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EFFECT OF INSERTION DEVICES ON BEAM PARAMETERS

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Abstract

Nowadays, insertion devices have installed in many storage rings to serve for synchrotron light experiments, or for other purposes. In the future, insertion devices will become major sources of synchrotron radiation. Therefore it is important to study the effects of the insertion devices on beam parameters of the rings, both theoretically and experimentally^{1,2,3,4}. In this paper we derive linear equations that describe the motion of an electron around the central orbit in the insertion devices. By using the equations, we can calculate the distortions of betatron and dispersion functions, the changes of the beam emittance and so on. A correction method of the tune shifts and the distortion of betatron function⁵ is also presented. More details have been given in Refs. 5 and 6, and will also appear in a KEK Report.

I. Equation of Motion in a Periodic Magnetic Field⁶

The magnetic field at the center of an insertion device is assumed to be given by,

$$B_{y} = C \sin \frac{2\pi}{\lambda} z \quad \text{at } y=0 , \qquad (1)$$

where z is the longitudinal direction, λ the period of the magnetic field. We further assume that the magnetic field is uniform in the x-direction perpendicular to y- and z-directions, so that we can take C as a constant. From Eq. (1) and the Maxwell equations, we can immediately obtain,

$$B_{y} = B_{0} \cosh(ky) \sin(kz), \qquad (2)$$

where $k{=}2\pi/\lambda,$ and it has been assumed that any uniform magnetic field does not exist.

The Central Orbit

when the velocity of an electron is almost equal to the velocity of light and the magnetic field is not so strong, the motion of the central orbit can be approximately described by,

$$X = \sin(kz)/\rho_{w}k^{2} \quad \text{for } X(0) ,$$

$$\frac{dX}{dz} = \theta = \cos(kz)/\rho_{w}k , \qquad (3)$$

where X is the x-coordinate of the central orbit, (Bp) the magnetic rigidity, and ρ the bending radius with respect to the maximum field^W at the center. From the second equation in (3), we have the well-known result of the maximum deflection angle of the orbit θ_{max} ,

$$\theta_{max} = X'_{max} = 1/\rho_{w}k = K/\gamma$$
, (4)

where K is so-called K-parameter given by,

$$K = \frac{\lambda}{2\pi} \frac{eB}{m_{c}c} ,$$

and γ is the Lorentz factor.

Equation of Motion around the Central Orbit

First we take new coordinates on the central orbit as shown in Fig. 1, while the new y-coordinate is the same as the old one. From the geometry in Fig. 2, B can be approximately written as,

$$B_{y} = B_{0} \cosh(ky) \sin(k(s-x \sin\theta))$$

$$\sum_{n=0}^{\infty} B_{0} \cosh(ky) (\sin(ks)-k x \theta \cos(ks)) .$$
(5)

When x and y are relatively small, we can retain only their first order terms, and in the same context put the s to be equal to the z. Thus we obtain the linear equations of motion around the central orbit,

$$\frac{d^2 x}{ds^2} - \frac{\cos(2ks)}{\rho_w^2} x = \frac{\sin(ks)}{\rho_w} \frac{\Delta p}{p},$$

$$\frac{d^2 y}{ds^2} + \frac{\cos^2(ks)}{\rho^2} y = 0.$$
(6)



II. Effects on the Orbit Parameters⁶

By using Eq. (6), We here list up the tune shift, the distortions of the β - and n-functions, the changes of the synchrotron integrals, and the change of the emittance.

Tune Shift Δv

From Eq. (6), the tune shifts due to an insertion device are approximately given by,

$$\Delta v_{\mathbf{x}} \sim 0 , \quad \Delta v_{\mathbf{y}} \sim L\overline{\beta_{\mathbf{y}}}/8\pi \rho_{\mathbf{w}}^{2} , \qquad (7)$$

$$\overline{\beta} = (\beta_{2} + \beta_{1} + \beta_{0} (1+\alpha_{1}\alpha_{2}))/3 , \qquad (7)$$

where it is assumed that β changes slowly in the device, while the terms such as cos(ks) are rapidly changing ones. The L is the length of the device, the β stands for the average of β in the device, and the subscripts 1 and 2 denote the ends of the device, and β_0 may be written as $\beta_0 = (1+\alpha_\perp^2)/\beta_1$.

