THE SIDEBAND INSTABILITY IN FREE ELECTRON LASER*

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Abstract

The one-dimensional fast-time averaged Hamiltonian H_o for an electron passing through a constant parameter wiggler and a radiation field is derived. Integration of the linearized Vlasov equation with perturbing sidebands over the unperturbed orbits found from H_o yields the sideband growth rates including both trapped and untrapped particles. The growth rates for the upper and lower sidebands are found absolutely symmetric, while stability is determined by the sign of $df_o/d\omega_b$, where f_o is the zeroth order distribution of the asymptotic wave particle equilibrium evolving along the unperturbed motion from some given initial distribution, and ω_b is the bouncing frequency in the ponderomotive well.

Introduction and Summary

The growth of parasitic modes at frequencies near the main signal frequency during high power FEL operation was theoretically predicted [1,2] in early 1980's. Since then there has been ample numerical [3,4] and experimental [5,6] evidence of sideband excitation in constant wiggler FELs. Unstable modes in variable wiggler FELs have also been observed in simulations [7-9] and recently in experiment [10]. Sidebands degrade the main signal efficiency and optical quality by channeling a considerable fraction of the power into parasitic frequencies. The performance of the mirrors in an oscillator can be harmed from the modulation of the wave envelope caused by the sidebands. Last, but not least, interaction among nearby sidebands above a certain amplitude may lead to chaotic particle motion, loss of trapping and incoherent radiation.

The above have stimulated a considerable amount of theoretical work focused on sideband growth. Simple one-dimensional configurations that are analytically tractable have been used to model the situation. Two lines of approach have been considered. The single particle picture regards the particle trajectories as functions of the initial conditions and computes the gain by ensemble averaging over initial distributions [7-9]. The alternative approach assumes some adiabatic equilibrium between the particles and the main signal and examines the stability of the perturbations induced by the sidebands, solving the kinetic equation [11,12]. Because of the equilibrium assumption the kinetic method is more appropriate for FEL operation as an amplifier. In both treatments so far, analytic results have been obtained only for particles localized near the bottom of the ponderomotive well. This implies the following limitations: The sideband spectrum becomes discrete

$$\omega_s = \omega_r \pm (k_r/k_w)n\omega_b(0), \quad k_r/k_w \simeq 2\gamma_z^2, \qquad (1)$$

where $\omega_b(0)$ is the bounce frequency at the bottom of the bucket, k_r k_w are the radiation and wiggler wave numbers respectively and $\gamma_z=(1-v_z^2/c^2)^{-1/2}$. The contribution from untrapped particles and trapped particles away from the bottom is neglected. The effect of the shear $d\omega_b/dJ$, where the action J parametrizes the distance from the centre of the separatrix J=0, is lost.

Here canonical formalism is introduced by expressing the unperturbed particle orbits in terms of action-angle variables. The unperturbed orbits, shown in Fig. 1(a), are the fast time averaged "synchrotron" oscillations of the electrons in the potential well formed by the combined action of the wiggler and the radiation signal. The perturbed kinetic equation is solved in action space, starting from an equilibrium extending over all trapped and untrapped electrons. We find that the normalized growth rate $g^{\pm}=(d/dt \ln a_s)\omega_r^{-1}$ is given by [13]

$$g^{\pm} = \mp a_w^2 \frac{\omega_r}{\omega_s} \frac{\pi^3\omega_p^2}{2\gamma_r\omega_r^2} \sum_n \frac{|Q_n^{\pm}(J)|^2}{|H_o(J_n)+\gamma_r|} \left(\frac{df_o}{dJ}\right)_{J_n} \left(\frac{d\omega_b}{dJ}\right)^{-1}_{J_n}, \qquad (2)$$

with ω_p the beam plasma frequency, ω_b the synchrotron frequency for trapped electrons, a_s and a_w the normalized radiation and wiggler amplitudes, ω_r and ω_s the radiation and sideband frequencies and Q_n the Fourier harmonics of the decomposition of the ponderomotive phase into synchrotron harmonics. The superscript + or - corresponds to upper $\omega_s>\omega_r$ and lower $\omega_s<\omega_r$ sideband respectively. J_n is given implicitly by the resonant condition

$$\pm n\omega_b(J_n) = (k_w/k_r)(\omega_s-\omega_r), \qquad (3)$$

as illustrated in Fig. 1(b). The sum over n on the right-hand side of Eq. (2) includes the contribution from all resonant groups of particles. The action J_n in Eq. (3) labels the orbit having the nth harmonic of the local bounce frequency in resonance with the sideband. The gain is determined by the slope of the distribution function f_o near these resonant orbits $J \simeq J_n$. Equation (2) has the following implication for the sideband growth.

