RADIATION FOCUSING, GUIDING AND STEERING IN FREE ELECTRON LASERS*

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Abstract

In a free electron laser (FEL), the radiation field, wiggler field and electron beam resonantly couple and modify the refractive index in the vicinity of the electron beam. The refractive index is modified such that the radiation beam will tend to focus upon the electron beam. A method for solving the 3-D wave equation for the FEL process is outlined. This approach, called the source dependent expansion method, provides an excellent analytical and numerical technique for studying optical focusing, guiding and steering in FELs. A radiation envelope equation is derived. Conditions and parameters necessary to achieve guided radiation beams (constant radius) in the exponential gain regime are obtained for FELs driven by either induction linacs or rf linacs. Immediately prior to saturation in the exponential gain region, the ponderomotive potential is large enough to trap the beam electrons. The wiggler field, at this point, could be tapered to further increase the operating efficiency. The possibility of bending or steering radiation beams in FELs is discussed and a condition necessary for radiation guiding along a curved electron beam orbit is obtained.

Introduction

In many short wavelength free electron laser devices the radiation beam will not be confined or guided by a structure such as a waveguide. Furthermore, in order to provide high gain and efficiency, it is usually necessary for the interaction length (length of wiggler field) to be long compared to the diffraction length (Rayleigh length) associated with the radiation beam. In the FEL the tendency of the radiation beam to diffract away over a distance of a few Rayleigh lengths can be overcome by a focusing phenomenon arising from the resonant coupling of the radiation and wiggler fields with the electron beam [1,2]. This radiation focusing effect plays a central role in the practical utilization of the FEL. This phenomenon was first analyzed for the low gain FEL with transverse effects where it was shown that the diffractive spreading of the radiation beam could be overcome by a focusing effect arising from the modified index of refraction [1]. Optical guiding in FELs operating in the small signal exponential gain regime has been studied for the asymptotic behavior of the radiation beam [3-6]. Recently, a general formalism for optical focusing, guiding and steering has been developed and applied to FELs [7].

In the following, we employ a modal expansion technique to examine the optical beam as it propagates through the wiggler. The formalism has the merit that with only a few modes it permits an accurate solution of the wave equation throughout the interaction region.

Model

In our model, the vector potential of an axially symmetric, linearly polarized, radiation field is

$$\underset{\sim}{A}_R(r,z,t) = A(r,z)\, e^{i(\omega z/c-\omega t)}\hat{e}_x/2 + \text{c.c.}, \qquad (1)$$

where $A(r,z)$ is the complex radiation field amplitude, ω is the frequency and c.c. denotes the complex conjugate.

The wave equation governing $\underset{\sim}{A}_R$ is

$$\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \partial^2/\partial z^2 - c^{-2}\partial^2/\partial t^2\right)\underset{\sim}{A}_R = -\frac{4\pi}{c}J_x\hat{e}_x, \quad (2)$$

where $J_x(r,z,t)$ is the driving current density. Substituting (1) into (2) leads to the following reduced wave equation,

$$\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + 2i\frac{\omega}{c}\frac{\partial}{\partial z}\right)a(r,z) = S(r,z,a), \qquad (3)$$

where $a(r,z) = |e|A/m_oc^2 = |a|\exp(i\phi)$ is the normalized complex radiation amplitude and we have assumed that $a(r,z)$ is a slowly varying function of z, i.e., $|(\partial a/\partial z)/a| << \omega/c$. The source function, S, is given by,

$$S = -\frac{4\omega}{c}\int_0^{2\pi/\omega} J_x(r,z,t)e^{-i(\omega z/c-\omega t)}dt. \qquad (4)$$