Distortion of the β -function, $\Delta\beta/\beta$

Using the averaged focusing strength in the device,

$$\Delta \tilde{K}_{\mathbf{x}} \gtrsim 0 , \quad \Delta \tilde{K}_{\mathbf{y}} \gtrsim 1/2\rho_{\mathbf{w}}^2 , \qquad (8)$$

we can obtain an approximate expression of $\Delta\beta/\beta$ as,

Table I. Numerical Examples of the PF ring

	L _u (m)	λ (cm)	в _о (т)	Δυ y		$\Delta \hat{\beta}_{y} / \beta_{y} (\%)$		$\frac{\Delta \varepsilon}{x} \frac{\varepsilon}{x} $ (%)	
				(#5)	(#4)	(#5)	(#4)	(#5)	(#4)
B01-B02 (U)	3.63	6	0.45	0.0054	0.0054	4	4	-0.15	-0.88
B15-B16 (MPW)	3.18	10	1.5	0.0525	0.0523	41	41	11.8	-7.12
B18-B19 (U)	2.35	10	0.5	0.0023	0.0041	2	3	-0.47	-0.51

$$(\Delta\beta/\beta)_{x} \stackrel{\sim}{\sim} 0 , (\Delta\beta/\beta)_{y} \stackrel{\sim}{\sim} \frac{-L\overline{y}\cos(2\nu_{y}(\pi - |\phi - \phi_{1}|))}{4\rho_{w}^{2}\sin(2\pi\nu_{y})} , (9)$$

where ϕ_1 may be taken as the betatron phase at the center of the device. Hence the maximum distortions around the ring are,

$$(\Delta \widehat{\beta} / \widehat{\beta})_{\mathbf{x}} \approx 0$$
, $(\Delta \widehat{\beta} / \widehat{\beta})_{\mathbf{y}} \approx |2\pi \Delta v_{\mathbf{y}} / \sin 2\pi v_{\mathbf{y}}|$. (10)

Distortion of the Π -function , $\Delta \eta$

First, we assume that before the installation of the device, $\eta = \eta' = 0$ at its place, and $s = -\lambda/4$ at the entrance of the device. Then the first-order expression of the η generated by the device is written as,

$$n_{x}^{W} = -(1 + \sin(ks))/\rho_{w}k^{2}, \qquad (11)$$
$$n_{x}^{W} = -\cos(ks)/\rho_{w}k.$$

The order of magnitude of the second-order one, Δn_{χ}^{W} , becomes,

$$\Delta n_{\mathbf{x}}^{\mathbf{W}} \sim 1/\rho_{\mathbf{w}}^{3} \mathbf{k}^{\mathbf{u}} \,. \tag{12}$$

Since the order of magnitude of η^w_x is $1/\rho_w^2$, $\Delta\eta^w_x$ is negligible compared with η^w_x if

$$1/\rho_{w}^{2}k^{2} = (KY)^{2} << 1$$
.

This condition would be usually satisfied.

In the case of non-zero n, the n may be distorted due to the installation of the device. The order of magnitude of the distortion is roughly given by $\max(n/k^2\rho^2)$, $\ln'/k^2\rho^2$). Therefore if $n<<\rho$, and $\ln'<<\rho$, then the n generated by the device, n_x^{\vee} , is larger than the distortion of n itself.

