

An Interference Wiggler for Precise Diagnostics of Electron Beam Energy

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Summary

Relativistic electrons passing through two identical magnetic sections generate synchrotron radiation whose spectrum is strongly modulated as the photon energy varies. The modulation is caused by the interference of radiation from each section, and has been observed [1] in the spectrum of spontaneous radiation from transverse optical klystron which utilizes two undulators. In this paper, we analyze and apply another device based on two simple wigglers. The device, which will be called the interference wiggler, can be used for precise diagnostics of electron beam energy: by analyzing the modulated spectrum with a monochromator, the electron energy can be determined up to an accuracy of 10^{-3} or 10^{-4} . In this paper we develop general design criteria for interference wigglers. We also give several example designs to measure the electron energy to an accuracy 10^{-4} for the planned electron beam facility at CEBAF [2], and to an accuracy 10^{-3} for the 1-2 GeV Light Source at Berkeley [3].

Spectrum of Interference Wiggler

The electron trajectory in an interference wiggler is shown in Fig. 1. The trajectory will be assumed to lie on the horizontal plane. In wiggler approximation [4], the radiation in the direction (ϕ, ψ) , where ϕ and ψ are respectively the horizontal and vertical angles, comes mainly from small segments of electron trajectory about the points where the slope is parallel to ϕ . For the trajectory in Fig. 1, there are in general four such points labelled 1, 2, 3 and 4. Among these, we will consider only 1 and 2, assuming either that 3 and 4 are sufficiently separated transversely from 1 and 2 so that they can be considered separately, or that the radiation intensity from 3 and 4 is much weaker due to weaker magnetic field. Computing the electric field from 1 and 2 and squaring it one obtains the angular density of flux. The result, when the effect of the electron beam angular divergence is taken into account, is as follows:

$$\frac{d^2 \mathcal{F}}{d\phi d\psi} = 2 \frac{d^2 \bar{\mathcal{F}}}{d\phi d\psi} (1 + f_x f_y \cos \alpha) \quad (1)$$

where

$$f_x = \exp[-(\sigma_x k L (1 + g) / \gamma_0 \delta_x^2)^2] \quad (2)$$

$$f_x = (1 + \delta_x^4)^{-1/4} \exp[-k L \phi^2 \delta_x^2 / 2(1 + \delta_x^4)] \quad (3)$$

$$f_y = f_x(\phi \rightarrow \psi, \delta_x \rightarrow \delta_y) \quad (4)$$

$$\delta_x = \sqrt{kL} \sigma_x, \quad \delta_y = \sqrt{kL} \sigma_y \quad (4)$$

$$\alpha = \frac{kL}{2} \left[\frac{1+g}{\gamma_0^2} + \frac{\phi^2}{1+\delta_x^4} + \frac{\psi^2}{1+\delta_y^4} \right] + \frac{1}{2} (\tan^{-1} \delta_x^2 + \tan^{-1} \delta_y^2) \quad (5)$$

and $k = 2\pi/\lambda$, λ = radiation wavelength, σ_x = electrons' relative energy spread (rms), L = the distance between two wigglers (see Fig. 1), γ_0 = the average electron energy in unit of rest energy, g is defined so that $L(1 + g/2\gamma_0^2)$ is the arc length of electron trajectory between the two crests in Fig. 1, σ_x and σ_y are respectively the horizontal and vertical angular spread of electron beam (rms).

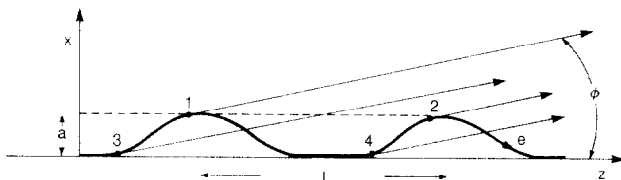


Fig. 1. A schematic representation of the electron trajectory in the interference wiggler. Radiation in the direction ϕ is generated from small segments of the trajectory around the tangent points, marked as 1, 2, 3 and 4.