(a) the spectrum becomes continuous replacing $\omega_b(0)$ by $\omega_b(J)$ in Eq. (1). The modes located at the peaks of the unstable spectrum grow faster, emerging as the discrete spectrum that is observed in simulations.

(b) More than one group of particles are in resonance with a given sideband frequency ω_s through different harmonics of their bounce freqeuncy and contribute to the growth rate.

(c) Upper and lower sidebands located symmetrically around the main signal frequency have opposite gains (complementary stability). Therefore one mode is always unstable. There is **no** stable distribution $f_o(J)$ except the trivial one $df_o/dJ=0$.

(d) The shear $d\omega_b/dJ$ is stabilizing. Distributions with gradients df_o/dJ localized near the separatrix are found to have the minimum growth rates because of the high shear there. This type of distribution is relevant to FEL's with tapered wigglers.

(e) The gain is proportional to $[df(J)/d\omega_b(J)]$, the relative population in oscillation quanta around resonance, in agreement with the quantum mechanical interpretation.

(f) For any **smooth** distribution, of finite df_o/dJ, electrons at the bottom of the well have a negligible effect on stability.

(g) Previous results, finding lower sidebands having an inherently larger gain than upper sidebands, are relevant only to the limiting case of a **singular** δ-function distribution $f_o(J)=\delta(J)$. This case is unrealistic because a wide, smooth initial distribution in action $f_o(J)$ corresponds to even an ideal cold beam distribution in momentum $f_o(p)=\delta(p-p_o)$.

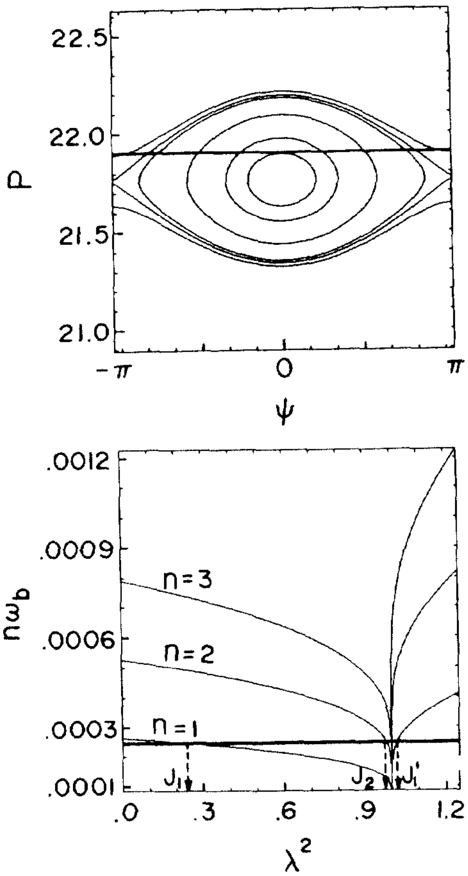
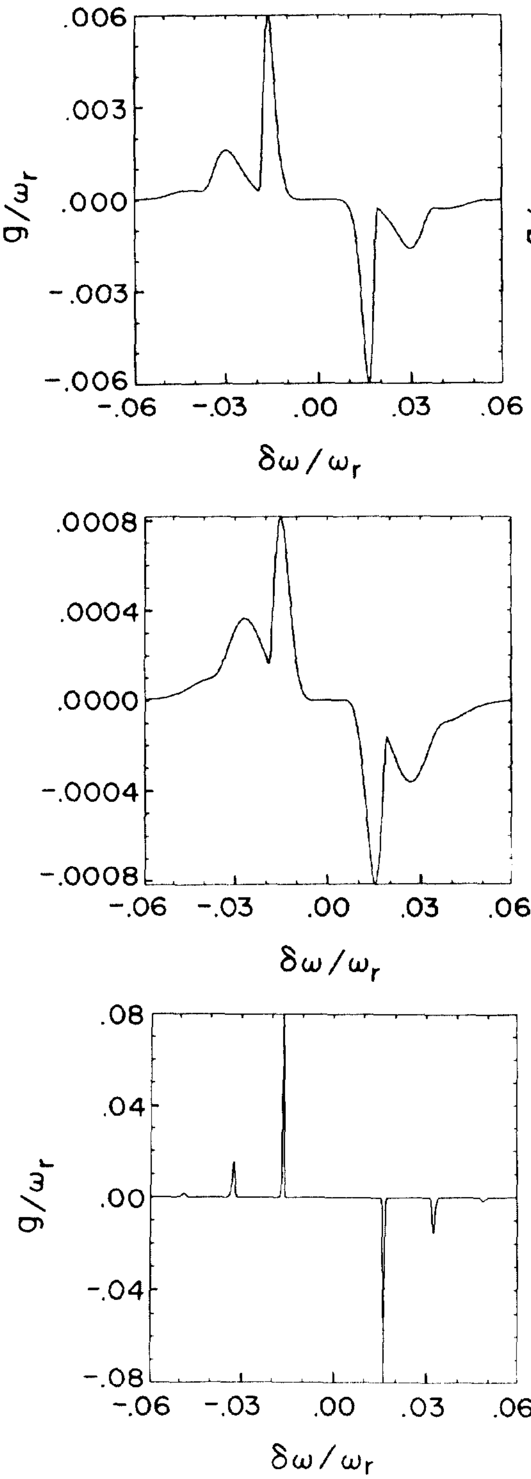