It is possible to relate the source function, S, to the index of refraction associated with the medium by noting that the wave equation for $\underset{\sim}{A}_R$ in a nonmagnetic, time-independent, nonlinear medium is $(\nabla^2 - (n^2(r,z,a)/c^2)\partial^2/\partial t^2)\underset{\sim}{A}_R = 0$, where n is the index of refraction associated with the medium and is, in general, complex and a nonlinear function of $a(r,z)$. Comparing the reduced wave equation written in terms of $n(r,z,a)$ with (3) we find that the source function can be written in terms of n,

$$S(r,z,a) = (\omega/c)^2\left(1-n^2(r,z,a)\right)a(r,z). \qquad (5)$$

Source Dependent Expansion Method

In order to solve (3) we will use the source dependent expansion (SDE) method [7]. In this method, we choose the following representation for $a(r,z)$ in terms of Laguerre-Gaussian functions,

$$a(r,z) = \sum_m a_m(z)L_m\left(\frac{2r^2}{r_s^2(z)}\right)e^{-(1-i\alpha(z))r^2/r_s^2(z)}, \qquad (6)$$

where $m = 0,1,2,\cdots$. In Eq. (6), $a_m(z)$ are the complex amplitude coefficients, $r_s(z)$ is the radiation spot size, $\alpha(z)$ is related to the radius of curvature of the radiation beam wavefront, $R = -(\omega/2c)r_s^2/\alpha$ and L_m is the Laguerre polynomial. Solving for the unknown quantities a_m, r_s and α in terms of the source term S allows us to completely describe the radiation dynamics. The representation in (6) is underspecified, since, when (6) is substituted into (3) and moments of the source function taken, there remain more unknown quantities than available equations. The additional degrees of freedom in our representation allow us to specify a particular functional relationship for the unknown quantities r_s and α in such a way that, if the radiation beam profile remains approximately Gaussian, the number of modes needed to accurately describe the radiation beam is small. This yields the following first order coupled differential equations for r_s and α,

$$r_s' - 2c\alpha/\omega r_s = -r_s H_I, \qquad (7a)$$

$$\alpha' - 2(1+\alpha^2)c/\omega r_s^2 = 2(H_R - \alpha H_I), \qquad (7b)$$

and a set of first order ordinary differential equations for the complex amplitudes $a_m(z)$,

$$a_m' + A_m a_m = -i\left[F_m - mBa_{m-1} - (m+1)B^* a_{m+1}\right], \qquad (7c)$$

where $H = F_1/a_0$, $' \equiv \partial/\partial z$, and $(\ )_{R,I}$ denotes the real and imaginary part of the enclosed function. In Eqs. (7), the functions A_m, B, and F_m are given by

$$A_m(z) = r_s'/r_s + i(2m+1)\left((1+\alpha^2)c/\omega_s^2 - \alpha r_s'/r_s + \alpha'/2\right),$$

$$B(z) = -\left(\alpha r_s'/r_s + (1-\alpha^2)c/\omega r_s^2 - \alpha'/2\right) - i\left(r_s'/r_s - 2\alpha c/\omega r_s^2\right),$$

$$F_m(z) = \frac{c}{2\omega}\int_0^\infty d\zeta\, S(\zeta,z)L_m(\zeta)\exp(-(1+i\alpha)\zeta/2),$$

where $\zeta = 2r^2/r_s^2$.

The merits of the SDE method can be demonstrated in a comparison between; a) the exact numerical solution of the wave equation in (3), (using 64x64 Fourier modes), b) the solution using a vacuum Laguerre-Gaussian modal expansion (10 modes) and c) the solution from the Laguerre-Gaussian SDE approach (10 modes). Figure 1 shows the radiation beam amplitude on-axis obtained from methods (a), (b) and (c) after four Rayleigh lengths for the FEL parameter in Table I. The SDE solution (c) is in excellent agreement with solution (a) while solution (b), beyond a Rayleigh length, grossly deviates from (a) and (c).