In Eq. (1), $d^2 \bar{\mathcal{F}} / d\phi d\psi$ is the angular density of flux from point 1 or 2 alone, which is a smooth function of photon energy represented by the dotted curve in Fig. 2. The term proportional to $\cos \alpha$ is due to interference and causes the modulation of the spectrum represented schematically as the solid curve in Fig. 2. An equation similar to Eq. (1) was first derived by Ellaume [1] in the analysis of the spontaneous radiation from transverse optical klystron which is a two undulator system.

For a complete characterization of source, it is necessary to calculate the flux density in phase space known as the brightness by using the method discussed in Ref. 5. The results are in accord with the expectation that the sources at 1 and 3, for example, appear to be separated transversely. In forward direction, the source separation is given by the maximum excursion amplitude a of electron trajectory (see Fig. 1).

Method of Determining γ_0 and σ_x

The modulated spectrum has peaks when $\alpha = 2\pi n$, n being an integer. In this paper, we consider only the forward direction $\phi = \psi = 0$. Using Eq. (5), and neglecting for the moment the last two terms, the location of n th peak k_n is found to be

$$k_n = \gamma_0^2 (4\pi / L(1 + g)) n \quad (6)$$

From this, it follows for any pairs of integers (n, m) that

$$n = m k_n / (k_{n+m} - k_n) \quad (7)$$

The location of peaks k_n and k_{n+m} can be determined by analyzing the spectrum with a monochromator. The integer m can be determined by counting the number of peaks between k_n and k_{n+m} . We can thus determine the integer n associated with k_n . The electron energy γ_0 is then determined from Eq. (6).

To discuss the measurement accuracy, let Δ indicate the error in the measurement. We obtain from Eqs. (7) and (6) that

$$\frac{\Delta n}{n} = \frac{\Delta k_n}{k_n} + \frac{\Delta(k_{n+m} - k_n)}{k_{n+m} - k_n} \quad (8)$$

$$\frac{\Delta \gamma_0}{\gamma_0} = \frac{1}{2} \left[\frac{\Delta k_n}{k_n} + \frac{\Delta L(1 + g)}{L(1 + g)} \right] \quad (9)$$

For an unambiguous determination of n it follows from Eq. (8) that the monochromator bandwidth $\Delta k_n / k_n$ needs to be smaller than $1/n$ and that the spectrum needs to be observed over a wide range of k so that $k_{n+m} - k_n$ is of order k_n . From Eq. (9), it follows that both the monochromator bandwidth and the errors in the magnet parameters should be about the

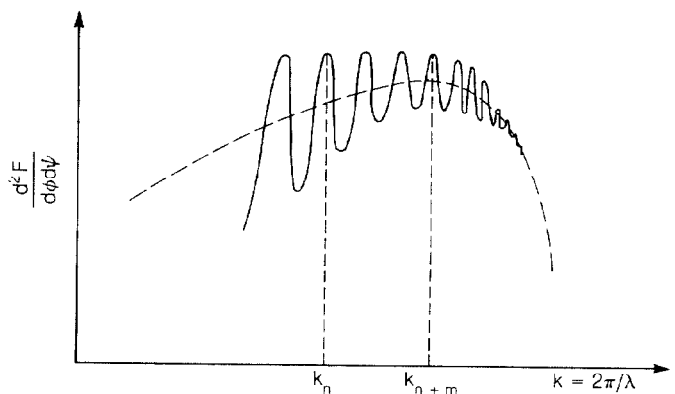


Fig. 2. A schematic representation of the spectrum from the interference wiggler. The solid line shows the modulated spectrum while the dotted line represents the smooth background due to radiation from individual wigglers. For the examples considered in the paper, hundreds of peaks would appear in the operating spectrum range.

desired accuracy in electron energy measurement, $\Delta\gamma_0/\gamma_0$. We take $\Delta\gamma_0/\gamma_0$ be of the order σ_e , the electron beam energy spread. Thus

$$\frac{\Delta\gamma_0}{\gamma_0} \sim \sigma_e \cdot \frac{\Delta k_n}{k_n} \leq \sigma_e \cdot \frac{\Delta L(1+g)}{L(1+g)} \leq \sigma_e \quad (10)$$

The electron energy spread σ_e can be determined from the wavelength dependence of the modulation amplitude through the factor f , given by Eq. (2).