Changes of Synchrotron Integrals

The useful synchrotron integrals are defined as,

$$I_{1} = \int n/\rho \, ds \quad , \quad I_{2} = \int 1/\rho^{2} \, ds \quad ,$$

$$I_{3} = \int 1/|\rho|^{3} \, ds \quad , \quad I_{4} = \int n/\rho^{3} \, ds \quad , \qquad (13)$$

$$I_{5} = \int H/|\rho|^{3} \, ds \quad , \quad H = \gamma n^{2} + 2\alpha nn' + \beta n'^{2} \quad .$$

In the following, the changes of the above integrals due to the device are designated by their small letters. The i_2 and i_4 would be usually small, and is expressed in the case of zero dispersion as follows;

$$i_1 = -L/2\rho_w^2 k^2$$
, $i_2 = -3L/8\rho_w^4 k^2$. (14)

The i_2 and i_3 are given by,

$$i_2 = L/2\rho_w^2$$
, $i_3 = 4 L/3\pi\rho_w^3$. (15)

The i5 consists of several terms,

$$i_5 = i_5^0 + i_5' + i_5^c$$
,

U : Undulator

MPW : Multipole Wiggler

with $i_5' = i_{51}' + i_{53}'$. and individual terms are written as,

$$\begin{split} \mathbf{i}_{5} &= 4 \mathbf{L} \ \overline{\mathtt{H}} / 3 \pi \rho_{w}^{3} \ , \quad \mathbf{i}_{51} \ = \ 4 \mathbf{L} \ \overline{\beta} / 15 \pi \rho_{w}^{5} \mathbf{k}^{2} \\ \mathbf{i}_{53} \ &= \ 12 \mathbf{L} \ \overline{\mathtt{Y}} / 5 \pi \rho_{w}^{5} \mathbf{k}^{4} \ , \ \mathbf{i}_{5}^{c} \ &= \ -8 \mathbf{L} \ (\overline{\mathtt{an}_{0}} + \mathtt{yn}_{0}) / 3 \pi \rho_{w}^{4} \mathbf{k}^{2} \ . \end{split}$$

Usually we can neglect the i_{53} ' compared with i_{51} ', because,

$$i_{53}/i_{51} \gtrsim 9(\lambda/\overline{B})^2/4\pi^2$$
.

Further in the case of non-zero $\eta, \ i_5'$ and $i_5^{\ c}$ would be negligible, if

$$\overline{\eta}/\overline{\beta} >> K/\gamma$$
 and $\overline{\eta}/\overline{\lambda} >> K/\gamma$.

Change of the Emittance

Using the synchrotron integrals, the change of the emittance, $\Delta \varepsilon / \varepsilon$, can be simply estimated by,

$$\Delta \varepsilon / \varepsilon = \mathbf{i}_{5} / \mathbf{I}_{5} - \mathbf{i}_{2} / \mathbf{I}_{2} \quad . \tag{16}$$

Nonlinear effects

The nonlinear effects of the device on the beam parameters are now under study. It implies that these effects would be serious even if the device are perfectly built, because its magnetic field has a form of (2). The order estimate of the octupole terms is k^2L/ρ^2 , so that when the field is strong and the period is short, this effect can not be ignored.

Numerical Examples

Some numerical examples for the PF Storage Ring are given in Table I. The MPW⁷ between B15 and B16 as well as the existing VW has serious effects on the beam parameters as seen in the table. When the MPW is installed in the ring, the emittance for the normal mode operation will grow up about 10 %, because of the non-zero n at its place of installation.

III. Correction Method⁵

We briefly describe a method of the simultaneous correction of the tune shifts and the distortions of β , which has already adopted at the PF Storage . In this method, the k-values of some Ring quadrupole magnets are changed by small amounts. The tune shifts and the distortions of β produced by these changes may counterbalance the effects of the device if we select an adequate set of quads. Generally, the correction of the tune shifts are more stringent requirement than that of the distortions of β . Therefore, we adopted a correction scheme in which the distortions of β are minimized by using the least squares method under the constraint that the tunes are kept constant or changed to desired values (the method of Lagrange's multipliers).

Some results of the computer simulations of this method are shown in Figs. 2 to 5. Figs. 2 and 3 show the simulated distortions of β caused by the VW and the MPW for the new low-emittance optics. The solid lines in the figures denote the results calculated by



transfer matrices, and the broken lines denote a thin-lens approximation of the insertion devices. Figs. 4 and 5 show the remains of the distortions after the corrections are made by using the quads indicated in the figures. The results of the actual application of this method are presented in Ref. 4.

Acknowledgements

We would like to thank Profs. K. Huke and M. Kihara for their encouragement.

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