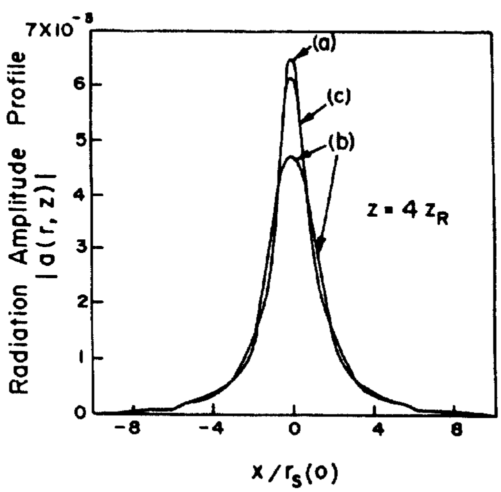

Fig. 1 Radiation amplitude profile, $|a(r,z)|$ for: a) exact numerical solution (64x64) Fourier modes), b) vacuum modal expansion solution (10 modes), and c) SDE solution (10 modes) at a distance of $z = 4Z_R$.

Refractive Index Associated with FELs

In the following derivation of the refractive index associated with the FEL, a number of simplifying assumptions are made. We assume, for example, that the beam electrons are monoenergetic without betatron oscillations and that the radiation is of a single frequency [8]. To obtain an expression for the refractive index we write the nonlinear driving current density, J_x, as

$$J_x = -|e|n_b(r)v_w(z)v_{oz}\int\delta(z-\tilde{z}(t,t_o))dt_o, \qquad (8)$$

where $n_b(r)$ is the ambient beam density, v_{oz} is the axial electron velocity at $z = 0$, t_o is the time a given electron crosses the $z = 0$ plane,

$$v_w(z) = |e|A_w/\gamma m_o c(\exp(ik_w z) + c.c.)\,\hat{e}_x/2,$$

is the wiggle velocity, γ is the Lorentz factor, A_w is the vector potential amplitude of the planar wiggler field and $k_w = 2\pi/\lambda_w$ is the wiggler wave number. Substituting (8) into the expression for S, (4), gives

$$S = \left(\frac{\omega_b(r)}{c}\right)^2 a_w\int_0^{2\pi/\omega} dt\omega/2\pi\int dt_o e^{-i\left(\left(\frac{\omega}{c}+k_w\right)z-\omega t\right)}\delta(t-\tau(z_o,t))/\gamma, \qquad (9)$$

where $a_w = |e|A_w/m_oc^2$, $\tau = t_o + \int_0^z dz'/v_z(z',t_o)$ and the t_o integration is over all entry times. Equating (9) with (5) and carrying out the integration over t, we find the index of refraction associated with the FEL to be given by

$$n_{fel}(r,z,a) = 1 + (\omega_b^2(r)/2\omega^2)\,\frac{a_w}{|a|}\left\langle\frac{e^{-i\psi}}{\gamma}\right\rangle_{\psi_o}, \qquad (10)$$

where

$$\psi = \int_0^z\left(\omega/c + k_w - i\ln(a/|a|) - \omega/v_z(z,\psi_o)\right)dz + \psi_o,$$

is the relative phase between the electron and the ponderomotive wave, $\psi_o = -\omega t_o$ is the initial phase of a given electron and $\langle\ \rangle_{\psi_o} = (2\pi)^{-1}\int_0^{2\pi} d\psi_o$ is an ensemble average over the initial phases. The radial profile of the index of refraction as given by Eq. (10) supports self-focusing of the radiation in an FEL. It should be noted, for completeness, that the relative phase satisfies the pendulum equation given by

$$\partial^2\psi/\partial z^2 = \partial k_w/\partial z - \gamma^{-2}(\omega/c)\left[4\partial a_w^2/\partial z - k_w a_w a\sin\psi\right]. \qquad (11)$$

Radiation Beam Envelope Equation

Equations (7a) and (7b) can be combined to give the following envelope equation for the radiation beam

$$r_s'' + K^2 r_s = 0, \qquad (12)$$

where

$$K^2 = (2c/\omega)^2\left(-1 + C^2\langle\sin\psi\rangle^2 + 2C\langle\cos\psi\rangle + (\omega/2c)r_s^2C'\langle\sin\psi\rangle\right)r_s^{-4}, \qquad (13)$$