Magnet Structure

The wiggler, two of which make the interference wiggler, is a three-pole device. We assume a simple step-function profile of magnetic field in the wiggler, as shown in Fig. 3. The wiggler consists of one full-pole of length d and field B_0 , two half-poles of length $d/2$ and field $-B_0$, field free space of $d/2$ between poles and $d/4$ at both ends, the total length being therefore $L = 3.5d$. For such a magnet, one can derive

$$g = 4.2 K^2, \quad a = 2.93 Kd/\gamma_0 \quad (11)$$

where

$$K = B_0(T) d(\text{cm}) \quad (12)$$

is a dimensionless parameter analogous to the usual deflection parameter in sinusoidal wigglers.

Design Considerations

We assume that the spectrum is to be observed in a range covering a factor two in wavelength, from $\bar{\lambda}$ to $2\bar{\lambda}$, or in wave numbers from $k = 0.5 \bar{k}$ to \bar{k} , where $\bar{k} = 2\pi/\bar{\lambda}$. The factor f should not be small so that the spectral peaks can be resolved easily and at the same time should have significant variation over the operating wavelength range to allow a measurement of σ_e . Hence we require

$$\sigma_e \bar{k} L(1+g)/\gamma_0^2 = 1 \quad (13)$$

The factor f then varies between 0.37 at $\lambda = \bar{\lambda}$ and 0.78 at $\lambda = 2\bar{\lambda}$. From Eqs. (11) and (13), and assuming $g \gg 1$, one obtains

$$K = 1.97 \times 10^{-6} \gamma_0 \sqrt{\bar{\lambda}(\text{\AA})/L(\text{m})} \sigma_e \quad (14)$$

To utilize large photon flux, it is sometimes advantageous to require in addition that λ corresponds to the critical photon energy

$$\epsilon_c(\text{keV}) = 0.665 B_0(T) E^2(\text{GeV}) = 1.24 \bar{\lambda}(\text{\AA}) \quad (15)$$

Combining (14) and (15), one obtains

$$K = 0.202/\sigma_e^{1/3} \quad (16)$$

We also require the following so that the factors f_x and f_y are not too small:

$$\delta_x^2 = kL\sigma_x^2 \leq 0(1) \quad \text{and} \quad \delta_y^2 = kL\sigma_y^2 \leq 0(1) \quad (17)$$

Finally, the sources at points 1 and 2 in Fig. 1 need to be sufficiently separated from those at 3 and 4. Thus, the excursion amplitude a should

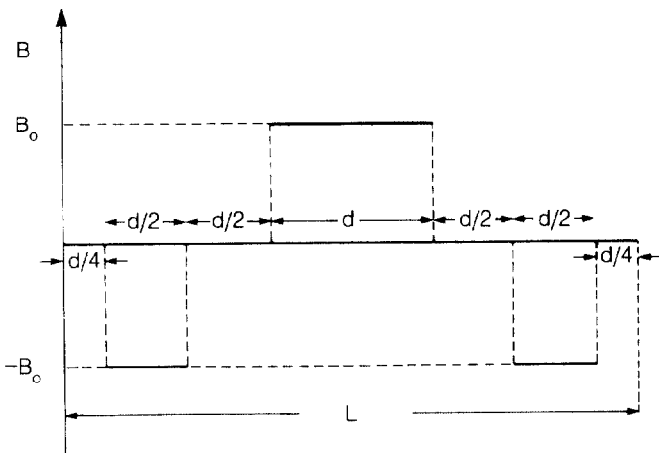


Fig. 3. Magnetic field configuration of the component wiggler.

be sufficiently larger than the rms beam dimension σ_x , say by a factor of 2. Thus we obtain from Eq. (11) that

$$a = 2.93 Kd/\gamma_0 \geq 2 \sigma_x \quad (18)$$

so far, we have not taken into account the finite angular acceptance of monochromators. Doing so would replace σ_x^2 and σ_y^2 in all expressions by $\sigma_x^2 + \Delta_x^2$ and $\sigma_y^2 + \Delta_y^2$, respectively, where Δ_x and Δ_y are the rms monochromator acceptances in the appropriate directions. Therefore the inequalities in Eq. (17) also determine the maximum monochromator acceptance: If the inequalities are marginally satisfied, $\Delta_x(\Delta_y)$ should be smaller than $\sigma_x(\sigma_y)$. If $kL\sigma_x^2$ and $kL\sigma_y^2$ are much smaller than unity, the monochromator acceptance is determined by $kL\Delta_x^2 = kL\Delta_y^2 \sim 1$.