Figure 1. Time averaged motion without the sidebands. (a)Plots in phase space of the unperturbed orbits $H_o(P,\psi)=K$. The intersections with the horizontal line P=const. mark the initial conditions for each orbit. (b) The normalized bounce frequency ω_b and the first two harmonics as functions of the trapping parameter $\lambda^2(J)$. The intersections with the horizontal line $\delta=(\omega_s-\omega_r)k_w/k_r$ determine the position J_n of the resonant orbits for a given ω_s.



The normalized gain g_s/ω_r is plotted against the percentage mismatch $(\omega_s-\omega_r)/\omega_r$ for both upper and lower sidebands in Fig. 2. The contribution up to the third harmonic $n\leq4$ in Eq. (2) is included in these plots. The parameters chosen correspond to a wiggler wavelength $\lambda_w=3$ cm, $a_w=5$, main signal strength $a_s=5\times10^{-4}$ beam energy of 11.43 MeV ($\beta=0.999$, $\gamma=22.37$) and current density j=100 A/cm^2 (beam density 6.25×10^{10} cm^{-3}). We have chosen two types of equilibrium distributions $f_o(J)$: (i) Two Gaussians $f_o(J)=(1/2\pi D^2)^{1/2}\exp(-J^2/2D^2)$ centered at the centre of the island and of characteristic lengths D equal to half the island width $D=J_s/2$ in Fig. 2(a) and the island width $D=J_s$ in Fig. 2(b). (ii) Two "step-like" distributions of the form $f_o(J)=(1/\alpha D)^{1/2}\exp(-(J/\alpha D)^N)$ with N=16. Selecting $\alpha=(N/N-1)^{1/N}$ places the sharp gradient at J=D and we plot the case $D=J_s/2$ in Fig. 2(c) and $D=J_s$ in Fig. 2(d).

The limit of a δ-function distribution $f_o=\delta(J-J_o)$, examined elsewhere [13], yields the fastest growth but is of small practical interest, because even the case of a monoenergetic beam distribution $p_z=p_o$ is described in J-space by a smooth distribution $f_o(J)$ of finite width ΔJ (see Fig. 1).

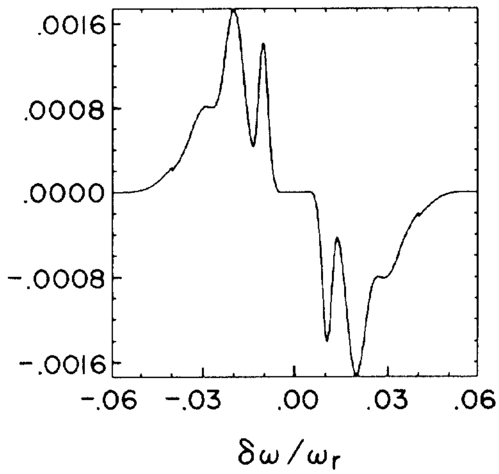


Figure 2. Normalized gain for monotonic distributions centered at the bottom of the well J=0 including the first three harmonics $n\leq 4$ in Eq. (2). (a) Gaussian distribution of width D equal to half the island width w, $D=J_s/2$ (b) Gaussian distribution with $D=J_s$. (c) Step-like distribution with $D=J_s/2$ and (d) Step-like distribution with $D=J_s$.

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