and $C(z) = (2\nu/\gamma)G(z)a_w/|a_o(z)|$, measures the coupling between the radiation and electron beam, $\nu = (\omega_{bo}\ r_b/2c)^2 = I_b/17\text{x}10^3$ is Budker's constant, I_b is the electron beam current in amperes, $G(z) = (1-f)/(1+f)^2$ and $f(z) = (r_b/r_s)^2$ is the filling factor associated with a Gaussian electron beam density profile. The first term on the right-hand side of (13) is the usual diffraction term, the second and third terms are focusing while the last term provides a focusing or defocusing contribution. In the high gain trapped particle regime, $\langle\sin\psi\rangle$ and $\langle\cos\psi\rangle$ are approximately constant, while $|a_o(z)|$ increases with z. Hence, K depends on z and a guided beam ($r_s' = 0$) cannot be exactly maintained in this regime, although, the radiation envelope is still reasonably well-confined. In the low gain trapped particle regime $|a_o(z)|$ increases slightly and, therefore, a guided beam can be approximately achieved. In either the Compton or Raman exponential gain regime, conditions for a stable guided beam can be found.

Guided Radiation Beams in the Exponential Gain Regime

In this section, we obtain the necessary conditions to achieve guided radiation beams in both the Compton (noncollective) and Raman (collective) exponential gain regimes. By considering the lowest order mode (Gaussian profile) we find that the source term appropriate for the high gain Compton and Raman regime is, respectively,

$$S(r,z)=\frac{(\omega_b(r)/c)^2(a_w k_w f_B)^2}{\gamma(1+a_w^2/2)}\,a(r,z)\begin{cases}\dfrac{1}{(\Delta k-i\Gamma)^2}\\[2ex] \dfrac{\gamma^{1/2}\gamma_z c}{2\omega_b(r)(\Delta k-i\Gamma)}\end{cases}, \tag{14a,b}$$

where Δk and Γ are the wave number shift and growth rate respectively and f_B is the usual difference of Bessel functions due to the linear wiggler. The lowest order mode is taken to have the form

$$a(r,z) = a_o(0)\exp\Big(i\int_o^z(\Delta k-i\Gamma)dz'-(1-i\alpha)r^2/r_s^2\Big). \tag{15}$$

For the purposes of illustration, we will consider the Compton FEL regime in which the electron beam has a Gaussian density profile, $n_b(r) = n_o\exp(-r^2/r_b^2)$. The conditions for a guided radiation beam require that the waist and curvature of the radiation beam remain constant, ($r_s' = \alpha' = 0$). Setting $r_s' = \alpha'=0$ in Eqs. (7a,b) and solving for Γ, Δk, r_s, and α, the following results for a guided beam are obtained.

$$\Gamma = (1+\alpha^2)^{-1}(1+2f)^{-1}\Gamma_o, \qquad \Delta k=\alpha\Gamma, \tag{16a,b}$$

$$r_s = \left(\frac{\gamma}{\nu}\right)^{1/4}\frac{\lambda_w}{2^{7/4}\pi\gamma f_B^{1/2}}\,\frac{(1+a_w^2/2)^{3/4}}{a_w^{1/2}}\,\frac{f^{1/4}(1+2f)^{3/2}}{(1+3f/2)^{3/4}}, \tag{16c}$$

$$r_s(f=1) = 0.25\,\lambda_w\left(\frac{\gamma}{\nu}\right)^{1/4}\frac{(1+a_w^2/2)^{3/2}}{\gamma f_B^{1/2}a_w^{1/2}}, \tag{16d}$$

$$\alpha = (f/(2+3f))^{1/2}, \tag{16e}$$

where $\Gamma_o=2f_B(\nu/\gamma)^{1/2}a_w k_w(1+a_w^2/2)^{-1/2}$ and $f=r_b^2/r_s^2$ is the filling factor.

Figure 2 shows the spatial evolution of the radiation waist for the induction linac driven FEL parameters in Table I. The parameters in Table I are consistent with Eqs. (16) and have been chosen to produce a guided radiation beam in the Compton exponential gain regime. The guided beam conditions can be shown to be stable [9], this is shown numerically by changing the spot size of the injected radiation beam. Figure 3 shows that irrespective of the initial value, the spot size asymptotes to the matched (guided) beam value. Figure 4 shows the evolution of the spot size for the rf linac-driven FEL parameters in Table II. As in Table I, the parameters in Table II have been chosen to produce a guided radiation beam in the Compton exponential gain regime and are consistent with Eqs. (16).