From Eqs. (6) and (13), one obtains $n = 1/4\pi\sigma_e$. The second inequality in (10) then becomes $\Delta k/k \leq 1/4\pi n$. Thus the monochromator resolution is at least ten times smaller than the relative spacing between spectral peaks. Such resolution is consistent with the requirement that the monochromator should be able to resolve the spectral peaks.

We have neglected the last two terms in Eq. (5). The condition that these terms does not introduce further errors in the energy determination is $\tan^{-1}\delta_x^2 \leq (kL(1+g)/\gamma_0^2)\Delta\gamma/\gamma_0$. In view of Eqs. (10) and (13), this inequality is always satisfied.

Design Examples

(I) CEBAF⁽²⁾. An accurate energy diagnostic is important for precise nuclear physics experiments at CEBAF. The electron beam parameters are $E = 4 \text{ GeV}$ ($\gamma_0 = 8000$), $\sigma_e = 10^{-4}$, $\sigma_x = \sigma_y = 10^{-2} \text{ mrad}$, $\sigma_x = \sigma_y = 10^{-2} \text{ mm}$ (we assume that $\sigma_x\sigma_y = 10^{-9} \text{ m-rad}$, β_x (beta function) = 10 m). For design at critical energy, we find from Eq. (16) that $B_0(T)d(\text{cm}) = 4.36$. Thus a possible set of magnet parameters are $B_0 = 0.436 \text{ T}$, $d = 10 \text{ cm}$, $L = 35 \text{ cm}$. Such magnet should be feasible with a gap less than 1 cm. The wavelength $\bar{\lambda}$ corresponding to the critical energy (4.64 keV) is 2.67 Å. Simple monochromators based on crystal can be built in the operating wavelength range between $\bar{\lambda}$ and $2\bar{\lambda}$ with resolution better than the required value $\Delta\lambda/\lambda = 10^{-4}$. The length of the interference wiggler is $2 \times 35 \text{ cm}$. We obtain $\delta_x^2 = 1.65$ at $\lambda = \bar{\lambda}$. Thus the inequalities (17) are marginally satisfied. The relative modulation amplitude ($f_x f_y$) varies between 20% to 70%, which is still substantial. The source separation requirement, Eq. (18), is well satisfied. It is expected that a photon flux of $2\eta \times 10^6 \text{ photons/sec}$ ($\Delta\lambda/\lambda = 10^{-4}$) will emerge at the monochromator exit, where η is the optical efficiency of the monochromator.

The short length of the magnet and the simplicity of the crystal monochromator makes the interference wiggler an attractive device for energy diagnostics at CEBAF.

(II) The 1-2 GeV Light Source at Berkeley.³ The electron beam parameters are $E = 1.5 \text{ GeV}$ ($\gamma_0 = 3000$), $\sigma_e = 10^{-3}$, $\sigma_x = 2 \times 10^{-2} \text{ mrad}$, $\sigma_y = 1 \times 10^{-2} \text{ mrad}$, $\sigma_x = 2 \times 10^{-1} \text{ mm}$, $\sigma_y = 4 \times 10^{-2} \text{ mm}$. It turns out that the design at critical energy requires monochromators based on grazing incident gratings, which is cumbersome. We shall instead set the wavelength range to be between $\bar{\lambda} = 500 \text{ \AA}$ and 1000 \AA , for which normal incidence monochromators with resolution well beyond the required 10^{-3} are readily available. Using Eq. (14), we obtain $B_0(T)d(\text{cm}) = 3.74/\sqrt{L(\text{m})}$. A possible magnet parameters are $d = 20 \text{ cm}$, $L = 70 \text{ cm}$ and $B_0 = 0.223 \text{ T}$. With these, Eqs. (17) and (18) are both satisfied and photon flux of $5\eta \times 10^{11} \text{ photons/sec}$ ($\Delta\lambda/\lambda = 10^{-3}$) is expected ($\eta = \text{optical efficiency}$). Accurate diagnostics of electron beam energy should be useful for the machine physics study.

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