in the approximation when only the lowest power in r of the focussing field is included, the radii of an elliptical K-V distribution beam were first analyzed by Gluckstern[11] for a straight beam transport line. With similar approximation, the $\ell = 2$ stellatron type beams have also been analyzed[9,12] for weak stellarator fields. Our analysis is carried out without any restrictions on r and the stellarator focusing field. We keep the exact ($\ell = 2$) field (Bessel function), together with a longitudinal field. Our treatment is in the spirit of a general formulation for beam envelope.[13]

For simplicity, we here develop the specialized beam envelope equation for the laminar flow of a cold beam of uniform density. Even without scattering, the canonical angular momentum is not conserved because of the presence of the non-axially symmetric helical field. In cylindrical coordinates, (r, θ, z), the externally applied field, being helically symmetric, can be written as a function of r and $\psi = \theta - z/\mu$, where $\mu = L/2\pi = 1/\alpha$, and L is the pitch of the helix:

$$B_r^e = 2\, b_2\, I_2'(2r/\mu) \sin 2\psi \tag{1}$$

$$B_\theta^e = \frac{\mu}{r}\, 2\, b_2\, I_2(2r/\mu) \cos 2\psi \tag{2}$$

$$B_z^e = B_0 - 2b_2\, I_2(2r/\mu) \cos 2\psi \tag{3}$$

where B_0 is the externally applied longitudinal field, b_2 is the strength of the $\ell = 2$ stellarator field and I_2 is the modified Bessel function.

The relativistic Lagrangian of a particle of charge q in an electromagnetic field $(\phi, \vec{A})$ is

$$L = - mc \sqrt{1-\beta^2} - q\,\phi + \frac{q}{c} \vec{A}\cdot\vec{V} \tag{4}$$

where $\beta = \frac{v}{c}$. Now,

$$\frac{\partial L}{\partial \dot{z}} \equiv P_z = \gamma\, m\, v_z + \frac{q}{c} A_z \tag{5}$$

and

$$\frac{\partial L}{\partial \dot{\theta}} \equiv P_\theta = r\,(\gamma m v_\theta + \frac{q}{c} A_\theta) \tag{6}$$

where $v_z \equiv \dot{z}$ and $v_\theta = r\dot{\theta}$. We can write $\vec{A} = \vec{A}^e + \vec{A}^s$, where the superscripts 'e' and 's' denote external and self-fields, respectively. The external fields, given by Eqs. (1)-(3), possess helical symmetry: They depend on r and $(\theta - z/\mu)$ only. When the $\ell = 2$ stellarator field is added to a uniform longitudinal guide field, the cross-section of the beam would become elliptical rather than circular. Although the beam does not then possess azimuthal symmetry, it is reasonable to assume that going along the z-axis the cross-section ellipse rotates with the externally applied fields, so that the self-fields have the same helical symmetry: $\vec{A}^s = \vec{A}^s\,(r,\ \theta - z/\mu)$, $\phi^s = \phi^s\,(r,\ \theta - z/\mu)$. Differentiation of Eq. (4) then gives

$$\mu\,\frac{\partial L}{\partial z} + \frac{\partial L}{\partial \theta} = 0$$

and Lagrange's equation can be combined to give

$$\mu\,P_z + P_\theta \equiv K = \text{constant.} \tag{7}$$

Thus,

$$\frac{q}{c}\,(\mu\,A_z^e + rA_\theta^e) + \frac{q}{c}\,(\mu A_z^s + rA_\theta^s) + + \mu\gamma m v_z + \gamma m v_\theta\, r = K \tag{8}$$

is a constant of the motion.

For the external fields given in Eqs. (1)-(3), one finds that

$$\mu A_z^e + rA_\theta^e = B_0\,\frac{r^2}{2} - \mu r b_2 I_2'\cos 2\psi\quad . \tag{9}$$

Substituting Eq. (9) and the self-fields of the beam of rotating elliptical cross-section into Eq. (8), we obtain, assuming that B_θ^s is the dominant magnetic self-field, for a particle at the beam surface

$$v_\theta = (\gamma r)^{-1}\left[\frac{K}{m} - \frac{\Omega}{2}\,r^2 + \mu r\Omega_2 I_2'\cos 2\psi - \mu\gamma\eta\, v_z\right] \tag{10}$$

where $\Omega = qB_0/mc$ and $\Omega_2 = qb_2/mc$;

$\eta = 1 - \frac{\omega_p^2}{4c^2\gamma}\,r^2\,(2 - \frac{r}{r_0})$, $\omega_p^{\,2} = \frac{4\pi n q^2}{m}$, and r_0 is the radius of the circular cross-section of the beam when only the longitudinal field, B_0, is applied. With the help of Eq. (10), the radial component of the transverse equation of motion

$$\ddot{r} + \dot{\gamma}\,\frac{\dot{r}}{\gamma} - \frac{v_\theta^{\,2}}{r} = \frac{e}{\gamma m}\left[E_r + \frac{1}{c}\,(v_\theta\, B_z - v_z\, B_\theta)\right] \tag{11}$$

then leads to the radial envelope equation of a cold (low emittance) beam, for the case of no acceleration ($\dot{\gamma} = 0$), in the paraxial approximation as

$$\ddot{R} + (\frac{\Omega}{2\gamma})^2\,R - \frac{\omega_p^{\,2}}{2\gamma^3}\,R\,(2 - \frac{R}{r_0}) = \frac{(K/m)^2}{\gamma^2 R^3} -$$
$$\frac{2\,(K/m)}{\gamma^2\,R}\left[(I_2 - \frac{\mu}{R}\,I_2')\,\Omega_2\cos 2\psi + \frac{\mu\zeta}{R}\right] -$$
$$\frac{2\mu}{\gamma^2}\,\Omega_2 I_2\cos 2\psi\,\left[\Omega_2 I_2'\cos 2\psi - \frac{\zeta}{R}\right] +$$
$$\frac{\mu^2}{\gamma R}\,\left[\Omega_2 I_2'\cos 2\psi - \frac{\zeta}{R}\right]^2 \tag{12}$$

where $\zeta = \gamma\beta c - \frac{\beta}{c}\,\frac{\omega_p^{\,2}}{4}R^2\,(2 - \frac{R}{r_0})$.

Applications of this envelope equation, together with its generalization for large emittance beams, and deductions therefrom, are the subject of another paper.[14] We here note that when the stellarator field is turned off ($\Omega_2 \to 0$, $\mu \to 0$), then, by Eq. (7), $K \to P_\theta$, and our envelope equation reduces to the familiar envelope equation of a high current beam in a longitudinal external magnetic field (with the emittance term dropped).

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