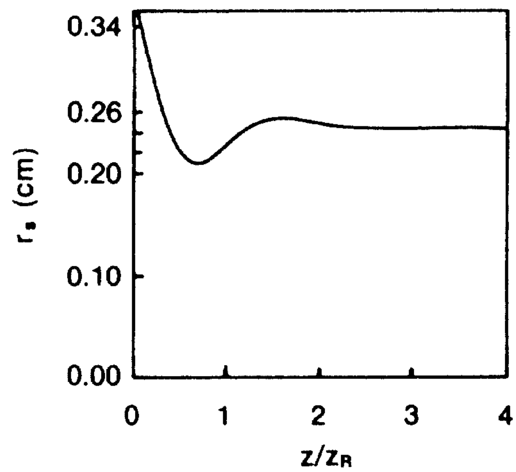

Fig. 2 Spatial evolution of the radiation spot size in the exponential gain regime for induction linac driven FEL parameters given in Table I.

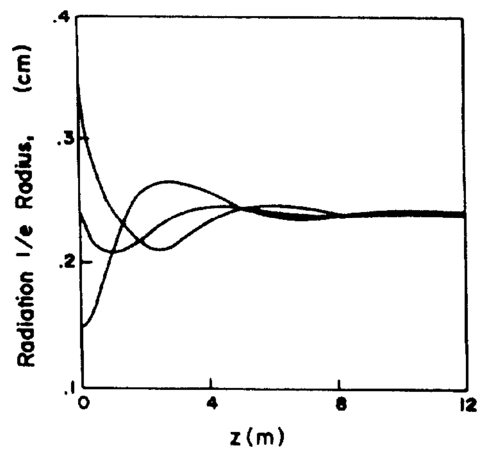

Fig. 3 Spatial evolution of the radiation spot size in the exponential gain regime for initial spot sizes; a) 0.35 cm, b) 0.24 cm, and c) 0.15 cm.

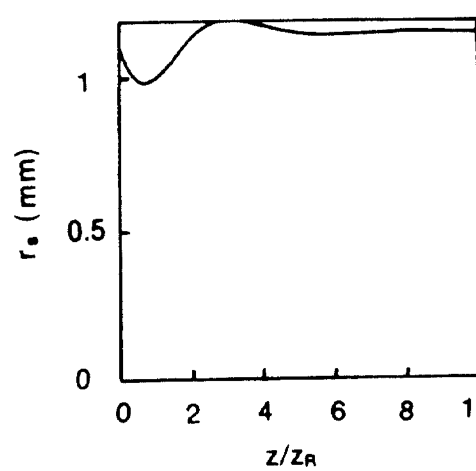

Fig. 4 Spatial evolution of the radiation spot size in the exponential gain regime for rf linac driven FEL parameters given in Table II.

Free Electron Lasers driven by either induction or rf linacs could initially operate in the guided, exponential gain regime until saturation occurs. Immediately prior to saturation, the ponderomotive potential can be large enough, as in the above illustrations, to trap a significant fraction of the beam electrons. At this point, the wiggler field can be spatially tapered to achieve a significant increase in the operating efficiency and a somewhat smaller input signal into the FEL amplifier.

To determine the viability of tapering the wiggler, prior to saturation, the trapping potential associated with the ponderomotive wave is needed. For linearly polarized waves, the fractional trapping potential is

$$\frac{|e|\phi_{trap}}{\gamma m_o c^2} = 2\sqrt{2}\left(\frac{aa_w}{1+a_w^2/2}\right)^{1/2}. \tag{17}$$

The radiation amplitude at saturation can be obtained from the intrinsic efficiency of the FEL. Using arguments based on electron trapping in the ponderomotive wave, we find that the intrinsic efficiency in the exponential (maximum) gain regime is

$$\eta = \Delta k/k_w. \tag{18}$$

Using the induction linac parameters in Table I as an illustration, we find that the intrinsic efficiency is $\eta = \Delta k/k_w = 0.66\%$. From this, the fractional trapping potential at the end of the exponential gain regime is $|e|\phi_{trap}/\gamma m_o c^2 = 6\%$, making it possible to trap the electron beam while tapering the wiggler field. In addition, the initial fractional energy spread of the electron beam must be somewhat less than η. This places a limitation on the fractional energy spread of the electron beam, $\delta E/E_b < 0.66\%$. One contribution to the beam energy spread is the transverse emittance, $\delta E/E_b = (1/2)(\varepsilon_n/r_b)^2$. Therefore, the normalized beam emittance must satisfy, $\varepsilon_n < (2\Delta k/k_w)^{1/2} r_b = 0.034$ cm-rad.

Bending and Guiding of Radiation Beams

Using the SDE formalism, it is possible to discuss the bending of a radiation beam by a curved electron beam in an FEL. For small displacements of the electron beam centroid, a nonaxisymmetric modal expansion similar to (6) can be performed and the spatial evolution of the centroid of the radiation beam found. Figure 5 shows the centroids of the electron and radiation beams for an FEL in the trapped particle regime with parameters given in Table I. Steering of the radiation beam by the electron beam is clearly demonstrated in this figure.

It is interesting to consider the conditions under which the radiation beam could be guided by a curved electron beam, as shown in Fig. 6. Such a situation could make possible a cyclic FEL driven by, for example, a betatron generated electron beam. In a cyclic FEL, the radiation beam would be guided by a circular electron beam. The wiggler field, which is along the circular orbit of the electron beam, cannot be spatially contoured. Therefore, in the trapped particle regime, enhancement of the FEL efficiency must be achieved by inducing an accelerating electric field along the beam orbit. For cyclic electron beams, the induced electric field can be generated by increasing the magnetic flux within the orbit of the electron beam.

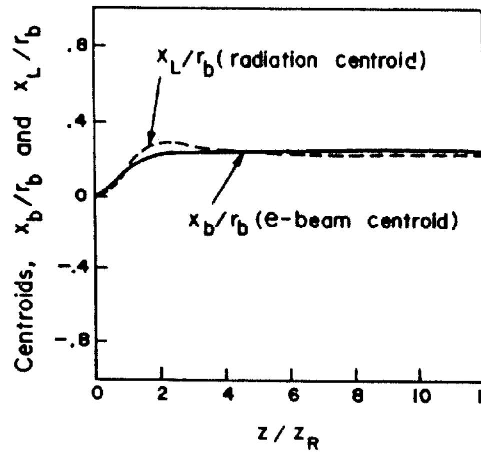

Fig. 5 Electron and radiation beam centroids, x_b and x_L for a displaced electron beam, $x_b = x_c(1-\mathrm{sech}(k_c z))$ with $x_c = r_b/4$ and $\lambda_c = 2\pi/k_c = 4Z_R$.

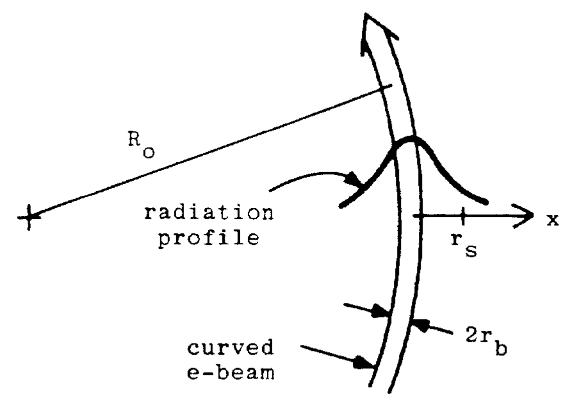

Fig. 6 Configuration showing guiding of radiation beam by a curved electron beam with radius of curvature, R_o.

To examine the conditions under which guiding can be achieved in the exponential gain regime, we denote the radial position by $r = R_o + x$, where R_o is the radius of curvature of the electron beam and x is the radial displacement from the center of the curved electron beam (see Fig. 6). The FEL refractive index (correct to order x/R_o) is

$$n = n_{fel} + x/R_o, \tag{19}$$

where n_{fel} is given by (10). In the exponential gain regime, a guided radiation beam in a curved FEL is possible if $R_o \gtrsim R_{min}$ where

$$R_{min} = r_s/|\mathrm{Re}(1-n_{fel})|. \tag{20}$$

Substituting the expressions for Γ, Δk and r_s, from Eqs. (16), into (20) yields

$$R_{min} = \frac{4(1+f)f\gamma^2 r_b}{(1+2f)(3f+2)^{1/2} f_B a_w (1+a_w^2/2)^{1/2}(\nu/\gamma)^{1/2}}, \tag{21a}$$

$$R_{min}(f=1) = \frac{1.2\ \gamma^2 r_b}{f_B a_w (1+a_w^2/2)^{1/2}(\nu/\gamma)^{1/2}}. \tag{21b}$$

For a numerical example of R_{min}, consider the following parameters, $\gamma = 100$, $I = 2$ kA, $r_b = 0.3$ cm, $a_w = 1.72$, $f = 1$ and $f_B = 0.85$ (Table I). For these parameters, the minimum turning radius required for a guided radiation beam is $R_{min} = 455$ m.

Table I

Parameters Associated with an Induction Linac Driven FEL in the Exponential Gain Regime

Electron Beam	
Current	I_b = 2kA, (ν = 0.118)
Energy	E_b = 50 MeV, (γ = 100)
Radius	r_b = 0.3 cm
Emittance	$\varepsilon_n < 34\times10^{-3}$ cm-rad
Wiggler Field	
Wavelength	λ_w = 8 cm
Wiggler Strength	B_w = 2.3 kG (a_w = 1.72)
Radiation Beam	
Wavelength	λ = 10.6 μm
Spot Size (guided beam)	r_s = 0.25 cm, (Z_R = 2 m)
e-folding length	$L_e = 1/\Gamma$ = 94 cm
Intrinsic Efficiency	$\eta = \Delta k/k_w = 0.66\%$
Saturated Power	P_{sat} = 660 MW ($a = 7\times10^{-4}$)
Trapping Potential	$\lvert e\rvert\phi_{trap}/\gamma m_o c^2 = 6.0\%$

Table II

Parameters Associated with an RF Linac Driven FEL in the Exponential Gain Regime

Electron Beam	
Peak Current	I_b = 500 A
Energy	E_b = 150 MeV
Radius	r_b = 1 mm
Emittance	$\varepsilon_n \lesssim 7\times10^{-3}$ cm-rad
Wiggler Field (planar)	
Wavelength	λ_w = 12 cm
Wiggler Strength	B_w = 900 G (a_w = 1)
Radiation Beam	
Wavelength	λ = 1 μm
Spot Size (guided beam)	$r_s(0)$ = 1.1 mm (Z_R = 3.8 m)
e-folding length	$L_e = 1/\Gamma$ = 196 cm
Intrinsic Efficiency	$\eta = \Delta k/k_w = 0.25\%$
Saturated Power	P_{sat} = 180 MW ($a = 7.25\times10^{-5}$)
Trapping Potential	$\lvert e\rvert\phi_{trap}/\gamma m_o c^2 = 2\%$

Conclusion

The source dependent expansion (SDE) method provides an excellent analytical and numerical technique for studying optical focusing, guiding and steering in FELs. We find that guided radiation beams in the FEL can be achieved both in the Compton and Raman exponential gain regimes but cannot be maintained in the high gain trapped particle (tapered wiggler) regime.

Free electron lasers driven by either induction linacs, such as the ATA, or high current rf linacs can operate in the guided, exponential gain regime until saturation occurs. At this point, the wiggler field could be spatially tapered so as to operate the FEL in the trapped particle regime in order to further increase the operating efficiency.

We also examined the possibility of bending or steering radiation beams in FELs. We find a condition which places a lower limit on the radius of curvation of the electron beam necessary for the radiation to be guided along a curved path.

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